Modeling and Identification of a Nuclear Reactor with Temperature Effects and Xenon Poisoning

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Abstract

This paper presents the modeling and identification procedure for a VVER-type pressurized water reactor. The modeling goal is to produce a mathematical description in nonlinear state-space form that is suitable for control-oriented model analysis and preliminary controller design experiments. The proposed model takes temperature effects and Xenon poisoning into consideration and thus it is an extension of formerly published simpler model structures. Real transient measurement data from the plant has been used for the identification that is based on standard prediction error minimization. It is shown that the model is fairly well identifiable and the newly inserted model components significantly improve the quality of fit between the measured and computed model outputs. Furthermore, the estimated parameter values fall into physically meaningful ranges.

Keywords: process modeling, parameter estimation, nuclear reactors, power systems

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1 Introduction

The gradually changing operating requirements, the ever stringent regulations related to performance, effectiveness and safety often necessitate the re-tuning or the re-design of different subsystems in nuclear power plants. This is particularly important in the case of elderly plants, such as the Paks Nuclear Power Plant (NPP) in Hungary, where the refurnishment of the control system is necessitated by the expiring of its originally designed lifetime. The Paks NPP operates four pressurized water (VVER-440/213 type) reactors with a total nominal electrical power of 1860 MWs.

The above mentioned important re-tuning or re-design tasks are supported by the improving quantity and quality of dynamic measurements as a result of developing hardware-software environment and modern sensor devices. In addition, it is well known from theory and engineering practice that the application of advanced feedback control can dramatically improve dynamical system properties often without the need to introduce significant changes in the technology.

Earlier efforts in developing advanced control schemes for nuclear reactors of different type, or for other key equipments of nuclear power plants focus on the application of soft-computing technologies (e.g. neural networks, fuzzy controllers) that do not need a reliable dynamic model of the plant to be controlled (see e.g. [3], [4], [18]).

For the dynamic analysis or controller synthesis, however, reliable dynamic models are needed where the level of detail and descriptive capability very much depend on the exact modeling goal [12]. Most modern controller design methods and the corresponding analysis techniques require that the mathematical model of the system is in the form of (a preferably low number of) ordinary differential equations [14]. Unfortunately, the traditionally available and commonly used dynamic models for nuclear power plants are much too complex and detailed for control purposes, see e.g. the papers [11, 15, 2, 21] applicable for NPPs with pressurized water reactors.

There are a few papers in the literature that use simple low dimensional dynamic models to design controllers or to analyze dynamic properties of the reactor or other equipments, such as the steam generator in nuclear power plants, see e.g. [17], [22]. However, the models and the dynamic properties of the modelled system are much dependent on the type of the nuclear power plant they belong to, that is, on the fact whether the plant operates pressurized water reactors (PWRs) or boiling water reactors, etc. The studies about the integrated control of pressurized water reactors (PWRs) are often based on linear state-space ([5]) or input-output models ([19]). However, these models usually do not give insight into the most important physical processes and sub-systems.

Therefore, a simple dynamic model in physical coordinates and the corresponding parameter estimation procedure for the primary circuit dynamics of VVER-type pressurized water reactors was presented in [9]. The primary uses of this model are control oriented dynamic model analysis and high level controller design. However, the reactor sub-model of this primary circuit model was too much simplified that has caused a mismatch in the predicted and observed signals related to the nuclear reactor itself. In this paper our aim is to extend this reactor model with temperature feedbacks and the dynamic of delayed neutron emitting nuclei. Beside the new model structure, the parameter estimation procedure is also presented using measured plant data.

The paper is organized as follows. In the second section, the extended model in state space form is presented. The third section describes the parameter estimation method and the measurements. The fourth section contains the results of the parameter estimation, while the conclusions can be found in the fifth section.

2 Reactor model

There are a few simple dynamic models of nuclear reactors reported in the literature that qualify to be suitable for control-oriented studies (see [6], [13]) but they have mainly been developed for training and/or demonstration purposes and they are not validated against real measured data. Below a simple dynamic model of the nuclear reactor in a pressurized water NPP is presented based on first engineering principles.

2.1 An oversimplified model

Earlier, we developed different versions of a dynamic model for the primary circuit of a VVER-type nuclear power plants [8, 9, 10]. The domain of these former models included the dynamic behavior in normal operating mode together with the load changes between the day and night periods, which is approximately the $80-100\%$ thermal power range. The reactor sub-model of our primary circuit model was a time-dependent, point kinetic model with a single type of delayed neutron emitting nuclei whose concentration was in a quasi steady-state [7, 16]. The effect of the control rod position on the reactivity was approximated by a quadratic function, i.e. the original simple reactor model was the following:

$$\frac{dN}{dt} = \frac{p_1 z^2 + p_2 z + p_3}{\Lambda} N + S \tag{1}$$

where N [%] is the neutronflux, z $[m]$ is the control rod position, S $[\%/s]$ is a virtual neutron source and p_i, $i = 1, 2, 3$ are estimated parameters. This model suffers from the following shortcomings.

