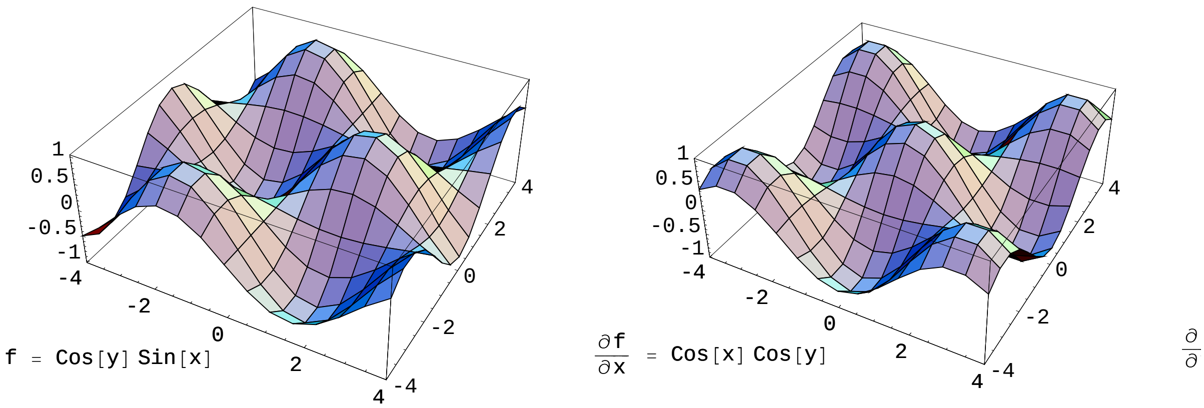
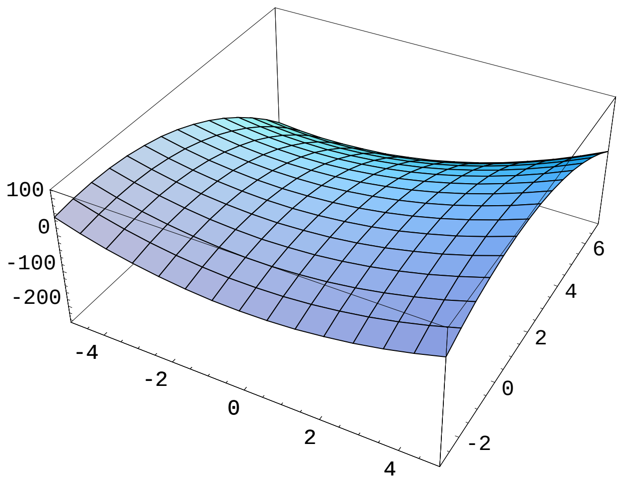
```
(**********************************************)
(************* parciális deriválás ***********)
(**********************************************)

f = x^2 + y^2;
f = Sin[x] Cos[y];
g = D[f, x];
h = D[f, y];
Show[GraphicsArray[
  {Plot3D[f, {x, -4, 4}, {y, -4, 4}, AxesLabel → {StringForm["f = ``", f], None, None},
    DisplayFunction → Identity], Plot3D[g, {x, -4, 4}, {y, -4, 4},
    AxesLabel → {StringForm["∂f/∂x = ``", g], None, None},
    DisplayFunction → Identity], Plot3D[h, {x, -4, 4}, {y, -4, 4},
    AxesLabel → {StringForm["∂f/∂y = ``", h], None, None},
    DisplayFunction → Identity]}], DisplayFunction → $DisplayFunction]
```



