MODEL IDENTIFICATION OF THE PRIMARY CIRCUIT AT THE PAKS NUCLEAR POWER PLANT

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ABSTRACT
Based on a simple process model of the primary circuit in physical coordinates, model parameter estimation has been performed on a unit of the Paks Nuclear Power Plant using measured transient data. The dynamic model is hybrid and nonlinear in its parameters and variables, therefore an estimation strategy based on the decomposition of the system has been applied. The estimated parameters fit well to their expected physical range, and the overall system response has also been reproduced in a satisfactory manner.

KEY WORDS
Energy and Power Systems Modeling , Dynamic Modeling, Parametric Identification, Estimation

Figure 1. The flowsheet of the primary circuit

1 Introduction

This paper presents model identification of the primary circuit system which is in current use in the Paks Nuclear Power Plant (Paks NPP) located in Hungary. The Paks NPP was founded in 1976 and started its operation in 1981. The plant operates four VVER-440/213 type reactor units with a total nominal (electrical) power of 1860 MWs that are of pressurized water reactor (PWR) type. About 40 percent of the electrical energy generated in Hungary is produced here. Considering the load factors, the Paks units belong to the leading ones in the world and have been among the top twenty-five units for years.

One of the main motivations of the present work is the successful modeling, identification [1], controller design [2] and implementation of the pressure control loop in the primary circuits of units 1, 3 and 4 of the Paks Nuclear Power Plant. Using this model-based design, the precise stabilization of the primary loop pressure (together with other significant safety and instrumentation developments) largely contributed to be able to safely increase the average thermal power of the units by an average of 1-2%.

While there are a number of papers about the dynamic modeling and integrated control of boiling water reactor (BWR) units (see, e.g. [3]), the modeling of VVER blocks is mainly based on coupled neutron kinetic/thermal codes
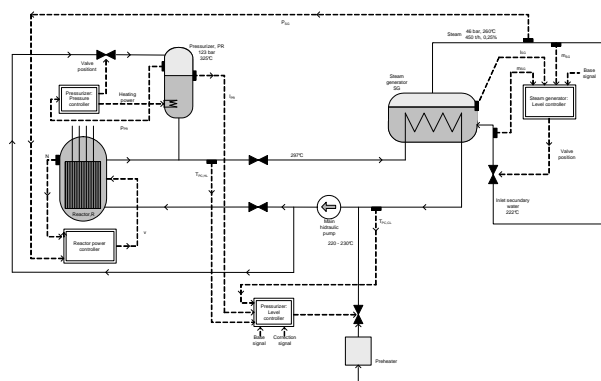
which are not suitable for advanced controller design in their original form [4].

The main aim of the present paper is to produce a simple process model in physical coordinates for the whole primary circuit dynamics that is simple enough, but is able to reproduce the main dynamic properties of the system and is applicable for distributed but coordinated control system design.

2 System description

Figure 1 shows the flowsheet of the primary circuit in Paks NPP, where the main equipments: the reactor, the steam generator(s), the main circulating pump(s), the pressurizer and their connections are depicted. The sensors that provide on-line measurements are also indicated in the figure by small full rectangles. The controllers are denoted by double rectangles, their input and output signals are shown by dashed lines.

The liquid in the primary circuit is circulated by a high speed, and it is under high pressure in order to avoid boiling. The energy generated in the reactor is transferred by the primary circuit to the liquid in the steam generator making it boiling. The generated secondary circuit vapor is

then transferred to the turbines.

3 Dynamic state-space model and its structure

There are a few papers in the literature that report on developing simple dynamic models for boiling water or pressurized water reactors for various purposes. A simple model was developed in [5] for the thermal-hydraulics part of a BWR reactor that is used for stability analysis of the reactor under different operating conditions. A relatively simple dynamic model used in a training course for simulation purposes is reported in [6].

A systematic modeling procedure suggested for constructing process models [7] has been followed to construct a simple dynamic model of the primary circuit. The detailed derivation of the model equations can be found elsewhere [8].

3.1 Engineering model

The construction of a dynamic engineering model starts with the identification of the so called *operating units*. Part of the primary circuit with clear functionality is considered as an operating unit (like the pressurizer). An operating unit may contain more than one physical units (pipes, containers, valves, etc.) but it is then regarded as a primary balance volume over which conservation balances can be constructed.

Modeling assumptions The overall modeling assumptions specify the considered operating units and their general properties.

