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Two-level controller design for an active suspension system

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Abstract— **In this paper, the design of a two-level controller is proposed for active suspension systems. The required control force is computed by applying a high-level controller, which is designed using a linear parameter varying (LPV) method. The suspension structure contains nonlinear components, i.e. the dynamics of the dampers, the springs and the actuator dynamics. The actuator generating the necessary control force is a nonlinear state dependent switching system, for which a low-level backstepping-based force-tracking controller is designed. The low-level controller is designed by both the backstepping and the feedback linearization methods. The operation of the two-level controller is illustrated through simulation examples.**

I. Introduction

Active suspensions are used to provide good handling characteristics and to improve ride comfort while harmful vibrations caused by road irregularities and on-board excitation sources act upon the vehicle. One of the difficulties in the control design is that the different control goals are usually in conflict and a trade-off must be achieved between them. The suspension problem is analyzed in fundamental papers such as [5], [7].

Several methods have been proposed to design active suspension systems. The vast majority of the papers assume that the suspension system can be approximated by a linear model and the control system is designed by linear methods, see e.g. [11], [17]. Another and smaller part of the papers assume that nonlinearity in suspension systems is dominant and the linearity assumption is not valid in the entire operation domain. The dynamic characteristics of suspension components, i.e. dampers and springs, have nonlinear properties, and they are not time-invariant, but change during the vehicle life cycles, see e.g. [1], [9], [12]. Some of the papers assume that the nonlinearities of suspension systems can be hidden by scheduling signals, which are assumed to be measured or achieved, and a Linear Parameter Varying (LPV) model-based control design is proposed, see [4], [6].

In this paper, the design of a two-level controller is proposed for active suspension systems. In the design of a high-level controller passenger comfort, road holding and tire deflection are taken into consideration as performance outputs and the control input designed is the control force. The designed control force is a required force, which must be created by the hydraulic actuator. The required force is tracked by a lower-level controller by setting the valve of the actuator. The advantage of this method is that the actuator dynamics and the suspension dynamics are handled in two independent control design steps. The design of the high-level controller is based on the parameter-dependent LPV method. Nonlinear methods are proposed for the design of the low-level controller, with which the output tracking problem is solved. The operation of the backstepping controller is compared with the controller which is based on the feedback-linearization method.

The structure of the paper is as follows. In Section II the modeling of the active suspension system for control design is presented. In Section III the model is augmented with the performance specifications and the design of the high-level controller. In Section IV the design of a lower-level controller, which is based on the backstepping method and the feedback-linearization method is also presented. In Section V the operation of the two-level controller is demonstrated through simulation examples.

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II. The control-oriented modeling of the suspension systems

In Figure 1 a two-degree-of-freedom quarter-car model is shown. The body mass m_s represents the sprung mass, which corresponds to one of the corners of the vehicle, and the unsprung mass m_u represents the wheel at one corner. The parameters k_t, k_s, b_s are the tyre stiffness, the suspension stiffness, and the damping rate of the suspension, respectively. The control signal F is generated by the actuator. x_1 and x_2 denote the vertical displacement of the sprung mass and the unsprung mass, respectively. The disturbance d is caused by road irregularities.

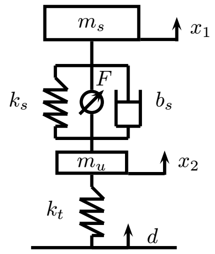

Fig. 1. Quarter-car model

In the modeling phase of the control design several specifications are used. The suspension structure is defined by the

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dynamics of the nonlinear components. The performance demands for ride comfort, road holding, suspension deflection, and input force are taken into consideration. The trade-off between performance specifications is defined by a nonlinear function. The model uncertainty is assumed to be in an output multiplicative structure.