- GraphicsArray -

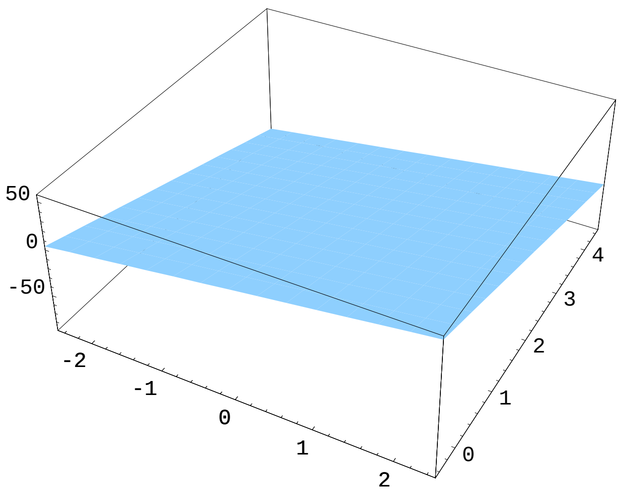
```
In[1]:= (**********************************************)
        (*************** Érintősík *****************)
        (**********************************************)

        ClearAll[f]; ClearAll[x]; ClearAll[y]; ClearAll[x0];
        ClearAll[y0]; ClearAll[Dx]; ClearAll[Dy]; ClearAll[Dα];
        ClearAll[nv]; ClearAll[dx];

        f[x_, y_] = 3 x^2 + 2 x y - 4 y^2 + 6 x - 2 y + 4;
        x0 = 0;
        y0 = 2;
        Dx = D[f[x, y], x] /. {x → x0, y → y0}
        Dy = D[f[x, y], y] /. {x → x0, y → y0}
        nv = -2 {Dx, Dy, -1}

Out[6]= 10

Out[7]= -18

Out[8]= {-20, 36, 2}
```

```
In[9]:= ErintoSik[x0_, y0_] = {z == f[x0, y0] + Dx (x - x0) + Dy (y - y0)};
        (*az (x0,y0) pontbeli érintősík egyenlete*)

        dx = 5;
        Plot3D[f[x, y], {x, x0 - dx, x0 + dx}, {y, y0 - dx, y0 + dx}]
```




```
Out[10]= - SurfaceGraphics -
```

```
In[11]:= Plot3D[ErintoSik[x0, y0][[1, 2]],
          {x, x0 - dx / 2, x0 + dx / 2}, {y, y0 - dx / 2, y0 + dx / 2}, Mesh → False]
         (*Adott x0 és y0 mellett ezt rakjuk majd a Plot3D-be,
          és akkor ábrázolja az érintősíkot.*)
```




```
Out[11]= - SurfaceGraphics -
```

```
(********************************************)
(********** Parc. deriv, érintősík ***********)
(********************************************)
(* felület implicit képlettel *)
G = {16 x^2 - 9 y^2 + 36 z^2 == 144};
x0 = 3;
y0 = -4;

f[x_, y_] = z /. Solve[G, z][[2]]
Dx = ∂x f[x, y]
Dy = ∂y f[x, y]
Dx = ∂x f[x, y] /. {x → x0, y → y0};
Dy = ∂y f[x, y] /. {x → x0, y → y0};
nv = {Dx, Dy, -1} (* normálvektor *)
ErintoSik[x_, y_] = f[x0, y0] + Dx (x - x0) + Dy (y - y0) (* érintősík egyenlete *)

dx = 10; (* rajzolási tartományhoz *)
dd = 3; (* normálvektor hossza *)

Show[Plot3D[f[x, y], {x, x0 - dx, x0 + dx},
   {y, y0 - dx, y0 + dx}, DisplayFunction → Identity, PlotPoints → 50],
 Plot3D[ErintoSik[x, y], {x, x0 - dx / 2, x0 + dx / 2}, {y, y0 - dx / 2, y0 + dx / 2},
  DisplayFunction → Identity, Mesh → False], Graphics3D[{Thickness[0.01], Hue[0],
   Line[{{x0, y0, f[x0, y0]}, {x0 - dd nv[[1]], y0 - dd nv[[2]], f[x0, y0] - dd nv[[3]]}}]}],
 DisplayFunction → $DisplayFunction]
```

$$\frac{1}{6}\sqrt{144-16x^2+9y^2}$$

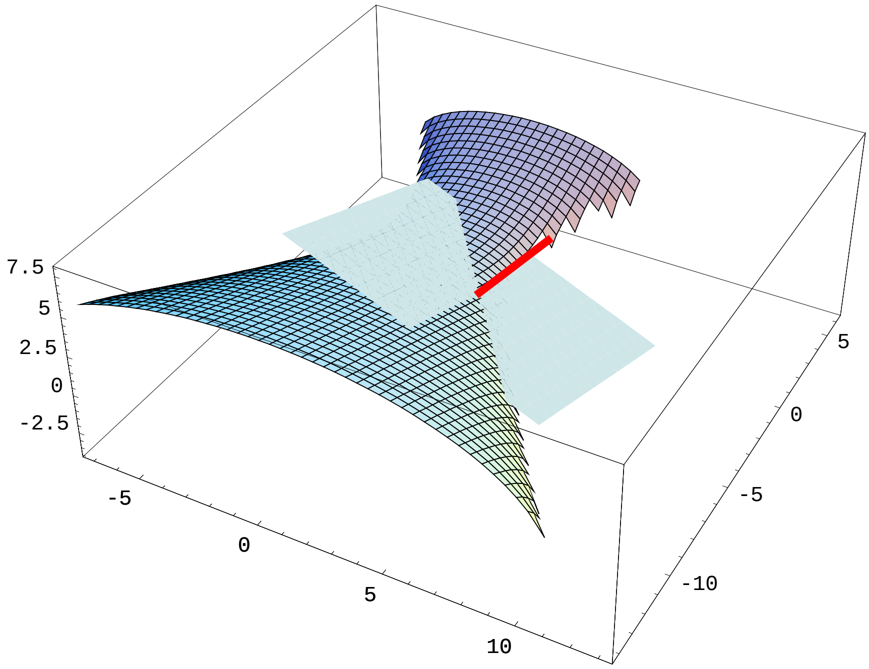
$$-\frac{8x}{3\sqrt{144-16x^2+9y^2}}$$

$$\frac{3y}{2\sqrt{144-16x^2+9y^2}}$$

$$\left\{-\frac{2}{3},\ -\frac{1}{2},\ -1\right\}$$

$$2-\frac{2}{3}(-3+x)+\frac{1}{2}(-4-y)$$

```
Graphics3D::nlist3 : {10.9592, -14., 0. + 0.615975 i} is not a list of three numbers.

Graphics3D::nlist3 : {10.9592, -13.5918, 0. + 1.78743 i} is not a list of three numbers.

Graphics3D::nlist3 : {11.3673, -13.5918, 0. + 2.69167 i} is not a list of three numbers.

General::stop : Further output of Graphics3D::nlist3 will be suppressed during this calculation.
```