G1 *The set of operating units*
considered in the simple dynamic model includes the reactor (R), the water in the primary circuit (PC), the pressurizer (PR) and the steam generator (SG).

G2 *The dynamic model of the operating units*
is derived from simplified mass, energy and neutron balances constructed for a single balance volume that corresponds to the individual unit.

G3 *The considered controller*
in the simplified model is the pressure controller. All the other controllers (including the level controller of the pressurizer and the power controller of the reactor, the level controller in the steam generator, and the controller of the turbines, main circulating pumps and other compressors and valves in the system) are assumed to be ideal, that is, they keep their reference values ideally, without any dynamics or delays.

Simplifying assumptions for the operating units In order to obtain a low dimensional dynamic model, the simplest possible set of operating units is considered in their simplest functional form. It is well possible to simplify the models of the operating units because the validity range of the model is restricted to the normal operating range (and excludes the start-up and shut-down, as well as the faulty operating modes).

Under these operating conditions we can assume constant physico-chemical properties (e.g. specific heats ($c_{p,\bullet}$), evaporation heat, nuclear parameters, heat transfer coefficients (K_{T,SG_1}), etc.) except for the temperature dependence of the density and saturation vapor pressure, that is taken into account. Because of the ideal operation of most of the controllers, the overall mass in the primary circuit (M_{PC}) and in the steam generator (M_{SG}) is assumed to be constant. Moreover, we have neglected any thermal effect to the reactor, and the thermal effect of the pressurizer to the primary circuit water (but not the reverse effect).

Conservation balances Dynamic conservation balances form the basis of our dynamic engineering model that are constructed for conserved extensive quantities over balance volumes (operating units). Such balances have been constructed for the number of neutrons (neutron flux, N) in the reactor, the internal energy of the water in the primary circuit, in the steam generators and in the pressurizer.

The terms in a conservation balance equation correspond to the different mechanisms causing the variation of the corresponding conserved extensive quantity in a balance volume, such as in- and outflows (convection), transfer and sources. Eq. (4) is the intensive form of the energy balance for the water in the primary circuit, where the terms in the bracket of its right-hand side correspond to the in- and outflow of the purge flow ($c_{p,PC}m_{in}(T_{PC,I} - T_{PC,CL})$), to the reactor's heating effect (W_R), to the energy transfer to the 6 steam generators ($K_{T,SG}(T_{PC} - T_{SG})$) and to the heat loss ($K_{loss,PC}(T_{PC} - T_{out})$). The terms in the brackets of the other two energy balance equations (5) and (6) have been constructed in a similar way.

Having constructed the balances, the intensive form of the energy balance equations have been computed to obtain differential equations for the measurable temperature $T_\bullet$ instead of its related internal energy $U_\bullet$, where $\bullet = PC, SG, PR$, by using the algebraic relationship $U_\bullet = c_{p,\bullet}M_\bullet T_\bullet$.

Constitutive equations There are additional algebraic equations that complement the differential conservation balance equations. In this model these are mainly equations describing the dependence of the physico-chemical properties on the temperature.

The dependence of the density φ on the temperature T is found in the literature [9] is approximated around $T^0 = 300^oC$ by a second order polynomial $\varphi(\widetilde{T}) = c_{\varphi,0} + c_{\varphi,1}\widetilde{T} + c_{\varphi,2}(\widetilde{T})^2$ with the coefficients $c_{\varphi,0} = 581.2$, $c_{\varphi,1} = 2.98$, $c_{\varphi,2} = -0.00848$, where $\widetilde{T}$ is the temperature measured in ^oC and the density is measured in kg/m^3.

The pressure of a saturated vapor p measured in kPa depends only on the temperature where the function p_*^T is assumed to be in the following form [9]:

$$p_*^T(\widetilde{T}) = 28884.78 - 258.01\widetilde{T} + 0.63\widetilde{T}^2$$

Pressurizer mass The water mass in the pressurizer serves as an indicator of the overall mass in the primary circuit:

$$M_{PR} = M_{PC} - M_{PC}^0 = M_{PC} - \varphi(T_{PC})V_{PC}^0 \quad (1)$$

where V_{PC}^0 is the constant volume of the primary circuit itself. The pressurizer serves as an 'overflow tank' for the primary circuit where there is a mass flow from the primary circuit to the pressurizer in the form

$$m_{PR} = -V_{PC}^0 \left(c_{\varphi,1} + 2c_{\varphi,2}T_{PC}\right) \frac{dT_{PC}}{dt} \quad (2)$$

