Stochastic Signals and Systems

Lecture 5.

Spectral Theory II

Review from last lecture.

16 October 2020.

The problem of prediction, review.

Let (y_n) be a completely regular process. Find its one-step ahead LSQ prediction given by:

$$\hat{y}_n = (y_n | H^y_{n-1}).$$

Formal solution of the prediction problem:

$$\widehat{y} = \left(H(q^{-1}) - 1\right) e \quad \text{and} \quad e = H^{-1}(q^{-1}) y$$

thus we get an expression of $\widehat{y}$ via y

Challenge: what is the meaning of the operator $H^{-1}(q^{-1})$?

Fourier methods for w.s.st. processes, review.

We were led to the formal objects:

$$\sum_{n=-\infty}^{\infty} y_n e^{-in\omega}, \qquad \omega \in [0, 2\pi). \tag{1}$$

The aim of spectral theory is to give a meaning to these formal objects.

Let (y_n) be a w.s.st. process. We can ask if the finite Fourier series

$$\xi_N := \sum_{n=-N}^{N} y_n e^{-in\omega}$$

has a limit in any sense?

At this point let us make the *assumption* that

$$\sum_{\tau=-\infty}^{+\infty} r^2(\tau) < +\infty. \tag{2}$$

Proposition

Under condition $\sum r^2(\tau) < +\infty$

$$\lim_{N\to\infty} \frac{1}{2N+1}\mathrm{E}\left|\sum_{n=-N}^{N} e^{-in\omega} y_n\right|^2 = f(\omega) \geq 0$$

exists a.s. on $[0, 2\pi)$ *w.r.t. the Lebesgue-measure, where*

$$f(\omega) = \sum_{\tau=-\infty}^{+\infty} r(\tau) e^{-i\omega\tau}.$$