Networked control approaches for a nuclear power plant pressurizer subsystem ⋆

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Abstract: This paper extends former results about the networked reference tracking control of a pressurizer in a pressurized water nuclear power plant. An L2-gain based method is used in a deterministic framework to compute the maximum allowable transmit intervals (MATIs) for the actually implemented dynamic inversion based controller that still guarantee the required performance. Furthermore, a stochastic linear quadratic regulator with a state estimator is also designed that can take packet losses into consideration. It is shown through the comparison that both methods are applicable for networked control but the LQ controller combined with a Kalman filter has better output variance properties.

Keywords: networked control, LQG control, dynamic inversion

1. INTRODUCTION

The requirement for the continuous improvement of process safety and effectivity often necessitates the dynamic analysis and/or re-design of certain subsystems in complex plants. The need for continuous development is particularly true for such a safety-critical application like a nuclear power plant. In many cases, the most advantageous and most economical way to substantially improve system dynamics is the detailed modelling and model-based advanced feedback design for the affected components of the system Hangos and Cameron [2001], Szederkényi et al. [2008].

One example for such a procedure is the successful modelling, identification and dynamic inversion based controller design for stabilizing the primary circuit pressure at the Paks Nuclear Power Plant in Hungary in 2004-2005 (see, e.g. Szabó et al. [2005], Varga et al. [2008]). This controller implementation largely contributed to the possibility that the average thermal power of the plant units could be increased by 1-2%. The implemented controller is a redundant networked control system (a redundant NCS), where the measurement results and the control commands are transferred to the computing units and actuators through an Ethernet network. The controller was originally designed in continuous time and then it was discretized using an appropriate sampling interval.

The modeling and parameter estimation procedure for the whole primary circuit has been described in Fazekas et al. [2007, 2008]. The same procedure for the pressurizer which is a subsystem of the primary circuit has been presented in Varga et al. [2006].

In recent years, significant and well-usable theoretical results appeared in the field of dynamic analysis (Nesic and Teel [2004b,a], Tabbara et al. [2007]) and controller design (Sinopoli et al. [2003, 2005]) of NCSs. Since many of these important results appeared only after the actual design of the pressurizer controller, the posterior analysis of the implemented control system and its comparison with other possible alternatives is definitely of interest.

The basic analysis for the pressurizer together with the implemented dynamic inversion based controller in the framework of Tabbara et al. [2007] has been performed in Szederkényi et al. [2008]. The aim of this paper is to extend these results and compare them in simulations to a networked LQ-servo design where the state estimator is a specially computed Kalman-filter (Sinopoli et al. [2005]).

2. BASIC NOTIONS

2.1 Dynamic inversion based control

In this section, the summary of the dynamic inversion based controller design is shown based on Szabó et al. [2005]. A more detailed desrciption of the design can be found in Szederkényi et al. [2008].

It is known that if an LTI system is invertible, then the maximal (A,B)-invariant subspace (V^*) contained in $kerC$, induces a decomposition of the linear system into:

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$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1 u \quad (1)$$
$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 \quad (2)$$
$$z = C_1 x_1, \quad (3)$$

where $ImB = ImB_1$ and $x_1 \in V^{*\perp}$, see Basile and Marro [1973], Wonham [1985].

By applying the feedback $u = F_1 x_1 + F_2 x_2 + v$, with $F = [F_1 \ F_2]^T$ that renders V^* $(A + BF, B)$ invariant, we can obtain the system:

$$\dot{x}_1 = A_{11}x_1 + B_1 v, \quad z = C_1 x_1. \quad (4)$$

The required input to track a desired output signal z_d is given by the dynamic system

$$\dot{\eta}_d = A_{22}\eta_d + A_{21}\zeta(z_d) \quad (5)$$
$$u_d = F_2\eta_d + \lambda(z_d), \quad (6)$$

provided that this input is applied to the original system started from the appropriate initial conditions.

In practice it seldom happens that the required initial conditions can be set precisely, therefore, there will be an error in the whole state. To close the loop, a suitable linear dynamical system of the tracking error is added to the linearizing control input. By examining the "open–loop" equations (5) one can observe that it is possible to introduce an "outer–loop" by applying an error feedback, that modifies the equations (6) that define the control input. This idea is shown by the dotted line part of Figure 1.