3.2 State-space model

The state-space model is obtained by substituting all of the algebraic constitutive equations into the differential ones (into the overall mass balances and to the intensive form of the energy balances):

$$\frac{dN}{dt} = \frac{\beta}{\Lambda} \left(p_1 v^2 + p_2 v + p_3\right) N + S \quad (3)$$

$$\frac{dT_{PC}}{dt} = \frac{1}{c_{p,PC}M_{PC}} \Big[c_{p,PC}m_{in} \left(T_{PC,I} - T_{PC,CL}\right) +$$
$$+ W_R - 6 \cdot K_{T,SG_1}(T_{PC} - T_{SG}) -$$
$$- K_{loss,PC}(T_{PC} - T_{out}) \Big] \quad (4)$$

$$\frac{dT_{SG}}{dt} = \frac{1}{c_{p,SG}^L M_{SG}} \Big[c_{p,SG}^L m_{SG}T_{SG,SW} -$$
$$- c_{p,SG}^V m_{SG}T_{SG} - m_{SG}E_{evap,SG} +$$
$$+ K_{T,SG_2}(T_{PC} - T_{SG}) - W_{loss,SG} \Big] \quad (5)$$

$$\frac{dT_{PR}}{dt} = \frac{1}{c_{p,PR}M_{PR}} \Big[\chi_{m_{PR}>0}c_{p,PC}m_{PR}T_{PC,HL} +$$
$$+ \chi_{m_{PR}<0}c_{p,PR}m_{PR}T_{PR} - c_{p,PR}m_{PR}T_{PR} -$$
$$- W_{loss,PR} + W_{heat,PR} \Big] \quad (6)$$

with $\chi_{condition}$ is the indicator function of $condition$ that is 1 when the condition is fulfilled and zero otherwise.

The output equations are as follows:

$$W_R = c_{\Psi 1}N \quad (7)$$
$$p_{SG} = p_*^T(T_{SG}) \quad (8)$$
$$\ell_{PR} = \frac{1}{A_{PR}} \left(\frac{M_{PC}}{\varphi_{PC}(T_{PC})} - V_{PC}^0\right) \quad (9)$$
$$p_{PR} = p_*^T(T_{PR}) \quad (10)$$

Identifier	Variable	Type
N	R neutron flux	s
v	R control rod position	i
W_R	R reactor power	o
m_{in}	PC inlet mass flow rate	i
$T_{PC,I}$	PC inlet temperature	d
$T_{PC,CL}$	PC cold leg temperature	(s)
$T_{PC,HL}$	PC hot leg temperature	(s)
p_{PR}	PR pressure	o,(s)
T_{PR}	PR temperature	s
ℓ_{PR}	PR water level	o,(s)
$W_{heat,PR}$	PR heating power	i
m_{SG}	SG mass flow rate	d
$T_{SG,SW}$	SG inlet water temperature	d
p_{SG}	SG steam pressure	o

Table 1. Measured variables with type (**s**tate, **i**nput, **o**tput, **d**isturbance)

3.3 Model variables

Given the state-space model above, the system variables can be classified as follows:

- *State variables*: differential variables in the differential equations, N, T_{PC}, T_{PR}, T_{SG}

- *Input variables*: manipulable variables affected by the considered controllers, v, m_{in}, m_{SG}, $W_{heat,PR}$

- *Disturbances*: all other possibly time-dependent variables appearing on the right-hand side of the differential equations, m_{PR}, $T_{SG,SW}$, $T_{PC,I}$

- *Output variables*: measurable variables that are regulated by the considered controllers, N (W_R), p_{SG}, ℓ_{PR} (M_{PC}), p_{PR}

Majority of the system variables above can be directly (or indirectly) measured on the units of the Paks Nuclear Power Plant, see Table 1 for the details. The 'measured' average temperature of the water in the primary circuit is approximated by

$$T_{PC} = \frac{T_{PC,HL} + T_{PC,CL}}{2} \quad (11)$$

3.4 Model parameters

The model parameters are the constants in the above state-space model equations. They can be classified according to the operating unit they belong to as follows:

(R): nuclear reaction parameters $\frac{\beta}{\Lambda}$, S; (p_1, p_2, p_3), efficiency parameter $c_{\Psi 1}$;

(PC): $c_{p,PC}$, M_{PC}, K_{T,SG_1}, $K_{loss,PC}$, T_{out} ;

(PR): $c_{p,PR}$, $W_{loss,PR}$; V_{PC}^0, cross section A_{PR};

(SG): $c_{p,SG}^L$, $c_{p,SG}^V$, M_{SG}, K_{T,SG_2}, $W_{loss,SG}$;

(phys-chem): parameters in functions φ and p_*^T.