The force equations of the quarter-car model are:

$$F_{m_s} = F_{k_s} + F_{b_s} - F, \quad (1)$$
$$F_{m_u} = -F_{k_s} - F_{b_s} - F_{k_t} + F, \quad (2)$$

where the forces from the sprung mass acceleration and the unsprung mass acceleration, the suspension damping force, the suspension spring force, the tire force, respectively, are as follows:

$$F_{m_s} = m_s\ddot{x}_1, \quad (3)$$
$$F_{m_u} = m_u\ddot{x}_2, \quad (4)$$
$$F_{b_s} = b_s^l(\dot{x}_2 - \dot{x}_1) - b_s^{sym}|\dot{x}_2 - \dot{x}_1| + b_s^{nl}\sqrt{|\dot{x}_2 - \dot{x}_1|}\,\mathrm{sgn}(\dot{x}_2 - \dot{x}_1), \quad (5)$$
$$F_{k_s} = k_s^l(x_2 - x_1) + k_s^{nl}(x_2 - x_1)^3, \quad (6)$$
$$F_{k_t} = k_t(x_2 - d), \quad (7)$$

and F is the force of the actuator. Here, parts of the nonlinear suspension damping b_s are b_s^l, b_s^{nl} and b_s^{sym}. The b_s^l coefficient affects the damping force linearly while b_s^{nl} has a nonlinear impact on the damping characteristics. b_s^{sym} describes the asymmetric behavior of the characteristics. Parts of the nonlinear suspension stiffness k_s are a linear coefficient k_s^l and a nonlinear one, k_s^{nl}.

The state vector x is selected as follows:

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T, \quad (8)$$

in which the components of the state vector x are the vertical displacement of the sprung mass x_1, the vertical displacement of the unsprung mass x_2, their derivatives $x_3 = \dot{x}_1$, $x_4 = \dot{x}_2$.

In the LPV modeling ρ parameters, which are directly measured or can be calculated from the measured signals, must be selected. In the LPV model of the active suspension system two parameters are selected. The relative velocity and the relative displacement are selected as scheduling parameters:

$$\rho_b = sgn(x_4 - x_3), \quad (9)$$
$$\rho_k = (x_2 - x_1)^2. \quad (10)$$

Parameter ρ_b depends on the relative velocity, parameter ρ_k is equal to the relative displacement. In practice, the relative displacement is a measured signal. The relative velocity is then determined by numerical differentiation from the measured relative displacement.

The nonlinear spring force in (6) can be reformulated in the following way:

$$F_{k_s}(\rho_k) = k_s^l(x_2 - x_1) + k_s^{nl}\rho_k(x_2 - x_1). \quad (11)$$

This force can be expressed by a linear combination of states allowing the force to have nonlinear ρ dependence. Similarly, the nonlinear damping force in (5) can be partitioned in the following way:

$$F_{b_s}(\rho_b) = b_s^l(x_4 - x_3) - b_s^{sym}\rho_b(x_4 - x_3) + b_s^{nl}\rho_b\sqrt{\rho_b(x_4 - x_3)}, \quad (12)$$

where the first and the second terms are the linear parts and the third term is the nonlinear part of the damping force.

The state space representation of the LPV model is as follows:

$$\dot{x} = A(\rho)x + hd + gu, \quad (13)$$

where $\rho = \begin{bmatrix} \rho_1 & \rho_2 \end{bmatrix}^T$ with $\rho_1 = \rho_k$, $\rho_2 = \rho_b$ and $u = F$.

The actuator which generates the necessary force for the suspension system is a four-way valve-piston system. The force balance of the actuator can be modeled by the equations: $F = A_P P_L$, where A_P is the area of the piston and P_L is the pressure drop across the piston with respect to the front and rear suspensions. The derivative of P_L is given by

$$\dot{P}_L = -\beta P_L + \alpha A_P\left(\dot{x}_2 - \dot{x}_1\right) + \gamma Q, \quad (14)$$

in which Q is the hydraulic load flow, α, β, γ are constants and $Q = sgn\left[P_S - sgn(x_v)P_L\right]x_v\sqrt{|P_S - sgn(x_v)P_L|}$, with the supply pressure P_S and the displacement of the spool valve x_v. The cylinder velocity acts as a coupling from the position output of the cylinder to the pressure differential across the piston. The displacement of the spool valve is controlled by the input to the servo-valve u:

$$\dot{x}_v = \frac{1}{\tau}\left(-x_v + u\right). \quad (15)$$

where τ is a time constant. It is assumed that during the operation $P_S > P_L$, which leads to the following equation:

$$Q = \begin{cases} x_v\sqrt{P_S - P_L}, & x_v \geq 0 \\ x_v\sqrt{P_S + P_L}, & x_v < 0 \end{cases} \quad (16)$$

which defines a state-dependent bimodal switching system for the actuator dynamics, see e.g. [2].