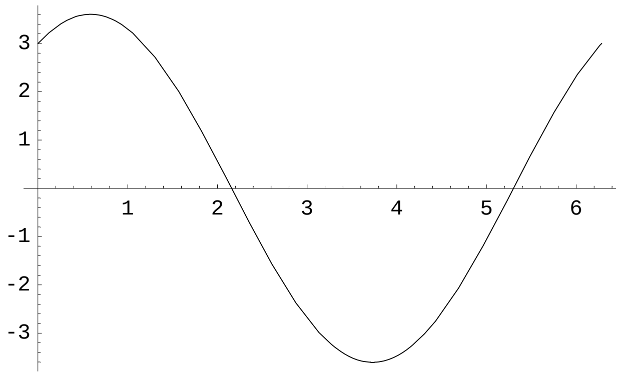
- Graphics3D -

```
(*******************************************)
(********** Iránymenti derivált *************)
(*******************************************)
ClearAll[f]; ClearAll[x]; ClearAll[y]; ClearAll[x0]; ClearAll[y0];
ClearAll[Dx]; ClearAll[Dy]; ClearAll[Dα]; ClearAll[dx];
f[x_, y_] = x^3 y^2;
x0 = 1;
y0 = 1;
Dx = ∂x f[x, y]
Dy = ∂y f[x, y]
Dα = Dx Cos[α] + Dy Sin[α]

dx = 2;
FuggolegesSik =
 Graphics3D[Polygon[{{#1[[1]], #2 /. x → #1[[1]], #3[[1]]}, {#1[[1]], #2 /. x → #1[[1]], #3[[2]]},
     {#1[[2]], #2 /. x → #1[[2]], #3[[2]]}, {#1[[2]], #2 /. x → #1[[2]], #3[[1]]}}]] &;
(* ez rajzol egy téglalapot egy megadott egyenes
  fölé. Használat: FuggolegesSik[{x1,x2},3 x+6,{z1,z2}] *)

Plot[Dα /. {x → x0, y → y0}, {α, 0, 2 π}]
p1 = Plot3D[f[x, y], {x, x0 - dx, x0 + dx}, {y, y0 - dx, y0 + dx}, PlotPoints → 60]
b = Plot3D[1, {x, x0 - dx, x0 + dx},
  {y, y0 - dx, y0 + dx}, PlotPoints → 60, Boxed → False, Axes → False]
Show[p1, b, Graphics3D[{PointSize[0.02], Hue[0], Point[{x0, y0, f[x0, y0]}]}]];

ps1 = FuggolegesSik[{x0 - dx, x0 + dx},
   y /. Solve[y - y0 == (Dy / Dx /. {x → x0, y → y0} ) (x - x0), y][[1]], {-1, 5}];
(*(Dx,Dy) irányvektorú egyenes fölé rajzolt sík. Mint tudjuk,
 a gradiens irányában a legnagyobb az iránymenti derivált.*)
Show[p1, ps1, b, Graphics3D[{PointSize[0.02], Hue[0], Point[{x0, y0, f[x0, y0]}]}]];
ps2 = FuggolegesSik[{x0 - dx, x0 + dx},
   y /. Solve[y - y0 == (-Dx / Dy /. {x → x0, y → y0} ) (x - x0), y][[1]], {-1, 2}];
(*(1,-Dx/Dy) irányvektorú egyenes fölé rajzolt
  sík. Lásd: implicit függvény tétel.*)
Show[p1, ps2, b, Graphics3D[{PointSize[0.02], Hue[0], Point[{x0, y0, f[x0, y0]}]}]];

(* Házi feladat: Legyen f (x,y)=3 x^2+2 x y-4 y^2+6 x-2 y+4,
 a ZH-ban szereplő függvény. Írd fel az (x0,y0)=(2,-1) pontban a
   legnagyobb iránymenti deriváltat! Rajzoltasd ki az ebbe az irányba álló,
 (2,-1,f (2,-1)) pontra illeszkedő függőleges síkot a függvény
  által meghatározott felülettel együtt egy közös ábrában. *)
```