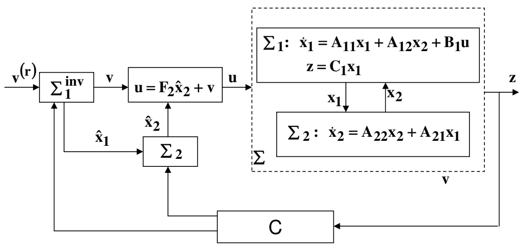

Fig. 1. Inversion based tracking

Based on this structure, an advanced (e.g. $\mathcal{H}_\infty$) controller can be designed in order to minimize the influence of the disturbances on the performance of the tracking error.

2.2 Deterministic performance analysis of networked control systems

The concepts and results summarized in this section are taken from Nesic and Teel [2004a] and Tabbara et al. [2007]. The basic configuration of a networked control system can be seen in Fig. 2, where x_p and x_c are the states of the plant and the controller, respectively, $y \in \mathbb{R}^r$ is the plant output, $u \in \mathbb{R}^p$ is the controller output, while $\hat{y} \in \mathbb{R}^r$ and $\hat{u} \in \mathbb{R}^p$ are the most recently transmitted plant and controller output values through the network. e is the error caused by network transmission that is defined as

$$e(t) = \begin{bmatrix} \hat{y}(t) - y(t) \\ \hat{u}(t) - u(t) \end{bmatrix} \quad (7)$$

Individual actuators and sensors connected to the networks are called nodes. We assume that node data are transmitted at time instants $\{t_0, t_1, \ldots, t_i\}$ where $i \in \mathbb{N}$. The transmission time instants satisfy $\epsilon < t_{j+1} - t_j \leq \tau$ for $j \geq 0$ where $\epsilon, \tau > 0$. The upper interval bound τ is called the *maximum allowable transfer interval* (MATI).

Using the notation $x = [x_p^T \quad x_c^T]^T \in \mathbb{R}^n$, the dynamic equations of a networked control system with disturbance vector $w \in \mathbb{R}^m$ between the transmission instants can be written as

$$\dot{x} = f(t, x, e, w), \quad t \in [t_{i-1}, t_i] \quad (8)$$
$$\dot{e} = g(t, x, e, w), \quad t \in [t_{i-1}, t_i] \quad (9)$$

The discontinuous change of e during transmission instants can be modeled as a jump system

$$e(t_i^+) = (I - \Psi(i, \hat{e}(t_i)))e(t_i) \quad (10)$$
$$\hat{e}(t_i^+) = \Lambda(i, \Psi(i, \hat{e}(t_i))e(t_i), \hat{e}(t_i)) \quad (11)$$

where $\hat{e}$ is the decision vector of the network scheduler, Ψ is the scheduling function and Λ is the decision update function. More details about the dynamics (10)-(11) in the case of different scheduling protocols can be found in Tabbara et al. [2007]. A key feature of network scheduling protocols from the point of view of closed loop stability is the so-called *persistently exciting* (PE) property. According to the definition, a protocol is *uniformly PE in time* T if it regularly visits every network node within a fixed period of time T.

In the LTI or linearized case Eqs. (8)-(9) will be used in the form

$$\dot{x} = \Phi_{11}x + \Phi_{12}e \quad (12)$$
$$\dot{e} = \Phi_{21}x + \Phi_{22}e \quad (13)$$

where Φ_{ij} are constant matrices of appropriate dimensions. Let us introduce the following notations. $\mathcal{A}_n^+$ denotes

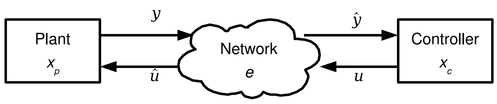

Fig. 2. Networked control system

the set of positive semidefinite symmetric $n \times n$ matrices with positive entries. For $x, y \in \mathbb{R}^n$, $x \preceq y \iff x_i \leq y_i$ for $i = 1, \ldots, n$. For an n-dimensional vector x, $\bar{x} = [|x_1|, \ldots, |x_n|]^T$. The following theorem from Tabbara et al. [2007] will serve as a theoretical basis for our forthcoming calculations in section 4.3.