- The neutron flux is independent of the temperatures, i.e. it does not contain a temperature feedback from the temperature of the moderator and/or the fuel.
- To be able to reproduce steady-states correctly with only one differential equation, a virtual neutron source term S has been included. This is accepted and used in the literature, but in a more detailed model the introduction of other elements with clear physical meaning would be desirable.
- The concentrations of the delayed neutron emitting nuclei are assumed to be in quasi steady-state, that is far from being realistic.
- The model does not describe the measured trends in the neutron flux in the neighborhood of the steady states and it has some inaccuracies when simulating load increase following a load decrease transient.

Process knowledge suggests that taking into consideration some additional physical details, a more accurate model could be obtained with a bit higher but still manageable number of state variables. Therefore, we extend the simple reactor model in Eq. (1) with temperature feedbacks, the dynamics of delayed neutron emitting nuclei and Xenon poisoning.

2.2 Modeling Assumptions

In order to have a low order dynamic model of the reactor the following simplification assumptions have been made.

- R1 The reactor is considered as a spatially homogeneous lumped parameter system. Therefore, the physical reactor model is a time-dependent, non-linear single-group model [16].
- R2 The dynamic model of the reactor is derived from the point kinetic equations.
- R3 Only a single "average" group of the delayed neutron emitting nuclei is assumed.
- R4 The reactor is composed of the fuel, the moderator and the control rod as modelling elements (balance volumes).
- R5 The reactivity dependence on the rod position is assumed to be quadratic.
- R6 The reactivity dependence on the temperatures is assumed to be linear.
- R7 The reactivity dependence on the Xenon concentration is assumed to be linear.
- R8 The boron concentration is regarded to be constant during the simulation together with the reactivity coefficients.
- R9 The mass flow rate of the moderator is assumed to be constant.
- R10 The heat loss of the reactor is neglected.

The input of the model is the control rod position z $[m]$ and the temperature of the water entering the reactor T_{in} $[°C]$. The outputs of the model are the neutron flux N $[\%]$ and the average temperature of the moderator

T_m [°C]. It is important to note that both the inputs and the outputs are measured variables.

The applied variables and the parameters with their units and definitions can be found in Table 1. The nominal values of the variables at the 100% operation mode are denoted by a subscript $_0$.

We have to note that the structure of the fuel rod is assumed to be homogeneous, i.e. the mass and the specific heat of the revetment of the fuel rod are taken into account in the mass and the specific heat of the fuel, respectively. The reason of this simplification is that the change of temperature of the revetment is proportional to the changes of the temperature of the fuel.

The rod effect has been modeled by a quadratic function. A widely used sinusoidal rod effect function might be more accurate, but its parameter dependence is less advantageous from an identification point of view.

Only a single averaged group of delayed neutron emitting nuclei has been assumed after our preliminary analysis of a model version with six groups [8], where we have found that their effects were negligible. The effective decay constant has been calculated as follows [7]:

$$\lambda_C = \frac{\sum_{i=1}^{6} \beta_i}{\sum_{i=1}^{6} \frac{\beta_i}{\lambda_{C,i}}} \tag{2}$$

2.3 Model Equations

The state equations are derived from the dynamic conservation balances for the neutron and delayed neutron emitting nuclei, as well as for the internal energy of the fuel and the moderator.

2.3.1 Neutron Dynamics

According to the assumptions R1, R2 and R3, the neutron dynamics and the delayed neutron emitting nuclei dynamics are described by the following equations [16]:

$$\begin{aligned} \frac{dN}{dt} &= \beta \frac{N}{\Lambda}\left(\rho - 1\right) + C\frac{\beta}{\Lambda} \\ \frac{dC}{dt} &= \lambda_C (N - C) \end{aligned} \tag{3}$$

Table 1: Variables and parameters

Identifier	**M. u.**	**Definition**
$z(t)$	m	Rod position
$N(t)$	$\%$	Neutron concentration
$C(t)$	$\%$	Concentration of the delayed neutron emitting nuclei
$n_I(t)$	cm^{-3}	Iodine concentration
$n_X(t)$	cm^{-3}	Xenon concentration
$\rho(t)$	$\$$	Reactivity
$T_f(t)$	$^\circ C$	Temperature of the fuel
$T_m(t)$	$^\circ C$	Average temperature of the moderator
$T_{out}(t)$	$^\circ C$	Temperature of the water leaving the reactor
$T_{in}(t)$	$^\circ C$	Temperature of the water entering the reactor
ϕ_0	$cm^{-2}s^{-1}$	Initial equilibrium neutron flux
Λ	s	Average generation time
Σ_f	cm^{-1}	Macroscopic fission cross section
β	-	Fraction of delayed neutron group
λ_C	s^{-1}	Decay constant of the delayed neutron emitting nuclei
λ_I	s^{-1}	Decay constant of Iodine
λ_X	s^{-1}	Decay constant of Xenon
σ_X	cm^2	Microscopic absorption cross section
Y_I	-	Iodine yield
Y_X	-	Xenon yield
α_f	$\$/^\circ C$	Temperature coefficient of the fuel
α_m	$\$/^\circ C$	Temperature coefficient of the moderator
A	m^2	Area of a fuel rod
U	$Wm^{-2}K^{-1}$	Heat transfer coefficient between the fuel and the moderator
$M_f c_{pf}$	J/K	Heat capacity of the fuel
$M_m c_{pm}$	J/K	Heat capacity of the moderator
m_p	kg/s	Mass flow rate of the moderator
F	$W/\%$	Reactor heat power per 1% of neutron flux
p_0, p_1, p_2	$\$$, $\frac{\$}{m}$, $\frac{\$}{m^2}$	Rod parameters