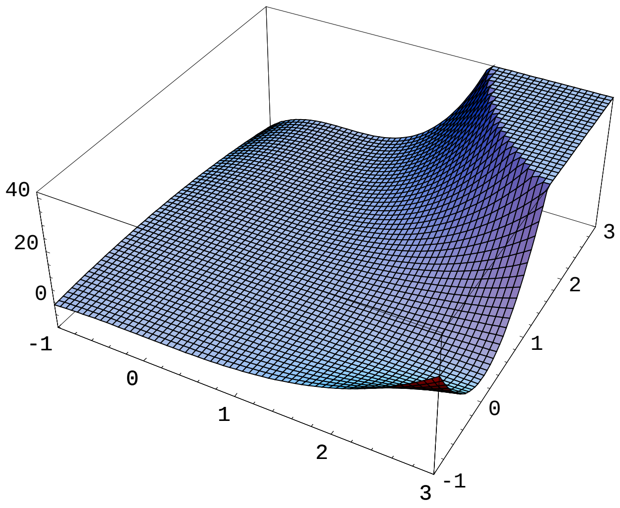
$3 x^2 y^2$

$2 x^3 y$

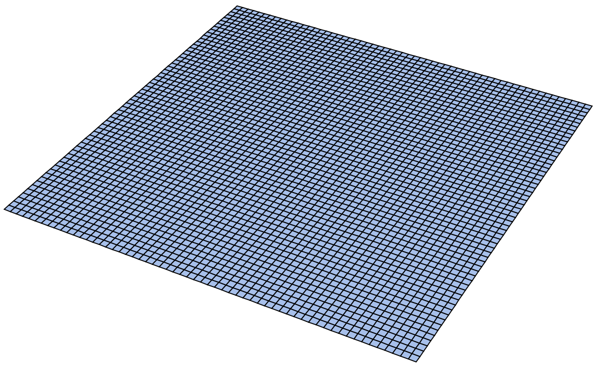
$3 x^2 y^2 \mathrm{Cos}[\alpha] + 2 x^3 y \mathrm{Sin}[\alpha]$