Theorem 1: Suppose that the NCS scheduling protocol of (8)-(11) is uniformly persistently exciting in time T and the following assumptions hold

(1) There exist $Q \in \mathcal{A}_n^+$ and a continuous output of the form $\tilde{y}(x, w) = G(x) + w$ so that the error dynamics (9) satisfies
$$\bar{g}(t, x, e, w) \preceq Q\bar{e} + \tilde{y}(x, w) \quad (14)$$
for all (x, e, w), for $t \in (t_i, t_{i+1})$, and for all $i \in \mathbb{N}$.
(2) (8) is $\mathcal{L}_p$ stable from (e, w) to $G(x)$ with gain γ for some $p \in [1, \infty]$.
(3) the MATI satisfies $\tau \in (\epsilon, \tau^*)$, $\epsilon \in (0, \tau^*)$, where
$$\tau^* = \ln(v)/(|Q|T), \quad (15)$$
and v is the solution of
$$v(|Q| + \gamma T) - \gamma T v^{1-1/T} - 2|Q| = 0. \quad (16)$$

Then the NCS is $\mathcal{L}_p$-stable from w to $(G(x), e)$ with linear gain.

Using Theorem 1, a sharp and practically usable estimation can be obtained for the acceptable upper bound of the MATI such that the $\mathcal{L}_p$ stability of the closed loop system is preserved.

2.3 LQG control over communication network

For comparative purposes a discrete-time LQG control has also been designed for the presented control problem. Since the communication channel can only guarantee the maximal transfer time (MATI–τ), the time t_k that is actually needed at a particular time instant k for a measurement packet to reach the controller can vary in time. A classical sampled controller can be implemented only if $t_k \ll T_s$, where T_s is the sampling time. This is equivalent to requiring that $\tau \ll T_s$. For the discrete-time controller to be applicable even if this condition does not hold, we implemented the modified version of the classical LQG controller. For this, suppose the length of the transfer period varies randomly and the probability of $t_k \ll T_s$ is known (The $\ll$ is defined practically e.g. by $t_k < 0.01T_s$). The case of $t_k \ll T_s$ means that the measurement is received on time and can therefore be used to compute the next control action. If $t_k \ll T_s$ does not hold (possibly $t_k > T_s$) we apply the following policy: the controller runs without the fresh measurement and the late packet will be dropped. This is equivalent to considering the measurement delay as a simple packet loss, which can be easily handled by the methods presented in Sinopoli et al. [2003],Sinopoli et al. [2005].

The system investigated in papers Sinopoli et al. [2003] and Sinopoli et al. [2005] can be given by the following discrete-time difference equations:

$$\begin{aligned} x_{k+1} &= Ax_k + \nu_k B u_k + w_k \\ y_k &= Cx_k + v_k \end{aligned} \tag{17}$$

where x_k, u_k, y_k are the state, input and output, as before and x_0, w_k, v_k are Gaussian, uncorrelated, white, zero mean random variables with covariance P_0, Q, R_k, respectively. The actual measurement is $\gamma_k \cdot y_k$, where $\gamma_k \in \{0, 1\}$ is a random variable with $P(\gamma_k = 1) = \bar{\gamma}$. (In view of our network configuration, $\bar{\gamma}$ is the probability of receiving the k-th packet on time.) The covariance R_k is time varying, because it depends on the random packet loss: $R_k = \gamma_k R + (1 - \gamma_k)\sigma^2 I$, where $\sigma \to \infty$ is taken to model the loss of information. By introducing an other random variable ν_k with $\nu_k \in \{0, 1\}$ and $P(\nu_k = 1) = \bar{\nu}$ it is possible to model a packet loss at the actuator side, which means in our case that the control input does not arrive to the actuators on time. The LQG controller is constructed to minimize the cost function

$$J_\infty = \mathbb{E}\left[\sum_{k=0}^{\infty} x_k^T W x_k + \nu_k u_k^T U u_k \mid \mathcal{I}_\infty\right] \tag{18}$$

where $\mathcal{I}_k = \{\mathbf{y}^k, \gamma^k, \nu^{k-1}\}$ denotes all available information and $\mathbf{y}^k = \{y_k, \ldots, y_0\}$, $\gamma^k = \{\gamma_k, \ldots, \gamma_0\}$, $\nu^{k-1} = \{\nu_{k-1}, \ldots, \nu_0\}$. By Sinopoli et al. [2005] the optimal control policy can be computed as follows:

$$\begin{aligned} u_k &= L_\infty \hat{x}_{k|k}, \quad L_\infty = -(B^T S_\infty B + U)^{-1} B^T S_\infty A \\ S_\infty &= A^T S_\infty A + W + \bar{\nu} A^T S_\infty B L_\infty \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + \gamma_k K_k (y_k - C\hat{x}_{k|k-1}) \\ K_k &= P_{k|k-1} C^T (C P_{k|k-1} C^T + R)^{-1} \\ \hat{x}_{k|k-1} &= A\hat{x}_{k-1|k-1} + B u_{k-1} \\ P_{k|k-1} &= A P_{k-1|k-1} A^T + Q \end{aligned} \tag{19}$$

where $P_{0|-1} = P_0$, $\hat{x}_{0|-1} = 0$. In Sinopoli et al. [2003] and Sinopoli et al. [2005] there are methods for determining the minimal probability of successful packet delivery required for the stability to be guaranteed. Since the pressurizer subsystem examined in the paper is open loop stable, the packet loss does not cause instability. The effect of network unreliability can be detected only in degradation of the performance, which is examined later via simulations.

3. SYSTEM DESCRIPTION

3.1 Task and operating principles of the pressurizer

The task of the pressurizer is to keep the primary circuit pressure within a predefined range. The pressurizer is a vertical tank and inside this tank there is hot water at a temperature of about 325°C and steam above. If the primary circuit pressure decreases, electric heaters switch on automatically in the pressurizer (see Fig. 3) Due to the heating more steam will evaporate and this leads to a pressure increase.

Using the old outdated pressurizer controller that applied a hystheresis-based switching algorithm, the primary circuit pressure was oscillating in an approximately 1 bar interval during normal operation. The high peaks of these oscillations prevented the possibility of operating the units at a slightly higher thermal power (because of safety limits). As a result of equipment modernization, the heating energy of the electric heaters can now be set in a continuous range of 0-360 kW and new pressure sensors were installed with a significantly lower measurement error. These changes made possible the design of a more advanced controller that can stabilize the pressure in a much narrower range.

The manipulable input variable of the model is directly proportional to the instantaneous electrical heating power. Furthermore, there is an approximately constant water in- and outlet to and from the tank. The time-varying temperature of the inflowing water will be treated as a disturbance in the model. The controlled and measured output is the pressure in the tank (see Fig. 3).

3.2 System model

The physical modelling of the primary circuit dynamics (including the pressurizer) has been carried out following the principles of Hangos and Cameron [2001] and is described in greater detail in Fazekas et al. [2007]. The variables and constants appearing in the model are listed in Table 1. The state-space model of the pressurizer model is given by

$$\dot{x} = Ax + Bu + Ed \tag{20}$$

where the state vector x, the input u and the external disturbance vector d are

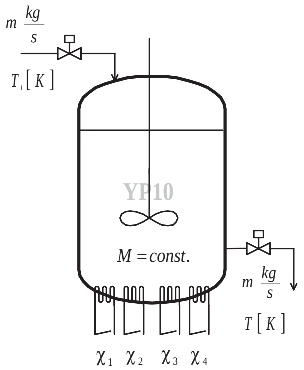

Fig. 3. Simplified structure of the pressurizer

T	water temperature	$^\circ$C
T_W	tank wall temperature	$^\circ$C
c_p	specific heat of water	$\frac{\mathrm{J}}{\mathrm{kg}^\circ\mathrm{C}}$
U	internal energy of water	J
U_W	internal energy of the wall	J
m	mass flow rate of inlet water	$\frac{\mathrm{kg}}{\mathrm{s}}$
T_I	inlet water temperature	$^\circ$C
M	mass of water	kg
C_{pW}	heat capacity of the wall	$\frac{\mathrm{J}}{^\circ\mathrm{C}}$
W_{HE}	power of one electric heater	W
W_l	environmental heat loss	

Table 1. Variables and constants of the pressurizer model

$$x = [T\ T_W]^T,\ u \in [0, 4],\ d = [T_I\ W_l]^T$$

The actually measured pressure is a nonlinear static function of the water temperature in the following form

$$p = h(T) = \frac{e^{\varphi(T)}}{100}$$

where $\varphi(T) = c_0 + c_1 T + c_2 T^2 + c_3 T^3$ with constants $c_0, \ldots, c_3$. Therefore the output equation can be written as

$$y = [\,1\ 0\,]\,x, \quad x_1 = h^{-1}(p) \tag{21}$$

The matrices of the model are

$$A = \begin{bmatrix} -\frac{m}{M} - \frac{K_W}{c_p M} & \frac{K_W}{c_p M} \\ \frac{K_W}{C_{pW}} & -\frac{K_W}{C_{pW}} \end{bmatrix} \tag{22}$$

$$B = \begin{bmatrix} \frac{W_{HE}}{c_p M} \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} \frac{m}{M} & 0 \\ 0 & \frac{1}{C_{pW}} \end{bmatrix}$$

4. CONTROL SYSTEM ANALYSIS

The main control goal is to stabilize the pressure at a prescribed reference value (typically around 123-124 bars which is equivalent to approximately 327 °C in terms of temperature). Moreover, the controller's additional task is to suppress the effect of measurement noise and that of the time-varying disturbances (W_l, T_I).