The reactivity depends on the temperatures, the control rod position and the Xenon concentration (assumptions R5, R6 and R7) [4]:

$$\begin{aligned}\rho &= \alpha_f(T_f - T_{f0}) + \alpha_m(T_m - T_{m0}) \\ &\quad + p_2 z^2 + p_1 z + p_0 \\ &\quad + \frac{\sigma_X}{\beta \Sigma_f}(n_X - n_{X0}) \end{aligned} \tag{4}$$

where $\alpha_f(T_f - T_{f0})$ describes the temperature feedback of the fuel, $\alpha_m(T_m - T_{m0})$ describes the temperature feedback of the moderator, $p_2 z^2 + p_1 z + p_0$ is the effect of the rod to the reactivity and $\frac{\sigma_X}{\beta \Sigma_f}(n_X - n_{X0})$ is the effect of the Xenon.

2.3.2 Equations of Thermodynamics

Energy balances are constructed for the fuel and the moderator (assumption R4). The energy balance for the *fuel* is

$$M_f c_{pf} dT_f = -UA(T_f - T_m)dt + FNdt \tag{5}$$

where $M_f c_{pf} dT_f$ is the inner energy change of the fuel due to the temperature change, $-UA(T_f - T_m)dt$ is the transferred heat to the moderator and FN is the heat power of the reactor per unit time. To describe the temperature of the fuel Eq. (5) is transformed to the

$$\frac{dT_f}{dt} = -\frac{UA}{M_f c_{pf}}(T_f - T_m) + \frac{F}{M_f c_{pf}} N \tag{6}$$

form.

The energy balance for the *moderator* is

$$\begin{aligned}M_m c_{pm} dT_m &= UA(T_f - T_m)dt + m_p c_{pm} T_{in} dt - \\ &\quad - m_p c_{pm} T_{out} dt \end{aligned} \tag{7}$$

where $M_m c_{pm} dT_m$ is the inner energy change of the moderator due to the temperature change, $m_p c_{pm} T_{in} dt$ is the energy of the inlet mass flow of the moderator and $m_p c_{pm} T_{out} dt$ is the energy of the outlet mass flow of the moderator (assumption R9). Applying a similar transformation as before we obtain that

$$\frac{dT_m}{dt} = \frac{UA}{M_m c_{pm}}(T_f - T_m) - \frac{m_p}{M_m}(T_{out} - T_{in}) \tag{8}$$

Let us group the parameters and introduce the following notations

$$A_1 = \frac{UA}{M_f c_{pf}} \qquad A_2 = \frac{F}{M_f c_{pf}} \tag{9}$$

$$A_3 = \frac{UA}{M_m c_{pm}} \qquad A_4 = \frac{m_p}{M_m} \tag{10}$$

With this we can transform (6) and (8) into the following form

$$\frac{dT_f}{dt} = -A_1(T_f - T_m) + A_2 N \tag{11}$$

$$\frac{dT_m}{dt} = A_3(T_f - T_m) - A_4(T_{out} - T_{in}) \tag{12}$$

However, the steady state determines the following relationships among these parameters:

$$\begin{aligned} 0 &= -A_1(T_{f0} - T_{m0}) + A_2 N_0 \\ 0 &= A_3(T_{f0} - T_{m0}) - A_4(T_{out0} - T_{in0}) \end{aligned}$$

From these we can express A_2 and A_4 as:

$$\begin{aligned} A_2 &= \frac{A_1(T_{f0} - T_{m0})}{N_0} \\ A_4 &= \frac{A_3(T_{f0} - T_{m0})}{(T_{out0} - T_{in0})} \end{aligned}$$

Furthermore, the expression $T_{out0} - T_{in0}$ in the denominator of A_4 can be transformed into $2\,(T_{m0} - T_{in0})$ using the equation $T_{m0} = \frac{T_{out0}+T_{in0}}{2}$.

2.3.3 Equations of Poisoning

In power reactors we cannot neglect the Xenon poisoning, because the duration of load changes (4-6 hours) between the day and night is close to the half life times of the Xenon (9 hours) and the Iodine (6 hours). The concentration change of Xenon is also relevant in shorter terms, because it has great microscopic absorption cross section. To describe the concentration change of Xenon we also have to describe the Iodine concentration [7, 16]:

$$\frac{dn_I}{dt} = Y_I \Sigma_f \frac{N}{N_0}\phi_0 - \lambda_I n_I \tag{13}$$

$$\begin{aligned} \frac{dn_X}{dt} &= Y_X \Sigma_f \frac{N}{N_0}\phi_0 + \lambda_I n_I - \lambda_X n_X \\ &\quad - \sigma_X n_X \frac{N}{N_0}\phi_0 \end{aligned} \tag{14}$$

where $Y_I \Sigma_f \frac{N}{N_0}\phi_0$ and $Y_X \Sigma_f \frac{N}{N_0}\phi_0$ is the effect of the Uranium fission, $\lambda_I n_I$ is the Iodine atoms decay to Xenon in one cm^3 per time unit, $-\lambda_X n_X - \sigma_X n_X \frac{N}{N_0}\phi_0$ is the decrease of the Xenon concentration because of the decay and neutron absorption. From now on, we use the following simplifications in the notations: $X = n_X/\Sigma_f$ and $I = n_I/\Sigma_f$.