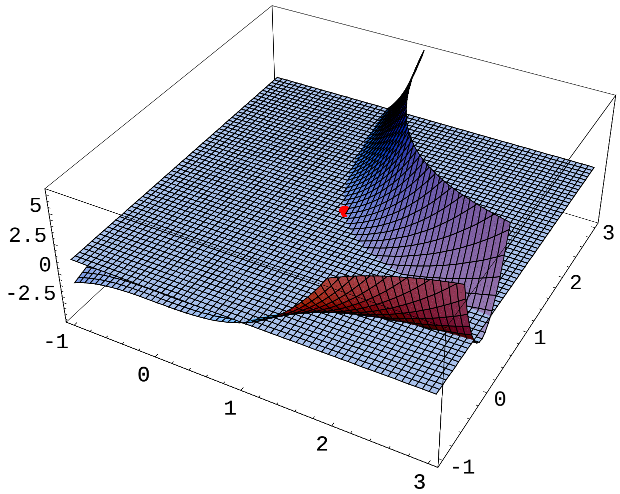
- Graphics -

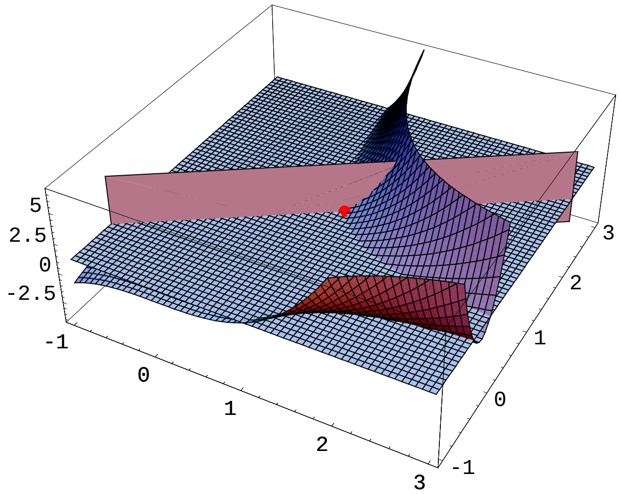


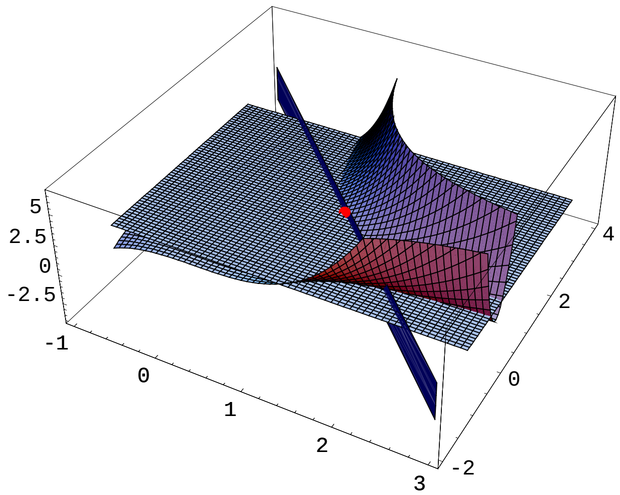
- SurfaceGraphics -



- SurfaceGraphics -


5
2.5
0
-2.5
-1
0
1
2
3
-1
0
1
2
3

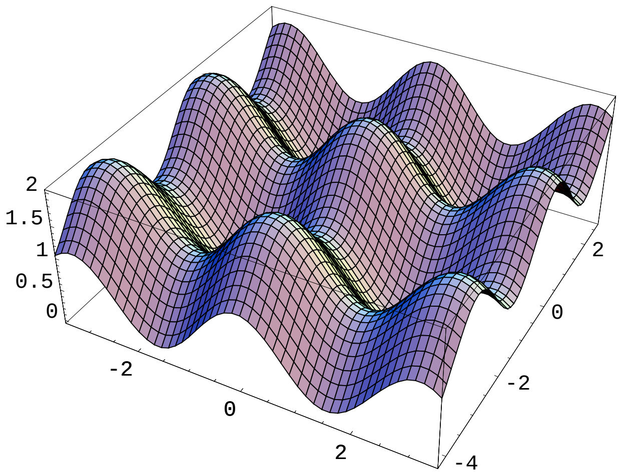

5
2.5
0
-2.5
-1
0
1
2
3
-1
0
1
2
3


5
2.5
0
-2.5
-1
0
1
2
3
-2
0
2
4

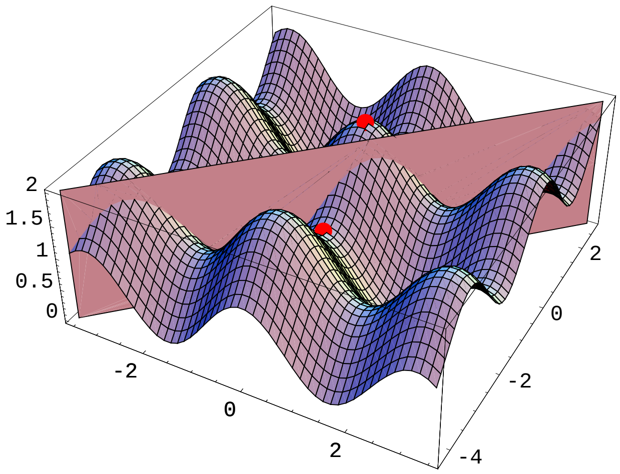
```
(*********************************************)
(***** Feltételes és lokális szélsőérték *****)
(*********************************************)

f[x_, y_] = Cos[x]^2 + Cos[y]^2; (* függvény *)
g[x_] = x - π/4; (* feltétel *)

pr = 3.5; (* plotrange *)
loksze = Solve[{∂x f[x, y] == 0, ∂y f[x, y] == 0}, {x, y}]; (* stacionárius pontok *)
loksze = Transpose[{x /. loksze, y /. loksze, f[x /. loksze, y /. loksze]}]
(* és hozzá a z koordináták *)

ClearAll[FuggolegesSik]; FuggolegesSik =
 Graphics3D[Polygon[{{#1[[1]], #2 /. x → #1[[1]], #3[[1]]}, {#1[[1]], #2 /. x → #1[[1]], #3[[2]]},
     {#1[[2]], #2 /. x → #1[[2]], #3[[2]]}, {#1[[2]], #2 /. x → #1[[2]], #3[[1]]}}]] &;

p1 = Plot3D[f[x, y], {x, -pr, pr}, {y, g[-pr], g[pr]}, PlotPoints → 50]
p1 = Plot3D[f[x, y], {x, -pr, pr},
   {y, g[-pr], g[pr]}, DisplayFunction → Identity, PlotPoints → 50];
p2 = FuggolegesSik[{-pr, pr}, g[x], {0, 2}];
p3 = Graphics3D[{PointSize[0.03], Hue[0], Point /@ loksze}];
Show[p1, p2, p3, DisplayFunction → $DisplayFunction,
 PlotRange → {Automatic, Automatic, Automatic}]

p1 = ContourPlot[f[x, y], {x, -pr, pr}, {y, g[-pr], g[pr]},
   ContourShading → True, Contours → 20, DisplayFunction → Identity];
p2 = Plot[g[x], {x, -pr, pr}, PlotStyle → Hue[0], DisplayFunction → Identity]
Show[p1, p2, DisplayFunction → $DisplayFunction];
```

```
Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.
```

$\{\{0, 0, 2\}, \{-\frac{\pi}{2}, 0, 1\}, \{\frac{\pi}{2}, 0, 1\}, \{0, -\frac{\pi}{2}, 1\}, \{-\frac{\pi}{2}, -\frac{\pi}{2}, 0\}, \{0, \frac{\pi}{2}, 1\}, \{\frac{\pi}{2}, \frac{\pi}{2}, 0\}\}$