Let x_5 and x_6 denote P_L and x_v, respectively. Then, the actuator model can be written separately as

$$\dot{x}_5 = -\beta x_5 + \alpha A_P(x_4 - x_3) + \gamma Q, \quad (17)$$
$$\dot{x}_6 = -\frac{1}{\tau}x_6 + \frac{1}{\tau}u_a. \quad (18)$$

It is clear that x_3 and x_4 appear as external disturbances in the above equations.

III. The design of a high-level controller based on an LPV method

The performance signals in the control design problem are the passenger comfort (heave acceleration) ($z_a = \ddot{x}_1$), the suspension deflection ($z_s = x_s - x_u$), the wheel relative displacement ($z_t = x_u$) and the control force (F). Weighting functions $W_{p,az}$, $W_{p,sd}$, $W_{p,td}$ and $W_{p,F}$ are applied in order to keep the heave acceleration, suspension deflection, wheel travel, and control input, respectively, small over the desired operation range. These weighting functions chosen for performance outputs can be considered as penalty functions, i.e. weights should be large in a frequency range where small signals are desired and small where larger performance outputs can be tolerated. Thus, $W_{p,az}$ and $W_{p,sd}$ are selected as

$$W_{p,az}(\rho_k) = \phi_{az}(\rho_k) \cdot 0.5 \frac{\frac{s}{350}+1}{\frac{s}{10}+1}, \tag{19}$$

$$W_{p,sd}(\rho_k) = \phi_{sd}(\rho_k) \cdot \frac{\frac{s}{350}+1}{\frac{s}{10}+1}. \tag{20}$$

Here, it is assumed that in the low frequency domain disturbances at the heave accelerations of the body should be rejected by a factor of ϕ_a and at the suspension deflection by a factor of ϕ_d. The other weighting functions are selected as $W_{p,td} = 1$ and $W_{p,F} = 1 \cdot 10^{-3}$.

The trade-off between passengers comfort and suspension deflection is due to the fact that is not possible to keep them together simultaneously. A large gain ϕ_{az} and a small gain ϕ_{sd} correspond to a design that emphasizes passenger comfort. On the other hand, choosing ϕ_{az} small and ϕ_{sd} large corresponds to a design that focuses on suspension deflection. In the LPV controller ρ_k is the relative displacement between the sprung and the unsprung masses: $\rho_k = x_1 - x_2$. ρ_k is used to focus on minimizing either the vertical acceleration or the suspension deflection response, depending on the magnitude of the vertical suspension deflection.

The parameter dependence of the gains is characterized by the constants ρ_1 and ρ_2 in the following way:

$$\phi_{az}(\rho_k) = \begin{cases} 1 & \text{if } \; |\rho_k| < \rho_1 \\ \frac{1}{\rho_1 - \rho_2}(|\rho_k| - \rho_2) & \text{if } \; \rho_1 \leq |\rho_k| \leq \rho_2 \\ 0 & \text{otherwise} \end{cases}, \tag{21}$$

$$\phi_{sd}(\rho_k) = \begin{cases} 0 & \text{if } \; |\rho_k| < \rho_1 \\ \frac{1}{\rho_2 - \rho_1}(|\rho_k| - \rho_1) & \text{if } \; \rho_1 \leq |\rho_k| \leq \rho_2 \\ 1 & \text{otherwise} \end{cases}. \tag{22}$$