Identifier	Parameter	Unit
(p_1, p_2, p_3)	control rod parameters	R
S	zero neutron flux	R
$c_{p,PC}$	specific heat	PC
M_{PC}	water mass	PC
$K_{T,SG_{1,2}}$	heat transfer coefficient	**PC**, SG
$K_{loss,PC}$	heat loss transfer coefficient	PC
T_{out}	containment temperature	PC
M_{SG}	water mass	SG
$W_{loss,SG}$	heat loss	SG
$c^L_{p,SG}$	liquid specific heat	SG
$c^V_{p,SG}$	vapor specific heat	SG
$c_{p,PR}$	liquid specific heat	PR
$W_{loss,PR}$	heat loss	PR

Table 2. Parameters to be estimated

Figure 2. The measured and estimated neutron flux

The parameters to be estimated have been selected from the above set by performing sensitivity analysis. Some of the parameters (equipment parameters and physico-chemical properties) were available in the literature, therefore only the unknown parameters in Table 2 have been selected for estimation.

3.5 Model decomposition

It is seen from the state equations (3)-(6) and the measured variables, that the parameters in the neutron flux balance equation (3) can be estimated independently of the others, thus the reactor forms an independent component of the model. Then the coupled equations (4) and (5) describing the dynamics of the water in the primary circuit and the steam generator form another component that uses the reactor power as its 'virtual input'. Finally, the third component is the pressurizer that depends on the dynamics of the water in the primary circuit.

4 Model parameter estimation

4.1 Estimation method

The parameter estimation has been carried out sequentially and component-wise following the dependencies outlined above. First the reactor unit described by equation (3) that is nonlinear in its parameters has been identified. The second block to be identified contains the coupled equations (4) and (5) that are linear in the parameters. The final dynamic sub-system is the pressurizer that is again nonlinear in its parameters.

If the dynamic model equation(s) is/are nonlinear in its/their parameters, an optimization-based parameter estimation method, the Nelder-Mead simplex method [10], [11] available in MATLAB has been used. For error value we measure the fit in terms of the 2-norm between the measured and the model-predicted output signals, i.e.

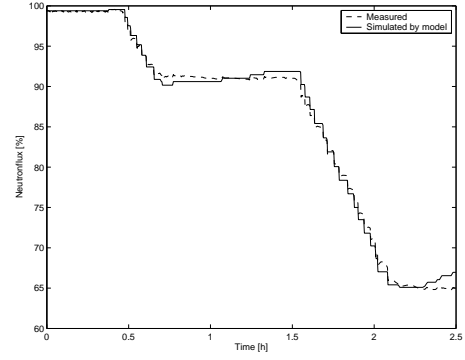$$e = \sqrt{\frac{\int_0^T (\hat{y}(t) - y(t))^2 dt}{\int_0^T y^2(t)dt}} \qquad (12)$$

	Parameter	Unit	Value
Reactor	p_1	$1/m^2$	-0.0191
	p_2	$1/m$	-0.00860
	p_3	1	-0.0305
	S	$\%/s$	1939
Pressurizer	$c_{p,PR}$	J/kgK	6873
	$W_{loss,PR}$	W	$1.68 \cdot 10^5$
	V^0_{PC}	m^3	239
PC and SG	K_{T,SG_1}	W/K	$8.0163 \cdot 10^6$
	K_{T,SG_2}	W/K	$4.7038 \cdot 10^6$
	$K_{loss,PC}$	W/K	$1.0066 \cdot 10^6$
	M_{PC}	kg	200100
	M_{SG}	kg	34713
	$c_{p,PC}$	J/kgK	6000
	$c^L_{p,SG}$	J/kgK	4093
	$c^V_{p,SG}$	J/kgK	2415
	$W_{loss,SG}$	W	99887
	$E_{evap,SG}$	J/kg	1658555

Table 3. The estimated physical parameters

where y is the measured output, $\hat{y}$ is the model-predicted (simulated) output signal and T denotes the time-span of the measurement/simulation.