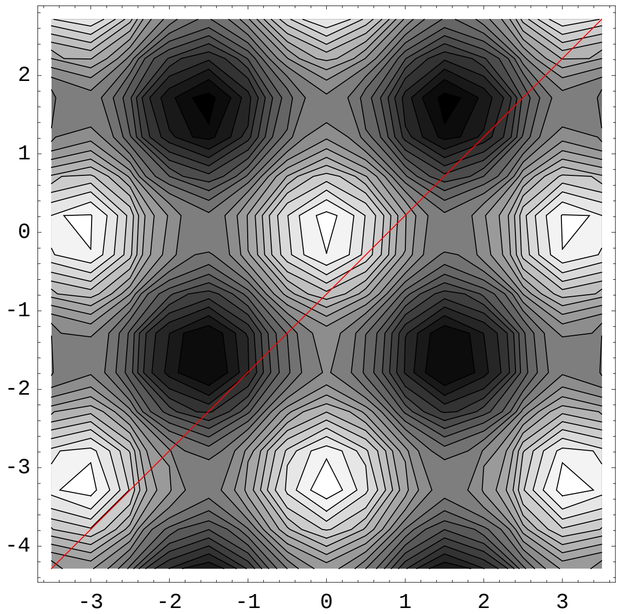



```
- SurfaceGraphics -
```

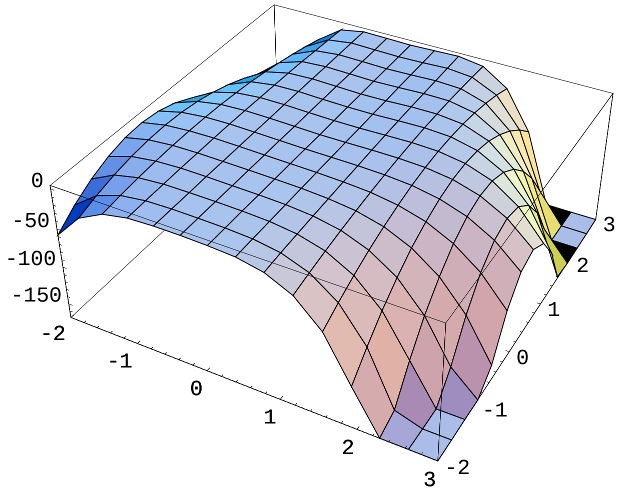


- Graphics3D -

- Graphics -

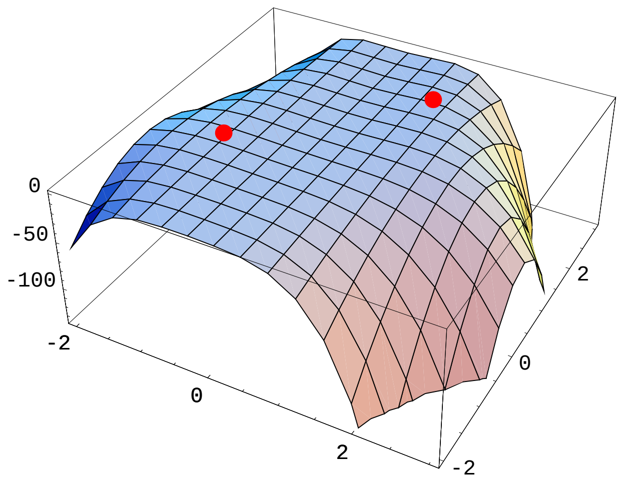


```
ClearAll[f]; ClearAll[F]; ClearAll[g]; ClearAll[G]; ClearAll[x];
ClearAll[y]; ClearAll[x0]; ClearAll[y0]; ClearAll[Dx]; ClearAll[Dy];
ClearAll[Dα]; ClearAll[nv]; ClearAll[dx]; ClearAll[pr];

f[x_, y_] = -(x^2 - 1)^2 - (x^2 y - x - 1)^2;

p1 = Plot3D[f[x, y], {x, -2, 3}, {y, -2, 3}]
loksze = Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}];
loksze = Transpose[{x /. loksze, y /. loksze, f[x /. loksze, y /. loksze]}]
p2 = Graphics3D[{PointSize[0.03], Hue[0], Point /@ loksze}];
Show[p1, p2, DisplayFunction → $DisplayFunction,
  AspectRatio → Automatic, PlotRange → {Automatic, Automatic, Automatic}];
(*Table[Show[p1,p2,ViewPoint->{0.1,k,1}],{k,-10,0,1}];*)
```



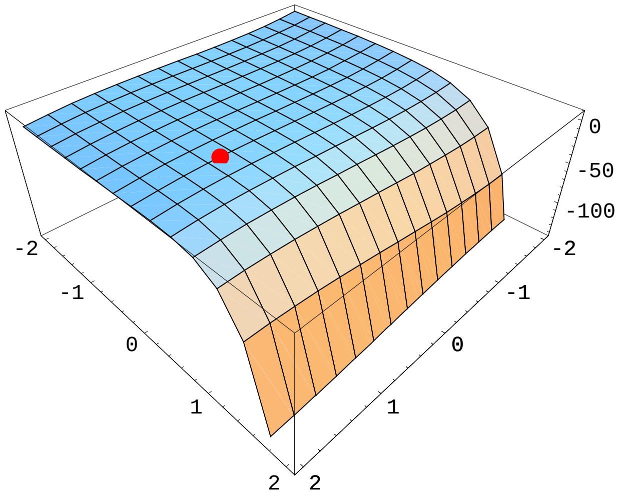
- SurfaceGraphics -

{{-1, 0, 0}, {1, 2, 0}}



```
ClearAll[f]; ClearAll[F]; ClearAll[g]; ClearAll[G]; ClearAll[x];
ClearAll[y]; ClearAll[x0]; ClearAll[y0]; ClearAll[Dx]; ClearAll[Dy];
ClearAll[Dα]; ClearAll[nv]; ClearAll[dx]; ClearAll[pr];

f[x_, y_] = 3 x Exp[y] - x^3 - Exp[3 y];

loksze = Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}];

loksze = loksze[[2]];
loksze = {x /. loksze, y /. loksze, f[x /. loksze, y /. loksze]};

p1 = Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2}, DisplayFunction → Identity];
p2 = Graphics3D[{PointSize[0.03], Hue[0], Point[loksze]}];
Show[p1, p2, DisplayFunction → $DisplayFunction,
  ViewPoint → {1, 1, 1}, PlotRange → {Automatic, Automatic, Automatic}];
```