The purpose of the control design is to minimize the the induced $\mathcal{L}_2$ norm of the weighted LPV system $G_{\mathcal{F}_\mathcal{P}}$, with zero initial conditions, is defined as

$$\|G_{\mathcal{F}_\mathcal{P}}\|_\infty = \sup_{\rho \in \mathcal{F}_\mathcal{P}} \sup_{\|w\|_2 \neq 0, w \in \mathcal{L}_2} \frac{\|z\|_2}{\|w\|_2}. \tag{23}$$

The solution of an LPV problem is based on the set of infinite dimensional LMIs being satisfied for all $\rho \in \mathcal{F}_\mathcal{P}$, thus it is a convex problem, [3], [10], [16]. In practice, this problem is set up by gridding the parameter space and solving the set of LMIs that hold on the subset of $\mathcal{F}_\mathcal{P}$. The number of grid points depends on the nonlinearity and the operation range of the system. For the interconnection structure, $\mathcal{H}_\infty$ controllers are synthesized for 5 values of ρ_1 in a range $[-2, 2]$ and 5 values of ρ_2 in a range $[0, 1]$.

IV. The design of a low-level controller based on nonlinear methods

We assume that the reference for F (which is a linear function of x_5) is given by the linear controller. The goal is to asymptotically track this reference with the actuator dynamics. Two solutions are proposed to solve this problem. Since the actuator subsystem and the suspension subsystem form a cascade of a nonlinear and a linear system, the backstepping methodology is an appropriate choice for our control goal. Backstepping is a control Lyapunov function-based nonlinear controller design method [14]. We will use the notations of [15] where backstepping is presented from the viewpoint of the theory of interconnected passive systems. As a second alternative, the reference tracking will be solved by the exact linearization of the actuator dynamics. For this part, the notations of [8] will be used.

A. *Backstepping design for the actuator subsystem*

The model of the whole suspension and actuator system with zero disturbance is written in the following form

$$\dot{z} = Az + B\xi_1, \tag{24}$$

$$\dot{\xi}_1 = a_1(z, \xi_1) + b_1(\xi_1)\xi_2, \tag{25}$$

$$\dot{\xi}_2 = a_2(\xi_2) + b_2 u_a, \tag{26}$$

where $z = [x_1 \; x_2 \; x_3 \; x_4]^T$, $\xi_1 = x_5$, $\xi_2 = x_6$, and

$$a_1(z, \xi_1) = -\beta x_5 + \alpha A_P(x_4 - x_3), \tag{27}$$

$$b_1(\xi_1) = \begin{cases} \gamma\sqrt{P_S - x_5}, & \xi_2 = x_6 \geq 0 \\ \gamma\sqrt{P_S + x_5}, & \xi_2 = x_6 < 0 \end{cases}, \tag{28}$$

$$a_2(\xi_2) = -\frac{1}{\tau}x_6, \quad b_2 = \frac{1}{\tau}. \tag{29}$$

Let us assume that there exists a smooth feedback function $K(z)$ (possibly in LPV form) such that the closed loop system

$$\dot{z} = Az + BK(z) \tag{30}$$

is asymptotically stable with control Lyapunov function $V(z)$.

The backstepping design for the actuator subsystem can be performed in two steps. In the first step, let us consider ξ_2 as a virtual input and $y_1 = \xi_1 - K(z)$ as a virtual output. Since ξ_1 is not a manipulable input, we would like to construct a feedback that guarantees the tracking of $K(z)$ with ξ_1. It is reasonable therefore to define the tracking error to be linear and stable, i.e., $\dot{y}_1 = -k_1 y_1$, $k_1 > 0$. From this (using eqs. (24)–(25)), the desired time-function for ξ_2 can be computed as a nonlinear feedback of the form

$$\xi_{2,des} = \alpha_1(z, \xi_1) = \frac{1}{b_1(\xi_1)}[-a_1(z, \xi_1) + \frac{\partial K}{\partial z} \cdot (Az + B\xi_1) - k_1(\xi_1 - K(z))]. \tag{31}$$

