Introduction: To the Student

Functional analysis was developed in the last years of the nineteenth century and during the first few decades of the twentieth century. Its development was, in large part, in response to questions arising in the study of differential and integral equations. These equations were of great interest at the time because of the vast effort by many individuals to understand physical phenomena.

The unifying approach of functional analysis is to view functions as points in some abstract vector (linear) space and to study the differential and integral equations relating these points in terms of linear transformations on these spaces. The term "functional analysis" is most often credited to Paul P. Lévy (1886–1971; France).[1] The rise of the field is consistent with a larger move toward generality and unification in mathematics. Indeed, this move can be viewed as part of a more general intellectual trend, and it is interesting to compare it to analogous movements in other fields such as philosophy, music, painting, and psychology.

Maurice Fréchet (1878–1973; France) is usually credited with the first major effort to develop an abstract theory of spaces of functions. Much of this work appears in his 1906 doctoral thesis. Many other names are associated with the birth and development of functional analysis, and you will read about them as you proceed through this text. The works of Stefan Banach (1892–1945; Austria–Hungary, now Hungary) and David Hilbert (1862–1943; Prussia, now Russia) have probably had the greatest influence.

It has been my goal to present the basics of functional analysis in a way that makes them comprehensible to a student who has completed first courses in linear

[1]See the biography of Fréchet for more on the origins of this phrase.

algebra and real analysis, and to develop the topics in their historical context. Bits of pertinent history are scattered throughout the text, including brief biographies of some of the central players in the development of functional analysis.

In this book you will read about topics that can be gathered together under the vague heading, "What everyone should know about functional analysis." ("Everyone" certainly includes anyone who wants to study further mathematics, but also includes anyone interested in the mathematical foundations of economics or quantum mechanics.) The first five chapters of the book are devoted to these essential topics. The sixth chapter consists of seven independent sections. Each section contains a topic for further exploration; some of these develop further a topic found in the main body of the text, while some introduce a new topic or application. The topics found in the sixth chapter provide good bases for individual student projects or presentations. Finally, the book concludes with two appendices that offer basic information on, respectively, complex numbers and set theory. Most of the material found in these two sections is not hard, but it is crucial to know before reading the book. The appendices can be read in advance and can be used as reference throughout your reading of the text.

There are plenty of exercises. There is much wisdom in the saying that you must *do* math in order to learn math. The level of difficulty of the exercises is quite variable, so expect some of them to be straightforward and others quite challenging.

There are many excellent books on functional analysis and the other topics that we discuss in this text. The bibliography includes references to classics by the "founding fathers" ([11], [80], [107], for example); some of the standard texts currently used for first-year graduate courses ([44], [47], [111], for example), treatments of historical aspects of our subject ([16], [17], [23], [34], [54], [61], [73], [76], [104], for example); books on related topics ([1], [22], [27], [77], [121], [99], for example); undergraduate real analysis texts ([25], [89], [109], [110], for example); and readable journal articles on topics we discuss ([13], [26], [31], [37], [64], [91], [96], [117], [119], [125], for example).

The list of references is meant to be used, and I hope that you take the opportunity to look at many of the referenced books and articles.

Finally, there is a very good history of mathematics web site run at St. Andrews University:
`http://www-groups.dcs.st-and.ac.uk/~history` (this address was good as of April 2001).