4.1 Controller description

Using the theory described in section 2.1, the brief summary of the controller design method is the following. The state-space equations of the open loop system (20) can be rewritten as

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + B_u u + E_T T_I \tag{23}$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 - E_W W_l \tag{24}$$

where $a_{ij} = A_{ij}$, $B_u = B_1$, $E_T = E_{11}$ and $E_W = E_{22}$ in (22). Let x_1^r denote the reference value for x_1 (i.e. $z_d = x_1^r$). Furthermore, let us denote the nominal (mean) values for the time-varying disturbances by $T_{I,n}$ and $W_{l,n}$, respectively.

Note that the system equations (23)-(24) are already in the form of eqs. (1)-(2) with extra disturbance terms. The dynamic equation of the inversion controller is given by

$$\dot{\eta} = a_{22}\eta + a_{21}x_1^r - E_W W_{l,n} \tag{25}$$

The input u is expressed as $u = u^r + v$, where

$$u^r = \frac{1}{B_u}(\dot{x}_1^r - a_{11}x_1^r - a_{12}\eta - E_T T_{I,n}), \tag{26}$$

and v is a new input term for additional feedback.

The state variables of the tracking error system are defined as

$$s_1 = x_1 - x_1^r, \quad s_2 = x_2 - \eta \tag{27}$$

Substituting (26) into (23) gives

$$\dot{x}_1 = a_{11}(x_1 - x_1^r) + a_{12}(x_2 - \eta) + E_T\tilde{d}_1 \tag{28}$$

where $\tilde{d}_1 = T_I - T_{I,n}$. For the tracking error dynamics, we get

$$\dot{s}_1 = a_{11}s_1 + a_{12}s_2 + E_T\tilde{d}_1 + B_u v \tag{29}$$

$$\dot{s}_2 = a_{21}s_1 + a_{22}s_2 - E_W\tilde{d}_2 \tag{30}$$

where $\tilde{d}_2 = W_l - W_{l,n}$.

Taking into consideration that x_1 is the measured state variable, we can shape the error dynamics with a dynamic (or static) controller of the general form

$$\dot{\xi} = M_{c1}\xi + M_{c2}s_1 \tag{31}$$

$$v = M_{c3}\xi + M_{c4}s_1, \tag{32}$$

for details, see Szabó et al. [2005].

4.2 Controller implementation

The hardware environment of the controller implementation is a distributed digital system. The functional units of the system are connected through an Ethernet network. The pressure is measured by a high precision instrument located in a hermetically sealed area. The data are transferred to a Siemens S300 control unit using the Profibus PA protocol. The pressure measurement loop has a redundant architecture. The S300 controller checks the status of the pressure measurements and transfers them to other nodes of the network. It can also check signal values during tests. The endpoints of the system are Wago intelligent controllers that actually operate the electric heaters and valves. These devices, located at different points of the power plant, are the real physical actuators in the system. The three controllers work cooperatively: their states are shared with each other and with the central computer system. They are also able to work in reduced mode independently in case of certain failures.

4.3 Computation of the MATI in an L2-gain framework

For the simplification of the forthcoming calculations, we will use the following assumptions for the analysis.

$\mathcal{A}1$ The time-varying disturbances T_I and W_l are constant. This assumption approximates reality quite well if we consider a few minutes to approximately one hour of system operation, because the change of these disturbances is usually rather slow compared to the system dynamics.
$\mathcal{A}2$ The nominal values of disturbances ($T_{I,n}$, $W_{l,n}$) are constant.
$\mathcal{A}3$ Zero order hold is assumed on the input.
$\mathcal{A}4$ The temperature reference x_1^r is (at least piecewise) constant.
$\mathcal{A}5$ Pressure measurement noise is not taken into consideration during the analysis.
$\mathcal{A}6$ Similarly to the examples in Tabbara et al. [2007], all the network induced errors are grouped to the output, i.e. $e = \hat{y}(t) - y(t)$.