In the second step, the following virtual output is defined: $y_2 = \xi_2 - \alpha_1(z, \xi_1)$. For the tracking error, a stable linear dynamics is also prescribed in this case: $\dot{y}_2 = -k_2 y_2$, $k_2 > 0$. Using eqs. (24)–(26), we can now express the physically manipulable actuator input u_a as a function of z and ξ in the following form

$$u_a = \alpha_2(z, \xi_1, \xi_2) = \frac{1}{b_2} \cdot [-a_2(\xi_2) + \frac{\partial \alpha_1}{\partial z} \cdot (Az + B\xi_1) + \frac{\partial \alpha_1}{\partial \xi_1}(a_1(z, \xi_1) + b_1(\xi_1)\xi_2) - k_2(\xi_2 - \alpha_1(z, \xi_1))]. \tag{32}$$

By applying the above design, the closed loop system will be asymptotically stable with control Lyapunov function $S(z) = V(z) + \frac{1}{2}y_1^2 + \frac{1}{2}y_2^2$ (see [14]). It is important to note that the obtained feedback law (32) is a state-dependent switching function because of the switching term $b_1(\xi_1)$ (see (28)).

Since the actual feedback law generated by the LPV controller is a rather complicated function of the state variables, and we do not know the road excitation disturbances in advance, the above controller design procedure cannot be implemented in its original theoretical form. Therefore in the next section we will consider the more realistic assumption, when the reference for x_5 is computed by the high level LPV controller, and for the trajectory tracking, the time derivatives of the reference signals are computed numerically.

The reference for x_5 computed by the LPV controller is denoted by $x_{5,ref}$. To simplify the forthcoming calculations, let us use the following notations

$$g_{a1}(x) = -\beta x_5 + \alpha A_p(x_4 - x_3), \quad (33)$$

$$f_{a1}(x_5) = \sqrt{P_S - x_5}, \quad (34)$$

$$f_{a2}(x_5) = \sqrt{P_S + x_5}. \quad (35)$$

This way, (17) can be written as

$$\dot{x}_5 = g_{a1}(x) + \gamma Q. \quad (36)$$

The required tracking error dynamics is defined as

$$\dot{x}_5 - \dot{x}_{5,ref} = -k_1(x_5 - x_{5,ref}) \quad \text{with} \quad k_1 > 0. \quad (37)$$

From (37) yield the following form:

$$\gamma x_6 f_{a1,2}(x_5) = -g_{a1}(x) + \dot{x}_{5,ref} - k_1(x_5 - x_{5,ref}). \quad (38)$$

The reference for x_6 is given by

$$x_{6,ref} = \begin{cases} \frac{-g_{a1}(x)+\dot{x}_{5,ref}-k_1(x_5-x_{5,ref})}{\gamma f_{a1}(x_5)} & \text{if} \quad x_6 \geq 0 \\ \frac{-g_{a1}(x)+\dot{x}_{5,ref}-k_1(x_5-x_{5,ref})}{\gamma f_{a2}(x_5)} & \text{if} \quad x_6 < 0 \end{cases}. \quad (39)$$

The tracking error dynamics for $x_{6,ref}$ is written as

$$\dot{x}_6 - \dot{x}_{6,ref} = -k_2(x_6 - x_{6,ref}) \quad \text{if} \quad k_2 > 0. \quad (40)$$

This gives

$$-\frac{1}{\tau}x_6 + \frac{1}{\tau}u_a - \dot{x}_{v,ref} = -k_2(x_6 - x_{6,ref}), \quad (41)$$

from which the following expression for the physical input u_a is deduced:

$$u_a = \frac{\frac{1}{\tau}x_6 + \dot{x}_{v,ref} - k_2(x_6 - x_{6,ref})}{1/\tau}. \quad (42)$$

In order to practically implement the control law, we need to compute the time derivatives of $x_{5,ref}$ and $x_{6,ref}$, which can be done in a number of ways depending on the measurement noise conditions and the required precision. In this method the controller parameters k_1 and k_2 determine the convergence speed of the virtual outputs.