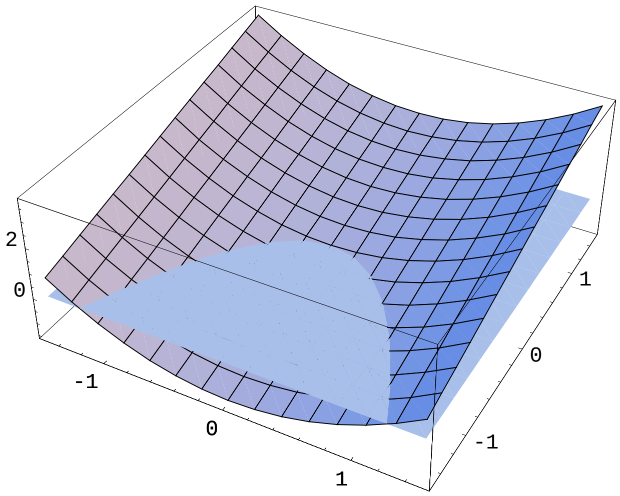
```
Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.
```



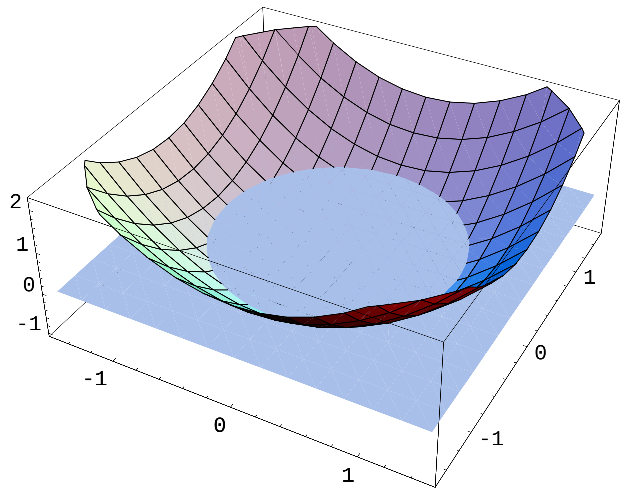
```
ClearAll[f]; ClearAll[F]; ClearAll[g]; ClearAll[G]; ClearAll[x];
ClearAll[y]; ClearAll[x0]; ClearAll[y0]; ClearAll[Dx]; ClearAll[Dy];
ClearAll[Dα]; ClearAll[nv]; ClearAll[dx]; ClearAll[pr];

<< Graphics`ImplicitPlot`
pr = 1.5;
F[x_, y_] = x^2 + y; (*Ennek a szélsőértékeit keressük*)
G[x_, y_] = x² + y² - 1; (*a G=0 feltétel mellett, azaz az egységkörön.*)
DF[x_, y_] = {D[F[x, y], x], D[F[x, y], y]}
DG[x_, y_] = {D[G[x, y], x], D[G[x, y], y]}

p1 = Plot3D[F[x, y], {x, -pr, pr}, {y, -pr, pr}, DisplayFunction → Identity];
p2 = Plot3D[G[x, y], {x, -pr, pr}, {y, -pr, pr}, DisplayFunction → Identity];
p = Plot3D[0, {x, -pr, pr}, {y, -pr, pr}, Mesh → False, DisplayFunction → Identity];
Show[p1, p, DisplayFunction → $DisplayFunction]
Show[p2, p, DisplayFunction → $DisplayFunction]
Show[p1, p2, p, DisplayFunction → $DisplayFunction]
g[x_] = y /. Solve[G[x, y] == 0, y][[2]]
f[t_] = {Cos[t], Sin[t]};
(*Kihasználom, hogy az egységkör a feltétel, és paraméterezem.*)


(*A lehetséges szélsőértékhelyek:*)
feltloksze = Solve[{D[F[x, y] - m G[x, y], x] == 0,
   D[F[x, y] - m G[x, y], y] == 0, D[F[x, y] - m G[x, y], m] == 0}, {x, y, m}]
```

