

## Stochastic Signals and Systems

Lecture 5.

## **Spectral Theory II** Review from last lecture.

16 October 2020.



## The problem of prediction, review.

Let  $(y_n)$  be a completely regular process. Find its one-step ahead LSQ prediction given by:

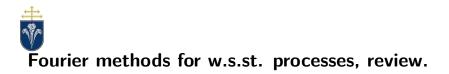
 $\hat{y}_n = (y_n | H_{n-1}^y).$ 

Formal solution of the prediction problem:

 $\widehat{y} = \left(H(q^{-1}) - 1\right) e$  and  $e = H^{-1}(q^{-1})y$ 

thus we get an expression of  $\hat{y}$  via y

Challenge: what is the meaning of the operator  $H^{-1}(q^{-1})$ ?



We were led to the formal objects:

$$\sum_{n=-\infty}^{\infty} y_n e^{-in\omega}, \qquad \omega \in [0, 2\pi).$$
(1)

The aim of spectral theory is to give a meaning to these formal objects. Let  $(y_n)$  be a w.s.st. process. We can ask if the finite Fourier series

$$\xi_N := \sum_{n=-N}^N y_n e^{-in\omega}$$

has a limit in any sense?



At this point let us make the assumption that

$$\sum_{\tau=-\infty}^{+\infty} r^2(\tau) < +\infty.$$
 (2)

## Proposition

Under condition  $\sum r^2( au) < +\infty$ 

$$\lim_{N\to\infty}\frac{1}{2N+1}\mathrm{E}\left|\sum_{n=-N}^{N}e^{-in\omega}y_{n}\right|^{2}=f(\omega)\geq0$$

exists a.s. on  $[0, 2\pi)$  w.r.t. the Lebesgue-measure, where

$$f(\omega) = \sum_{\tau=-\infty}^{+\infty} r(\tau) e^{-i\omega\tau}$$