B. Feedback linearization method

The purpose of feedback linearization is to transform a nonlinear input-affine system through a nonlinear coordinates transformation and a nonlinear state feedback to a linear and controllable system (see [8]). It is easy to see from eqs (17)-(18) that the relative degree of the actuator subsystem with input u_a and output x_5 is 2 around any point in the state space, since u_a appears explicitely only in the second derivative of x_5. Therefore, the system can be exactly linearized by applying an appropriate nonlinear state feedback.

Let $f_5(x)$ denote the right-hand side of (17) (i.e. $\dot{x}_5 = f_5(x)$). Furthermore, let us use the following notations

$$\xi_1 = x_5, \quad \xi_2 = \dot{\xi}_1 = \dot{x}_5$$

Then the time derivative of ξ_2 can be written as

$$\begin{aligned}\dot{\xi}_2 &= -\beta \dot{x}_5 + \alpha A(\dot{x}_4 - \dot{x}_3) + \gamma \dot{Q} \\ &= -\beta f_5(x) + \alpha A(\dot{x}_4 - \dot{x}_3) + \gamma \dot{Q}. \quad (43)\end{aligned}$$

Note that f_5 is a switching function, since it contains the hybrid dynamics of the actuator.

In case $x_6 \geq 0$ the time-derivative of Q with $f_5(x)$ is calculated as

$$\begin{aligned}\dot{Q} = &-\frac{x_6(-\beta x_5 + \alpha A(x_4 - x_3) + \gamma x_6\sqrt{P_S - x_5})}{2\sqrt{P_S - x_5}} \\ &- \frac{x_6\sqrt{P_S - x_5}}{\tau} + \frac{\sqrt{P_S - x_5}}{\tau}u_a \quad (44)\end{aligned}$$

Using (44) the time derivative of ξ_2 can be computed as

$$\begin{aligned}\dot{\xi}_2 = &-\beta(-\beta x_5 + \alpha A(x_4 - x_3) + \gamma x_6\sqrt{P_S - x_5}) \\ &+ \alpha A(\dot{x}_4 - \dot{x}_3) \\ &+ \gamma\Bigg(-\frac{x_6(-\beta x_5 + \alpha A(x_4 - x_3) + \gamma x_6\sqrt{P_S - x_5})}{2\sqrt{P_S - x_5}} \\ &- \frac{x_6\sqrt{P_S - x_5}}{\tau} + \frac{\sqrt{P_S - x_5}}{\tau}u_a\Bigg). \quad (45)\end{aligned}$$

The linearizing feedback can be calculated from (45) as

$$\begin{aligned}u_a = \Big(&-2\beta^2\sqrt{P_S - x_5}\tau x_5 + 2\beta\sqrt{P_S - x_5}\tau\alpha A x_4 \quad (46) \\ &-2\beta\sqrt{P_S - x_5}\tau\alpha A x_3 + 2\beta\tau\gamma x_6 P_S - 3\gamma x_6\tau\beta x_5 \\ &- 2\alpha A\sqrt{P_S - x_5}\tau\dot{x}_4 + 2\alpha A\sqrt{P_S - x_5}\tau\dot{x}_3 \\ &+ \gamma x_6\tau\alpha A x_4 - \gamma x_6\tau\alpha A x_3 + x_6^2\tau\gamma^2\sqrt{P_S - x_5} \\ &+2\gamma x_6 P_S - 2\gamma x_6 x_5 + +2\tau\sqrt{P_S - x_5}v\Big) / (2\gamma(P_S - x_5)).\end{aligned}$$