{2 x, 1}

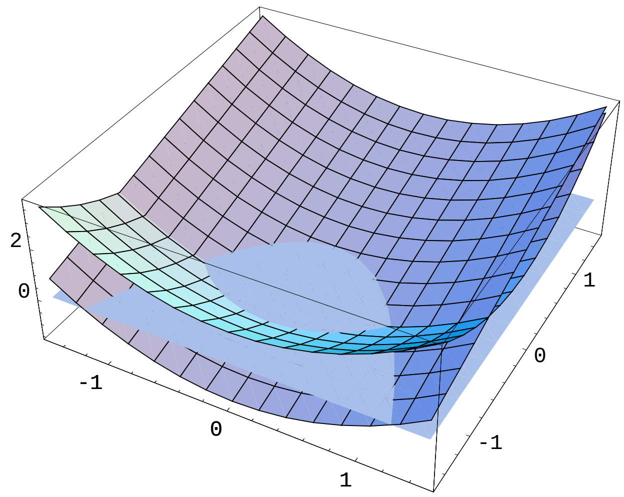
{2 x, 2 y}



- Graphics3D -



- Graphics3D -

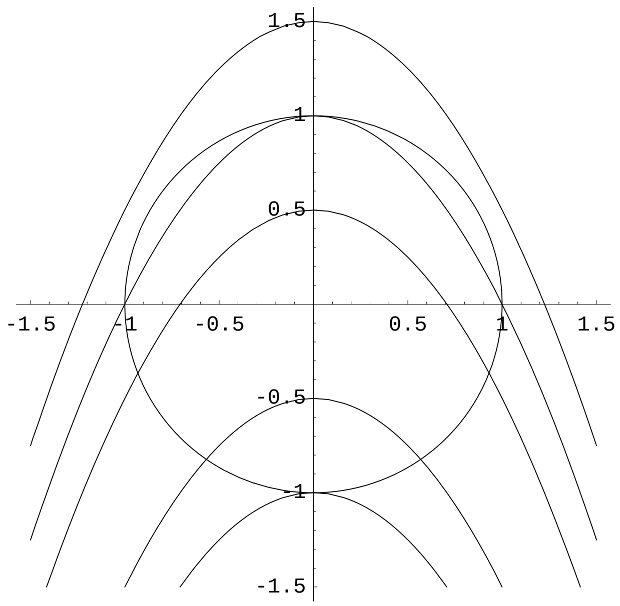


- Graphics3D -

$\sqrt{1-x^2}$

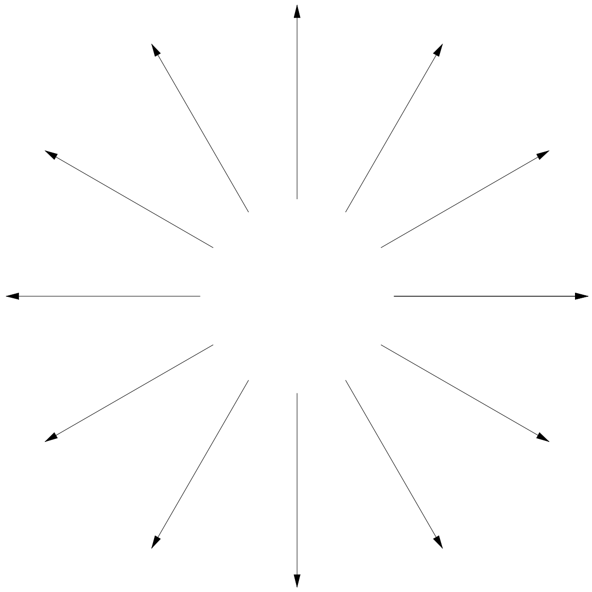
$\left\{\left\{m\to -\frac{1}{2},\ x\to 0,\ y\to -1\right\},\ \left\{m\to \frac{1}{2},\ x\to 0,\ y\to 1\right\},\ \left\{m\to 1,\ x\to -\frac{\sqrt{3}}{2},\ y\to \frac{1}{2}\right\},\ \left\{m\to 1,\ x\to \frac{\sqrt{3}}{2},\ y\to \frac{1}{2}\right\}\right\}$

```
(*Ábrázolom a kört és néhány F=konstans görbét.*)
p3 = ImplicitPlot[G[x, y] == 0, {x, -pr, pr},
   {y, -pr, pr}, AxesOrigin → {0, 0}, DisplayFunction → Identity];
p4 = ImplicitPlot[F[x, y] == 1.5, {x, -pr, pr}, {y, -pr, pr},
   AxesOrigin → {0, 0}, DisplayFunction → Identity];
p5 = ImplicitPlot[F[x, y] == 1, {x, -pr, pr}, {y, -pr, pr},
   AxesOrigin → {0, 0}, DisplayFunction → Identity];
p6 = ImplicitPlot[F[x, y] == 0.5, {x, -pr, pr}, {y, -pr, pr},
   AxesOrigin → {0, 0}, DisplayFunction → Identity];
p7 = ImplicitPlot[F[x, y] == -0.5, {x, -pr, pr}, {y, -pr, pr},
   AxesOrigin → {0, 0}, DisplayFunction → Identity];
p8 = ImplicitPlot[F[x, y] == -1, {x, -pr, pr}, {y, -pr, pr},
   AxesOrigin → {0, 0}, DisplayFunction → Identity];
Show[p3, p4, p5, p6, p7, p8, DisplayFunction → $DisplayFunction];
(*Ebből leolvasható, hogy F feltételes minimuma -1,
 míg a feltételes maximum nagyobb egynél, de kisebb másfélnél.*)
```




```
(*A kör paraméterezését használva felírhatom Pi/6-onként F és G gradiensét.*)
dfelt = Table[{f[t], DG[f[t][[1]], f[t][[2]]]}, {t, 0, 2 Pi, Pi / 6}];
dfv = Table[{f[t], DF[f[t][[1]], f[t][[2]]]}, {t, 0, 2 Pi, Pi / 6}];
<< Graphics`PlotField`
pdfelt = ListPlotVectorField[dfelt, VectorHeads → True];
pdfv = ListPlotVectorField[dfv, VectorHeads → True];
(*A lehetséges szélsőértékhelyeken a gradiensek párhuzamosak. Ha f gradiense nem
  merőleges a kör érintőjére, akkor az egyik irányba (kicsit) elmozdulva nagyobb,
 a másik irányba (kicsit) elmozdulva kisebb értékeket vesz föl f,
 tehát nem lehet szélsőérték.*)
Show[p3, pdfelt, pdfv, DisplayFunction → $DisplayFunction, Axes → None];
(*Látható az előbb kiszámolt négy szélsőértékhely. Az
  alsó és a felső lokális minimum, a két oldalsó maximum.*)
```

```
General::spell1 : Possible spelling error: new symbol name "pdfelt" is similar to existing symbol "dfelt".
```



```
General::spell1 : Possible spelling error: new symbol name "pdfv" is similar to existing symbol "dfv".
```