2.4 State Space Form

In order to be able to apply standard identification and controller design methods, the above engineering model equations have been transformed into a state-space model form.

State equations

$$\frac{dN}{dt} = \beta\frac{N}{\Lambda}\Bigg(\alpha_f(T_f - T_{f0}) + \alpha_m(T_m - T_{m0}) + + p_2 z^2 + p_1 z + p_0 + \frac{\sigma_X}{\beta}(X - X_0) - 1\Bigg) + C\frac{\beta}{\Lambda} \quad (15)$$

$$\frac{dC}{dt} = \lambda_C(N - C) \quad (16)$$

$$\frac{dT_f}{dt} = -A_1(T_f - T_m) + A_1\frac{T_{f0} - T_{m0}}{N_0}N \quad (17)$$

$$\frac{dT_m}{dt} = A_3(T_f - T_m) - A_3\frac{T_{f0} - T_{m0}}{T_{m0} - T_{in0}}(T_m - T_{in}) \quad (18)$$

$$\frac{dI}{dt} = Y_I\frac{N}{N_0}\phi_0 - \lambda_I I \quad (19)$$

$$\frac{dX}{dt} = Y_X\frac{N}{N_0}\phi_0 + \lambda_I I - \lambda_X X - \sigma_X X\frac{N}{N_0}\phi_0 \quad (20)$$

Output equation

$$y = [N, T_m]^T \quad (21)$$

3 Parameter identification

This section is devoted to the parameter identification including the description of the measurements and that of the parameter estimation method.

3.1 Measurements

Measured data from unit 1 of the Paks Nuclear Power Plant in Hungary were collected for parameter estimation purposes. To extract as much dynamic information as possible, load change periods were selected for identification.

The measured data that are needed for the identification included the neutron flux N, and the average temperature of the moderator T_m as outputs, the rod position z and the temperature of the inlet moderator T_{in} as inputs. The data source was the Verona system (see [2]) that is a reactor monitoring system storing also reactor data. The stored values are uniformly sampled, the sampling time is 10 s.

The time-span of the raw measurements was between 2 and 72 hours. The selected data sequences had to contain steady state values together with power increase/decrease without any significant disturbances and operating mode changes. After the investigation of the measured data, a time interval of 7 hours was chosen for parameter estimation.

It is important to note that these data are passive measured data from the nuclear power plant under closed-loop control where the excitation was provided only by the load changes.

3.2 Identification Method

First, the parameters of the model have been grouped based on the knowledge of their values.

- *Known parameters.* They are the parameters of nuclear processes, β, Y_I, Y_X, λ_C, λ_I, λ_X. The value of these parameters is in principle known from the literature, but because of the model simplification, the estimated value can be different from the literature value. We accept $\pm 5\%$ differences.

- *Partially known parameters.* A reliability domain is given to their value, they are ϕ_0, σ_X, α_m, α_f, Λ, A_1, A_3 and the rod parameters.

The value of these parameters have been estimated from measurements using a *constraint* during the estimation: the estimated value of the parameters

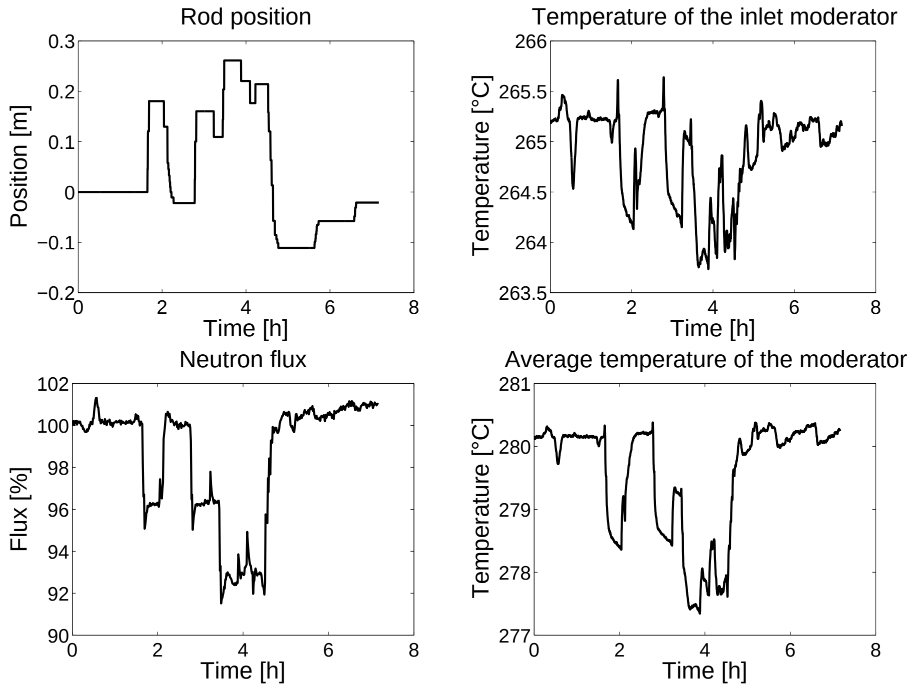


Figure 1: The measured signals in unit 1.