4.2 Reactor

In the case of the reactor unit, the input variable is the control rod position (v), the output is the neutron flux (N) while the estimated parameters are p_1, p_2, p_3, S. (Values of β and Λ are assumed to be known.) Because of the nonlinearity in the parameters, the Nelder-Mead simplex method has been used.

The estimated parameters are given in Table 3. The fitting of the model predicted and measured outputs can be seen in Fig. 2.

4.3 Liquid in the primary circuit and the steam generators

As it has been mentioned, the dynamics of the coupled equations (4)-(5) can be identified separately from the neutron flux and pressurizer dynamics. It is important to ob-

serve, that the model equations are linear in their parameters but these parameters depend on the physical parameters in nonlinear way. Therefore, the parameter estimation has been carried out by a combination of a standard least squares estimation and a constrained optimization procedure.

Discretizing equations (4)-(5) with using the variable transformations $z_1 = c_{p,PC}M_{PC}T_{PC}$, $z_2 = c_{p,SG}^L M_{SG}T_{SG}$ and a standard Euler-approximation for the derivatives gives

$$\frac{z_1(k+1) - z_1(k)}{t_s} = c_{p,PC}m_{in}(k) \cdot (T_{PC,I}(k) - T_{PC}(k))$$
$$+W_R(k) - 6(K_{T,SG_1})(T_{PC}(k) - T_{SG}(k))$$
$$-K_{loss,PC}(T_{PC}(k) - T_{out}) \tag{13}$$
$$\frac{z_2(k+1) - z_2(k)}{t_s} = -m_{SG}(k)\left(c_{p,SG}^V T_{SG}(k) + E_{evap,SG}\right.$$
$$\left. -c_{p,SG}^L T_{SG,SW}(k)\right) + K_{T,SG_2}(T_{PC}(k) - T_{SG}(k))$$
$$-W_{loss,SG} \tag{14}$$

where t_s denotes the sampling time (10s). The physical parameters in this case can be divided into two groups according to the available a'priori information. We had a relatively good initial guess for the parameters

$$\theta_1 = [c_{p,PC} \ c_{p,SG}^L \ c_{p,SG}^V \ M_{SG} \ M_{PC} \ W_{loss,SG} \ T_{out}]$$

from data tables and technical documentation. However, we had no knowledge about the heat transfer coefficients and the heat loss coefficient

$$\theta_2 = [K_{T,SG_1} \ K_{T,SG_2} \ K_{loss,PC}]$$

It is visible that equations (13)-(14) are linear in θ_2. Starting from the acceptable initial guess, the constrained optimization algorithm was searching in the space of θ_1 in such a way, that in each evaluation step, a least squares estimate was computed for θ_2. The constrained algorithm minimized the prediction error

$$e = \sqrt{\sum_{i=1}^N (T_{PC}(i) - \hat{T}_{PC}(i))^2} + \sqrt{\sum_{i=1}^N (T_{SG}(i) - \hat{T}_{SG}(i))^2}$$

where $T_{\{PC,SG\}}$ denotes the measured temperatures and $\hat{T}_{\{PC,SG\}}$ are the model predicted temperatures. The number of samples N was 900 in this case. Using the proposed method, the more-or-less known and unknown parameters have been separated in the optimization procedure and the final estimates have physically meaningful values as it is visible in Table 3. The measured and computed primary circuit and steam generator temperatures can be seen in Fig. 3.

4.4 Pressurizer

Since the model equation (6) of the pressurizer is hybrid, that is, it contains a discrete switching term, the Nelder-Mead simplex method has been used with the error function (12). The input variables are the pressurizer heating
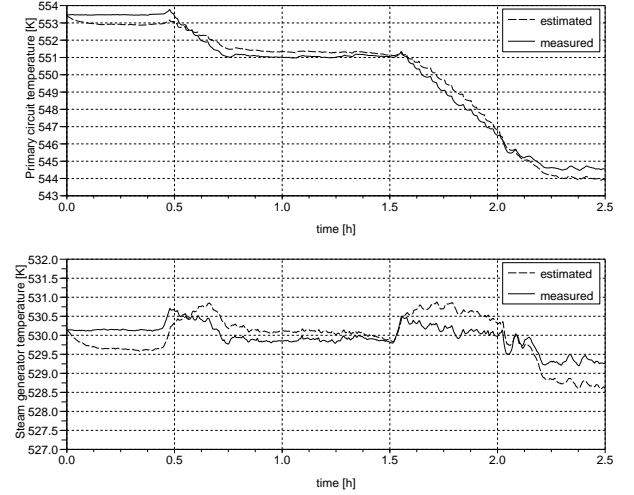

Figure 3. The measured and estimated primary circuit and steam generator temperatures

power ($W_{heat,PR}$) and the average temperature in the primary circuit (T_{PC}), the output variables are the pressure in the primary circuit (p_{PR}), while parameters to be estimated are the specific heat ($c_{p,PR}$) and the heat loss ($W_{loss,PR}$) in the pressurizer. Here the measured data from unit 2 have been used where an old, on-off type pressure controller has been operating that provided sufficient excitation for the parameter identification.