Similarly, the linearizing feedback can be calculated in case $x_6 < 0$ case as the following form:

$$\begin{aligned}u_a = \Big(&-2\beta^2\sqrt{P_S + x_5}\tau x_5 + 2\beta\sqrt{P_S + x_5}\tau\alpha A x_4 \quad (47) \\ &-2\beta\sqrt{P_S + x_5}\tau\alpha A x_3 + 2\beta\tau\gamma x_6 P_S + 3\gamma x_6\tau\beta x_5 \\ &- 2\alpha A\sqrt{P_S + x_5}\tau\dot{x}_4 + 2\alpha A\sqrt{P_S + x_5}\tau\dot{x}_3 \\ &- \gamma x_6\tau\alpha A x_4 + \gamma x_6\tau\alpha A x_3 - x_6^2\tau\gamma^2\sqrt{P_S + x_5} \\ &+2\gamma x_6 P_S + 2\gamma x_6 x_5 + 2\tau\sqrt{P_S + x_5}v\Big) / (2\gamma(P_S + x_5)).\end{aligned}$$

Assuming that we exactly know the disturbance variables x_3, x_4 and their derivatives, the linearized actuator dynamics reads $\dot{\xi}_1 = \xi_2$ and $\dot{\xi}_2 = v$, where ξ_1 is the controlled output

x_5 in the subsystem. With the linearizing feedback, the dynamics of the whole system is linear, where the actuator dynamics is a simple double integrator. It can be seen from (46)-(47) that the derivative of x_3 and x_4 is needed for the computation of the linearizing feedback.

Let us again denote the reference for x_5 by $x_{5,ref}$. Then the output tracking can be solved by the following linear feedback

$$v = \ddot{x}_{5,ref} - k_{f1}(\xi_2 - \dot{x}_{5,ref}) - k_{f2}(\xi_1 - x_{5,ref}), \quad (48)$$

where the controller parameters are $k_{f1}, k_{f2} > 0$.

C. Controller implementation and tuning

As it has been mentioned before, the derivatives of reference signals and state variables have to be computed for the implementation of the controllers described above. Knowing the open-loop and the desired closed-loop frequency response of the suspension system, it is a straightforward choice to use proper transfer functions of the form

$$H_d(s) = \frac{s}{k_d s + 1} \quad (49)$$

where k_d is a sufficiently small positive number. The effect of the approximation of derivatives and the choice of k_d in (49) for the reference tracking performance is not straightforward to quantify. The asymptotic stability of the closed loop system in the original theoretical case follows from the structure of the controller and the control Lyapunov function can be easily determined [15].

The first solution is to write an LPV state-space realization of the whole closed loop system and find a parameter-dependent or a parameter-independent Lyapunov function by trying to solve the corresponding set of LMIs (see, e.g. [13]). The second (more conservative) method uses the fact that the closed loop system is a standard feedback interconnection of two systems: the mechanical suspension subsystem together with the LPV controller and the linearized actuator subsystem together with the reference tracking controller. In this case, we can apply the well-known small-gain theorem (see, e.g. [15]) to prove the overall stability of the closed loop system.

The computation of the $\mathcal{L}_2$-gain of the controlled actuator subsystem is straightforward. Because of the reference tracking configuration, this $\mathcal{L}_2$-gain is expected to be around 1. The controller parameters in the feedback linearization method were $k_d = 10^{-3}$, $k_{f1} = 80$ and $k_{f2} = 1600$. In the backstepping method the controller parameters were set to $k_1 = k_2 = 20$.

V. Simulation examples

In this section the operation of the two-level controller is presented. The controlled systems are tested on a bad-quality road, on which four bumps disturb the vehicle motion: the bumps are 8 cm, 6 cm, 2 cm and 4 cm. The time responses of the wheel travel with the control force are illustrated in Figure 2. The solid line corresponds to the force required by the LPV controller in which the performance specifications are taken into consideration. The dashed lines illustrate the result of the controllers designed by feedback linearization and backstepping methods. The relative error of the tracking is below 5 % in these methods as it is shown in Figure 2.

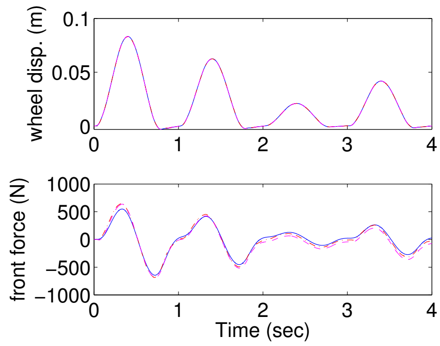

Fig. 2. Time responses of the required force

The realization of the control force by using feedback linearization method is illustrated in Figure 3. The figure shows the pressure drop across the piston, the displacement of the spool valve and the control signal, see equation (15). The realization of the control force by using backsteping method is illustrated in Figure 4.