must be in their *reliability (physically meaningful) domain*. The domains were determined using available technical documentation and through consultations with reactor operation experts of the plant and the Institute of Nuclear Techniques, Budapest University of Technology and Economics. Table 2 shows the reliability domain of the estimated parameters in a separate column.

In addition to the parameters, the initial value of the non-measured state variables, such as the concentrations of the delayed neutron emitting nuclei C, the Xenon X and Iodine I, as well as the fuel temperature T_f has also been estimated, i.e. they were considered as additional partially known parameters. Particularly, $X(0)$ and $I(0)$ have a significant long term effect on the system dynamics that is not vanishing through the whole examined operation interval.

The above parameter estimation problem is basically an optimization problem with objective function f_{obj} which is bound constrained to keep some estimated parameter values in a physically meaningful range.

For the evaluation of f_{obj}, the simulation of the system dynamics with some parameter vector θ is required which is a computationally expensive operation. This means that the numerical approximation and evaluation of the gradient of f_{obj} requires much computational effort and moreover, it can often be unreliable because of the noise of some measurements. These facts motivated us to choose a simple yet effective numerical optimization method that does not need the computation of the gradient of the objective function.

The Parameter Estimation Tool of the Matlab is applied to implement the identification algorithm. The applied identification method is the Pattern search/Nelder-Mead search method. It is an optimization-based parameter estimation method based on the Nelder-Mead simplex method [20]. The objective function is the SSE (sum of square of error) which measures the data fit in terms of the 2-norm between the measured and the model-computed output signals, i.e.

$$f_{obj} = \sqrt{\frac{\int_0^T (\hat{N}(t) - N(t))^2 dt}{\int_0^T N^2(t) dt} + \frac{\int_0^T (\hat{T}_m(t) - T_m(t))^2 dt}{\int_0^T T_m^2(t) dt}} \tag{22}$$

where N and T_m are the measured outputs, $\hat{N}$ and $\hat{T}_m$ are the model-computed (simulated) output signals and T denotes the time-span of the measurement/simulation.

The brief operational principle of the Nelder-Mead algorithm is the following (for details, see [20]). A simplex is the convex hull of $n+1$ vertices in an n-dimensional space. The method starts from an initial working simplex which is created using the given initial parameter value. The algorithm then performs a sequence of transformations (that can be reflection, expansion, contraction or shrink) of the working simplex, to decrease the objective function values at the vertices. The algorithm is terminated when the size of the simplex is sufficiently small, or when the function values at the vertices are close to each other in some norm. In each iteration step, the algorithm typically needs only one or two objective function evaluations which is quite low compared to most other methods.

It is important to note that the simplex search algorithm (similarly to many nonlinear optimization techniques) does not guarantee that the obtained point is a global minimum on the whole parameter domain. Therefore it is very important to use as much prior information about the modeled process as possible to choose proper initial parameter values for the method.

To use the simplex method, suitable initial values are needed. They are taken from [1] and from the discussion with nuclear power plant experts.

4 Results

The parameter estimation was based on the state-space model equations (15)-(20). The input variable to the model was the rod position $z(t)$ and the input temperature $T_{in}(t)$, the model output variables were the neutron flux $N(t)$ and the moderator temperature $T_m(t)$.

The values of the parameters β, Y_I, Y_X, λ_C, λ_I and λ_X were assumed to be known from the literature, while A_1, A_3, α_f, α_m, ϕ_0, σ_X and Λ were estimated using Nelder-Mead simplex algorithm.

The applied measured signals can be seen in Fig. 1. We have to note that the rod position is measured as the difference between the nominal position of the rod and the current rod position. If the rod is inserted from there, then the rod position becomes positive.

The estimated parameter values can be seen in Table 2, while the neutron flux and the temperature of the moderator fitting can be seen in Fig. 2 and Fig. 3, respectively.

The quality of the parameter estimation is investigated by the analysis of normalized error function as a function of each parameter, and by the analysis of the normalized error function as a function of some pairs of parameters. The normalized error function is defined as in Eq. (22). Some typical results can be seen in Figs. 4 and 5. The circle shows the estimated values in Fig. 4.

4.1 Discussion

The analysis of the fit In Fig. 2 one can see the measured neutron flux, together with the simulated one of our new model.