The analyzed dynamic inversion based controller uses a static error feedback, therefore the controller equations (25), (26) and (32) can be summarized in the following simple state-space model containing only one state variable (denoted by x_3):

$$\dot{x}_3 = A_c x_3 + B_{c1} x_1 + (B_{c1} - B_{c4}) x_1^r + B_{c3} W_{l,n}$$
$$y_c = D_{c4} x_1 + C_c x_3 + (D_{c1} - D_{c4}) x_1^r + D_{c2} T_{I,n}$$

where A_c, B_{ci}, C_c and D_{ci} are the controller parameters, and x_1 is the temperature in the pressurizer. The actual system input can be computed as

$$u = y_c + e \tag{33}$$

Using (33), the equations of the closed loop system can be written as

$$\dot{x}_1 = (a_{11} + B_u D_{c4}) x_1 + a_{12} x_2 + B_u C_c x_3 + E_T T_I$$
$$+ B_u (D_{c1} - D_{c4}) x_1^r + B_u D_{c2} T_{I,n} + B_u e \tag{34}$$
$$\dot{x}_2 = a_{21} x_1 + a_{22} x_2 + E_W W_l \tag{35}$$
$$\dot{x}_3 = B_{c4} x_1 + A_c x_3 + (B_{c1} - B_{c4}) x_1^r + B_{c3} T_{I,n} \tag{36}$$
$$y_c = D_{c4} x_1 + C_c x_3 + (D_{c1} - D_{c4}) x_1^r + D_{c2} T_{I,n} \tag{37}$$

Observe, that we have a LTI closed loop system model. Therefore, from Eqs. (34)-(36), matrices Φ_{11} and Φ_{12} in (12) are obtained as

$$\Phi_{11} = \begin{bmatrix} a_{11} + B_u D_{c4} & a_{12} & B_u C_c \\ a_{21} & a_{22} & 0 \\ B_{c4} & 0 & A_c \end{bmatrix}, \tag{38}$$

$$\Phi_{12} = \begin{bmatrix} B_u \\ 0 \\ 0 \end{bmatrix} \tag{39}$$

Let us denote the ith row of Φ_{11} by Φ_{11}^i. Using assumptions $\mathcal{A}1$–$\mathcal{A}4$, the time derivative of e can be written as

$$\dot{e} = -\dot{y}_c = -D_{c4}\dot{x}_1 - C_c \dot{x}_3 =$$
$$-D_{c4}\Phi_{11}^1 x - C_c \Phi_{11}^3 x - D_{c4} B_u e - D_{c4} E_T T_I -$$
$$D_{c4} B_u (D_{c1} - D_{c4}) x_1^r - D_{c4} B_u D_{c2} T_{I,n} -$$
$$C_c (B_{c1} - B_{c4}) x_1^r - C_c B_{c3} W_{l,n} \tag{40}$$

From (40), matrices Φ_{21} and Φ_{22} of (13) are the following

$$\Phi_{21} = -D_{c4}\Phi_{11}^1 - C_c \Phi_{11}^3 \tag{41}$$
$$\Phi_{22} = -D_{c4} B_u \tag{42}$$

The controller parameters were the following

$$A_c = -0.0029,\ B_{c1} = 0.0029,\ B_{c3} = -0.0020,$$
$$B_{c4} = 0.011,\ C_c = -2.1031,\ D_{c1} = 2.11, \tag{43}$$
$$B_{c2} = -0.007,\ B_{c4} = -7.58$$

Using the model of the closed loop system (34)-(37), the model parameters and the controller parameters (43), the matrices Φ_{11}, Φ_{12}, Φ_{21} and Φ_{22} are easy to compute (see Szederkényi et al. [2008] for the details).

Since the system is SISO, the number of network links can chosen to be one, i.e. $T = 1$.

The $\mathcal{L}_2$ gain between the error e and the output $\tilde{y} = \Phi_{21} x$ is $\gamma = 9.595 \cdot 10^{-3}$. For the estimation of τ^*, first we solve Eq. (16) that yields $z = 1.499$. The norm of Φ_{22} is easy to compute and thus: $|\Phi_{22}| = |Q| = 9.5902 \cdot 10^{-3}$.

From these results and by solving (16), we obtain the following estimate for the MATI: $\tau^* = 42.27$ s. This proves, that the present sampling time of 10 s is a safe value from the point of view of $\mathcal{L}_2$ stability even if there are some network induced delays that do not violate τ^*.