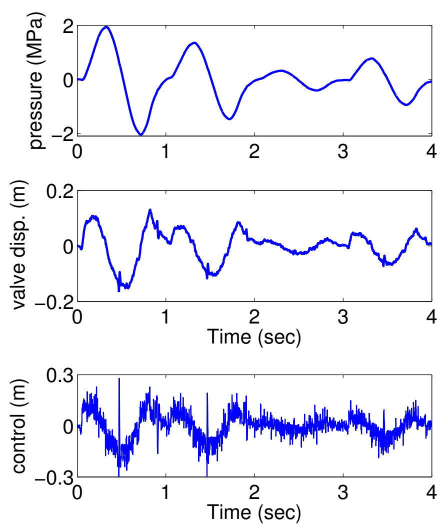

Fig. 3. Time responses of the actuator based on feedback linearization method

In the second example the tracking properties are tested in an uncertain case, i.e., when the parameters b_s^l, k_s^l and k_t are assumed to be uncertain and the percentage of the variation around their nominal value is 10 %. The solid line corresponds to the force required by the LPV controller in Figure 5. The dashed line illustrates the force of the controller designed by the backstepping method, while the dashed-dotted line illustrates the force of the controller designed by the feedback linearization method. As a result of the uncertainty the controller designed by the feedback linearization method generates an error relative to the required force. Although theoretically the linearization-based actuator

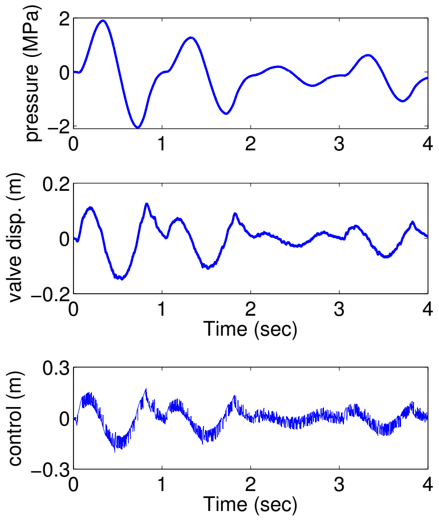

Fig. 4. Time responses of the actuator based on backstepping method

is able to handle the approximate derivatives of signals, it is sensitive in model uncertainties. The controller designed by the backstepping method is able to generate the required force even in uncertain cases, i.e., it is also able to handle the parametric uncertainties. As the handling of uncertainty is an important consideration we recommend the backstepping control method for carrying out the low-level task.

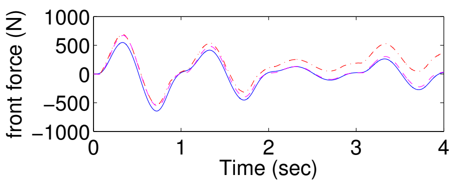

Fig. 5. Time responses of the controlled system with parametric uncertainties

VI. Conclusions

In this paper a two-level controller is proposed for the design of active suspension systems, one for the suspension and the other for the actuator. It is advantageous for several reasons. First, the the fast dynamics of the actuator and the relatively slow dynamics of the suspension can be handled independently. The performance requirements guaranteed by the controller in the upper level can be achieved by solving tracking task with the low level controller. Second, the complexity of the model does not increase significantly. Only the complexity of the upper level controller increases if the full-car model is applied and all of the performance specifications are taken into consideration such as minimization of the rolling during cornering, or minimization of the pitching during braking. At the same time the complexity of the low-level controllers is the same since they are designed by using the quarter-car model. The main advantage of the proposed solution is its ability to meet complex control performance criteria together with the handling of switching nonlinear actuator dynamics.

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