The steady states are reproduced very well, and the the dynamics of the measurements are also described in an excellent way. This is particularly

Table 2: Values of estimated parameters and known constants

Identifier	**Estimated value**	**Acceptable domain**
ϕ_0	$1.3 \cdot 10^{13}$	$[10^{13}, 10^{14}]$
σ_X	$2.805 \cdot 10^{-18}$	$[2.8 \cdot 10^{-18}, 3.2 \cdot 10^{-18}]$
α_f	$-5.362 \cdot 10^{-3}$	$[-5.5 \cdot 10^{-3}, -3.8 \cdot 10^{-3}]$
α_m	$-2.075 \cdot 10^{-2}$	$[-3.5 \cdot 10^{-2}, -1.8 \cdot 10^{-2}]$
A_1	0.1056	$[0.1, 1]$
A_3	0.8757	$[0.1, 1]$
p_0	0.0401	$[-0.1, 0.1]$
p_1	-0.44	$[-1, -0.1]$
p_2	-0.966	$[-1, -0.1]$
Λ	$2.18 \cdot 10^{-5}$	$[1.5 \cdot 10^{-5}, 3.5 \cdot 10^{-5}]$
λ_I	$2.849 \cdot 10^{-5}$	
λ_X	$2.150 \cdot 10^{-5}$	
λ_C	$7.728 \cdot 10^{-2}$	
β	0.0065	
Y_I	$6.39 \cdot 10^{-2}$	
Y_X	$2.2 \cdot 10^{-3}$	

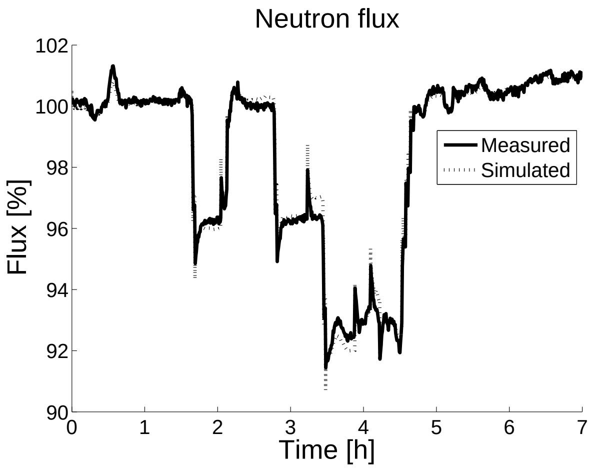


Figure 2: The measured and the simulated neutron flux in unit 1.

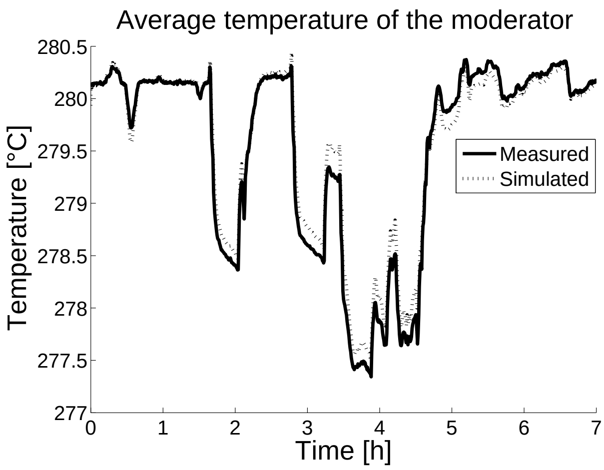
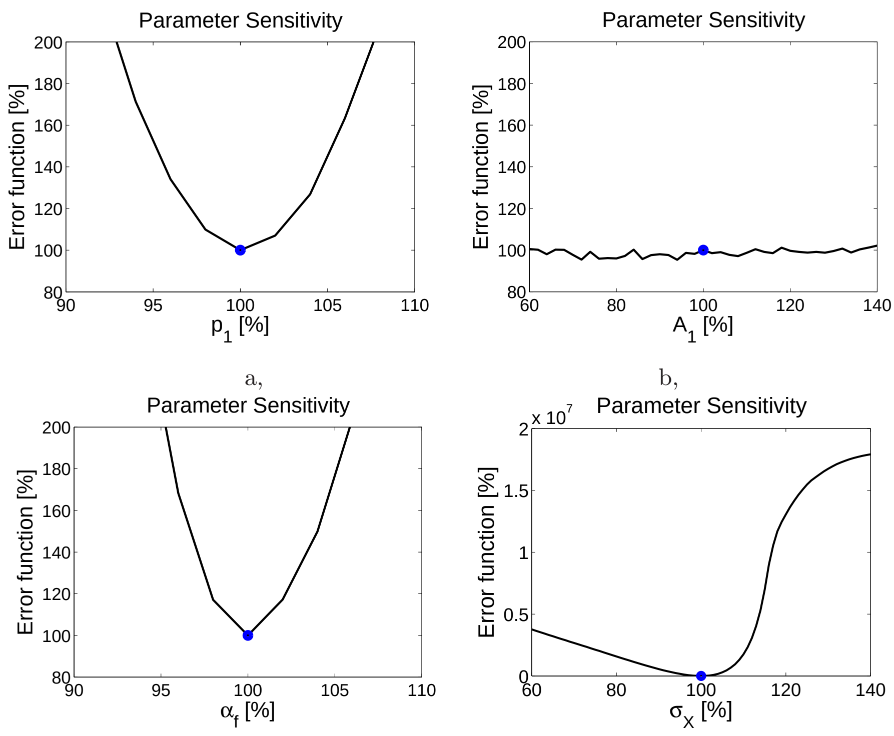


Figure 3: The measured and the simulated temperature of the moderator in unit 1.

visible at the end of the power increase after 5 hours of examined operation, where the drifting is caused by the poisoning [9]; here our model fits well, too. In addition, the time constants of the new model correspond well to the dynamics of the real system shown by the measurements.