The estimated parameters are given in Table 3 together with an example of the fit in the output signal shown in Fig. 4.
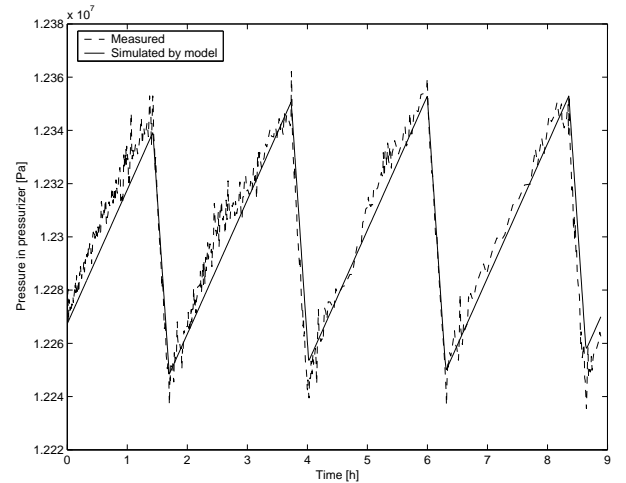

Figure 4. Measured pressurizer heating power and temperature, and the fit, unit 2

The level function To compute the level of pressurizer the volume of primary circuit (V_{PC}^0) has to be known. From

Eq. (9) one can see that if the mass of the liquid in the primary circuit is constant then the level in the pressurizer depends only on the temperature in primary circuit. To estimate V_{PC}^0 a linear equation is formed from Eq. (9) between the level (ℓ_{PR}) and the temperature (T_{PC}):

$$\ell_{PR}(T_{PC}) = c_1 T_{PC} + c_0 \tag{15}$$

The parameters (c_1 and c_0) in the above linear model have been estimated by standard least squares method using the measured level (ℓ_{PR}) and temperature (T_{PC}) data, where the following parameter values have been obtained:

$$c_1 = 0.1095 \, m/C \ , \ c_0 = -24.9365 \, m$$

The fit of measured and simulated level is shown in fig. 5. Comparing the estimated parameters of Eq. (15) with a first degree Taylor polynomial approximation of the original equation (9) we obtain the estimated value of the volume of primary circuit, that is $V_{PC}^0 = 239 m^3$.
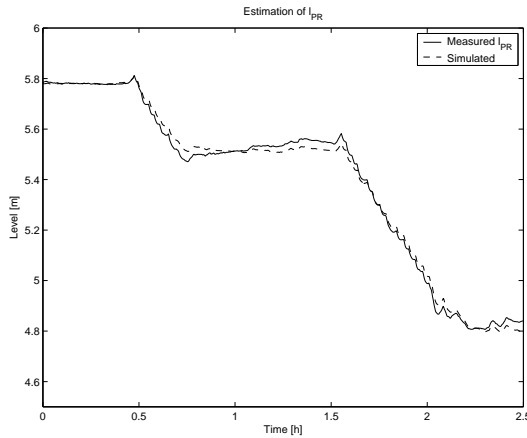

Figure 5. Level of pressurizer, unit 3

5   Conclusion

Based on a simple nonlinear concentrated parameter process model of the primary circuit in physical coordinates model parameter estimation has been performed on a unit of the Paks Nuclear Power Plant using measured transient data. The dynamic model is nonlinear in its parameters and variables, therefore an estimation strategy based on the decomposition of the system has been applied.

The parameter estimation has been carried out sequentially estimating first the reactor parameters by the Nelder-Mead simplex method, then the parameters of the liquid in the primary circuit and the steam generators by an LS method followed by a regularization procedure, and finally the parameters of the pressurizer by the Nelder-Mead simplex method. The estimated parameters fit well to their expected physical range, and the overall system response has also been reproduced in a satisfactory manner.

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