4.4 Simulation results and comparison

To compare the obtained results, 24 hours of system operation have been examined on simulations in Matlab/Simulink. The time function of the disturbance T_I is shown in Fig. 4, while W_{loss} was constant. The reference value for the pressure was 124 bars. White measurement noise was assumed on the output with a variance of 0.01. The noisy pressure measurements were converted to temperature using the same spline functions as in the real implementation. The sampling time of the controller was 10 s with zero order hold on the input. The effect of various transmit intervals for the dynamic inversion based controller have been compared to different probabilities of packet losses in the case of the networked LQ controller. Fig. 5 shows the pressure values in the case of the dynamic inversion controller for 10s, 40s delay and 100s transmit intervals, respectively. The output variances for these three cases were 0.0053, 0.0063 and 0.0093.

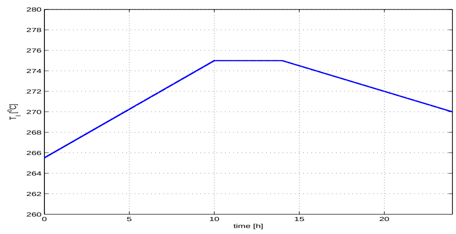

Fig. 4. Time function of disturbance T_I

The discrete-time LQG controller presented in subsection 2.3 has been designed with the following parameters:

$$W = \mathrm{diag}([10^{-6}\ 10^{-6}\ 10^{-3}]); \quad U = 10$$
$$P_0 = 10^{-3} I_4; \quad Q = 0.1 I_4$$
$$R = \mathrm{diag}([10^{-2}\ 10^{-8}\ 10^{-8}]); \tag{44}$$

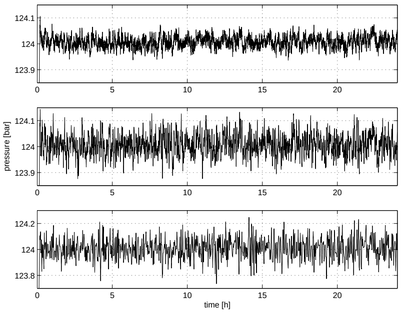

Fig. 5. Pressure in the controlled system with dynamic inversion based controller: a) 10s TI, b) 40s TI, c) 100s TI

The probability $\bar{\gamma}$ of successful measurement packet delivery has been chosen to be 1 (no network delay), 0.25 (only every 4-th packet arrives averagely), 0.1 (9 are failed averagely form 10 consecutive transmissions). To obtain the discrete time model required by the LQG controller, the system (22) was centered around the reference state and the nominal disturbance values $(T_{I,n}, W_{l,n})$ and then it was discretized with sampling time $T_s = 10s$. The simulation results can be seen in figure 6. The output variances in the three different cases were 0.0044, 0.0056, 0.0065, respectively.

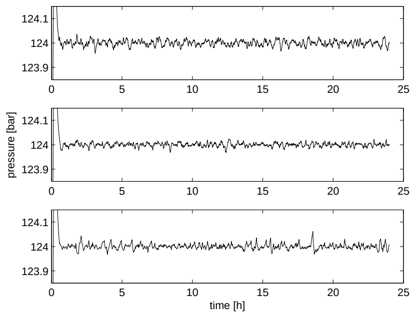

Fig. 6. Pressure in the system controlled by the discrete-time LQ controller with a) $\bar{\gamma} = 1$, b) $\bar{\gamma} = 0.25$, c) $\bar{\gamma} = 0.1$

The disturbance rejection properties of the two controllers are similar. It is also visible from the results that in the case of the dynamic inversion based controller, the output has significantly greater variance than that of the LQ controller, if the MATI is bigger than the standard sampling interval. However, it must be taken into consideration that the LQ design explicitely takes into account the output noise characteristics. Moreover, the initial overshoot of the response (that is also of concern because of strict upper pressure limits) tends to be much bigger in the case of the LQ controller which is an expectable fact from practice.

5. CONCLUSIONS

The properties of two control design approaches for a nuclear power plant pressurizer system have been analyzed and compared in this paper. The MATI for the actually implemented dynamic inversion based controller was computed and a network based LQ controller have been designed and tested in simulations for the same system. Simulation results show that the variance properties of the output are better in the case of the LQ controller, if the operating conditions are similar.

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