The fit of the moderator temperature (see Fig. 3.) is a bit worse, but the maximal error is below 0.2^oC that is well within the acceptable range. Here again, the dynamics of the system shown by the measurements are reproduced very well.



a, b, c, d,

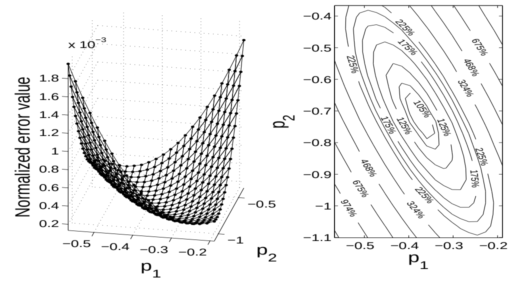
Figure 4: Normalized error values vs. parameter. Circles show the estimated values.

Analysis of the estimated values The estimated values are acceptable, they are in their physically meaningful reliability domains.

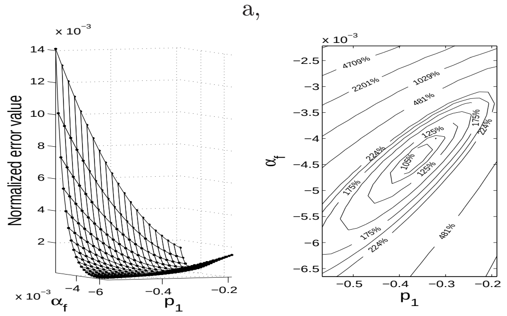
The parameters can be classified based on the *shape of the error function* as follows.

- p_0, p_1, p_2, α_f, α_m (Fig. 4 a, and c,).
  The error value as a function of the parameter is similar to a quadratic function, therefore a unique minimum exists. One can see that the estimated parameter values are close to the minimum value.

- A_1, A_3 (Fig. 4 b,).
  The error value function as a function of the parameter is close to a constant. It contains a lot of local minima but their values are similar. This means that these parameters cannot be estimated properly, the result is not sensitive to these parameters.



a,



b,

Figure 5: Normalized error values vs. two parameters and their contours.

- σ_X (Fig. 4 d,).

  The error value function is asymmetric. On one side its gradient is high, while in the other side it is much lower. A unique minimum exist, that can be relatively well determined.

Based on the analysis of the *error function as a function of two parameters* (see Fig. 5) one can see that the estimation of some variable pairs are strongly correlated. For example, one can see from Fig. 5 a, and b, that there is a unique minimum of the error function as a function of p_1 and p_2, as well as that of p_2 and α_m but there is a correlation between these two parameters, i.e. they cannot be estimated totally independently.

The uncertainty of the well-identifiable parameter estimates was also studied by analyzing the parameter dependence of the error function. For those parameters (and initial conditions), where the shape of the error function permitted, the minimal and maximal parameter values corresponding to the 110% of the minimal error function value were determined. Fig. 6 shows the results for α_f, α_m, p_0 and σ_x. The obtained intervals (which were

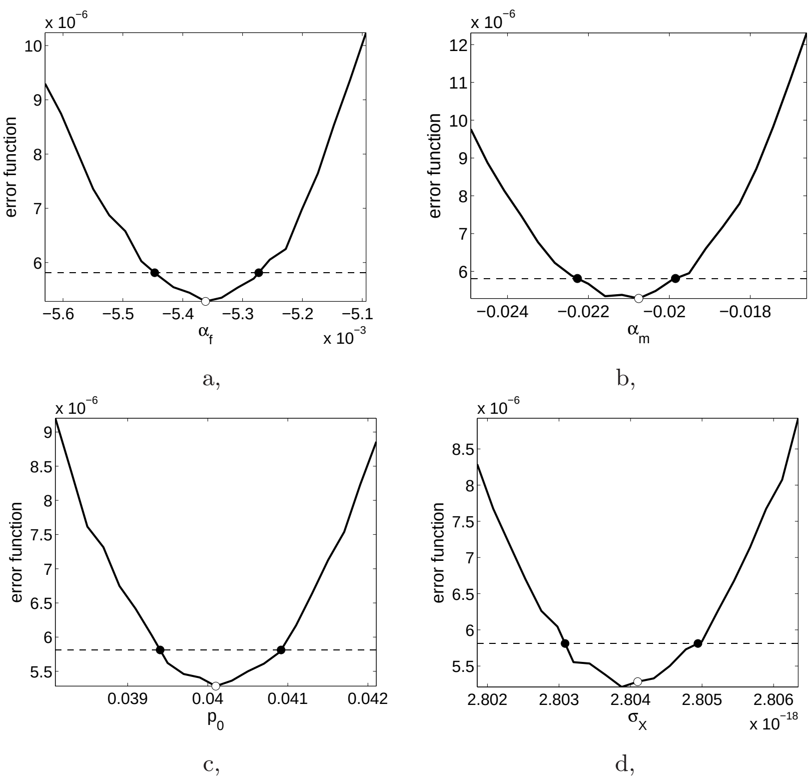


Figure 6: Error function dependence on parameters. Empty circles show the estimated values, the horizontal dashed lines show 110% of the minimal error.

symmetrized by a simple averaging) together with the estimated parameter values and initial conditions are shown in Table 3. It is clearly visible from the table that the reliability of the examined parameters is fairly good.

4.2 Model validation

For the validation of our model a time interval of 4 hours was chosen. The measured data was also collected in unit 1. and close to the data used for the identification. It is important because the values of the parameters change through the campaign (the time period in a nuclear power plant between two fuel-changes). The change of the moderator temperature coefficient (α_m) is significant during the typically 48 weeks of full power operation. During this time the boron concentration decreases from its original value to zero, and

Table 3: Sensitivity of parameters and estimated initial values

Identifier	**Estimated value**	**Symmetrized 110% interval**
ϕ_0	$1.3018 \cdot 10^{13}$	$\pm 2.6 \cdot 10^9$
σ_X	$2.8041 \cdot 10^{-18}$	$\pm 9.25 \cdot 10^{-22}$
α_f	$-5.362 \cdot 10^{-3}$	$\pm 1 \cdot 10^{-4}$
α_m	$-2.018 \cdot 10^{-2}$	$\pm 1.3 \cdot 10^{-3}$
p_0	0.0401	$\pm 7.8 \cdot 10^{-4}$
p_1	-0.44	$\pm 10^{-2}$
p_2	-0.976	$\pm 4.4 \cdot 10^{-2}$
T_{f0}	$6.19 \cdot 10^2$	± 6.1
Xe_{init}	$1.49 \cdot 10^{16}$	$\pm 2.9 \cdot 10^{12}$
I_{init}	$2.927 \cdot 10^{16}$	$\pm 9.7 \cdot 10^{12}$

this causes the decrease of α_m from about -0.017 to -0.076. Certain other parameters (β, rod parameters, α_f) are also changing slightly during the campaign because the components of the fuel are changing, but their effect on the model quality is less significant. In summary, we can say that the reactor model can be considered time-invariant within a few weeks interval.

We only used an initial value estimation of the Xenon and Iodine concentrations for the validation. In Fig. 7 one can see the measured neutron flux and the measured temperature of the moderator together with the simulated one. While the simulated neutron flux fits very well to the measured data, on the temperature graph, a maximum of $0.2°C$ difference can be seen.

5 Conclusions

A new extended reactor model for the control oriented modeling of the primary circuit of a nuclear power plant has been presented in this paper that describes the dynamic behaviour of the reactor under normal operating conditions and load changes. It is important to emphasize, however, that the

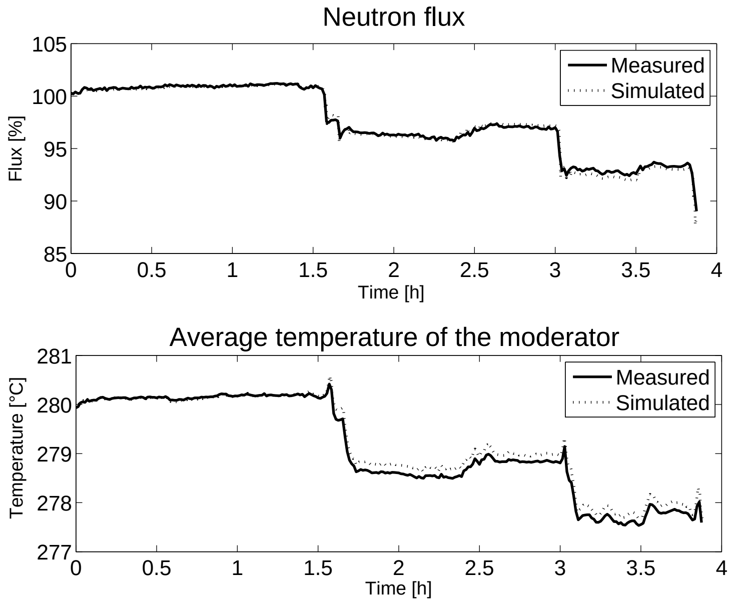


Figure 7: The measured and the simulated neutron flux in unit 1.

model is not suitable (and not intended) for describing dynamics under non-standard operating conditions, such as faults.

In the new reactor model, the reactivity depends on the control rod position, the average temperature of the moderator, the temperature of the fuel, and on the poison processes. The model parameters have been classified appropriately, and the partially known and the unknown model parameters together with the initial condition of the non-measurable state variables have been estimated using a quadratic objective function and a nonlinear optimization algorithm, namely, the Nelder-Mead simplex search method. The necessary measurement data were collected from a unit of the Paks Nuclear Power Plant, located in Hungary. The quality of estimates has also been investigated by the analysis of the objective function.

The introduction of previously unmodeled effects resulted in the fact that this more detailed model describes the system dynamics more precisely than before, and a very good fit has been achieved even for the load changing transients.

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