Linear and Nonlinear Control of Musculoskeletal Systems

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Aim and Application

- Examine different kind of controls of nonlinear musculoskeletal system
- Application: designing and controlling artificial limbs, muscle prosthesis and FES systems

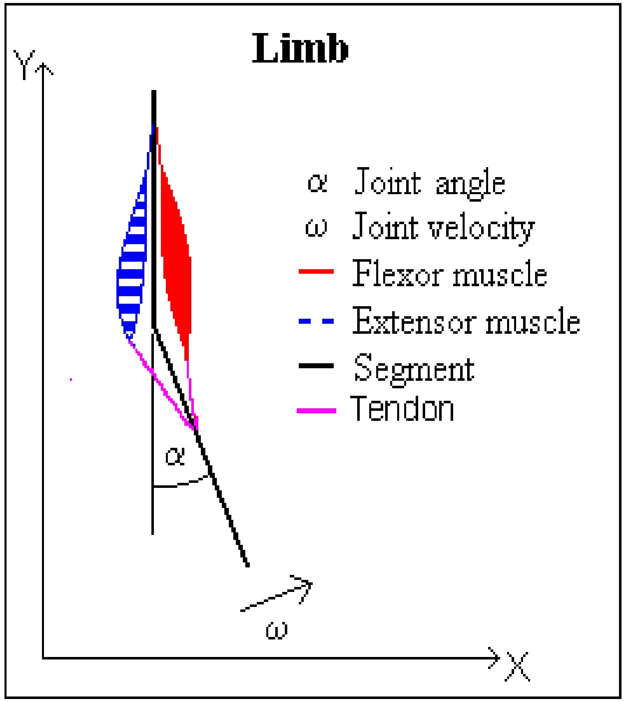
Musculoskeletal System

- Like a robot arm. Contains:
  - Rigid segments
  - Actuators are musculotendons
- Muscle can pull the segment and cannot push it.
- Nonlinear because muscle and movement dynamics are nonlinear

Simple Model

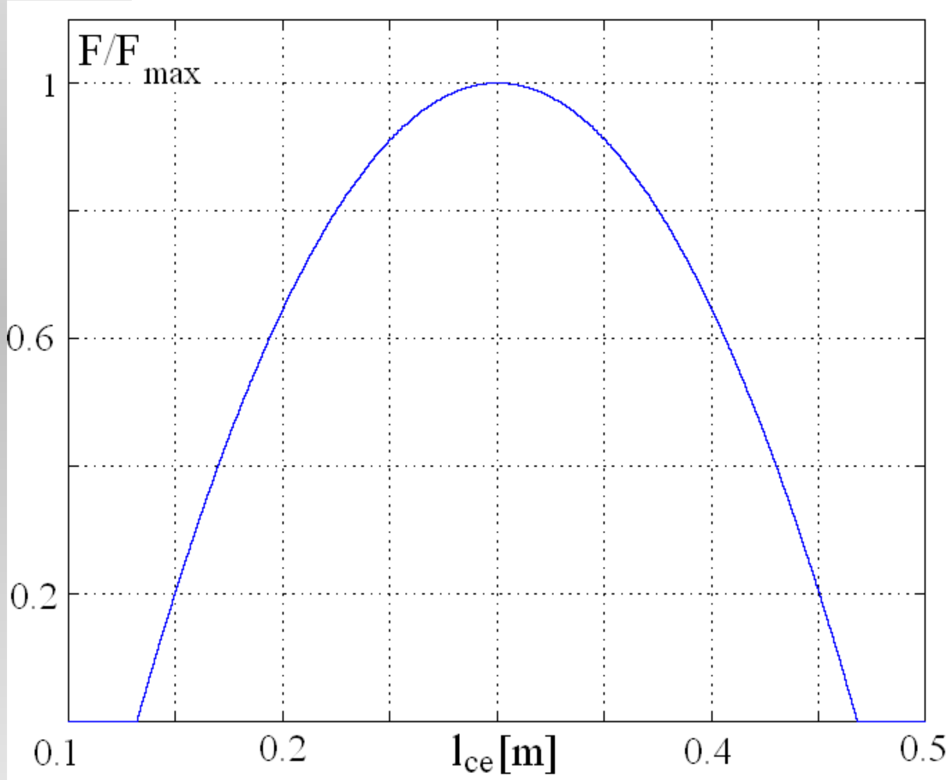
- One joint and two muscles.
- Takes into account the activation, muscle, tendon and movement dynamics.
- Like an elbow.
- 8 states, 2 inputs and 1 output.
- Hard constraints on the physical input of the system:

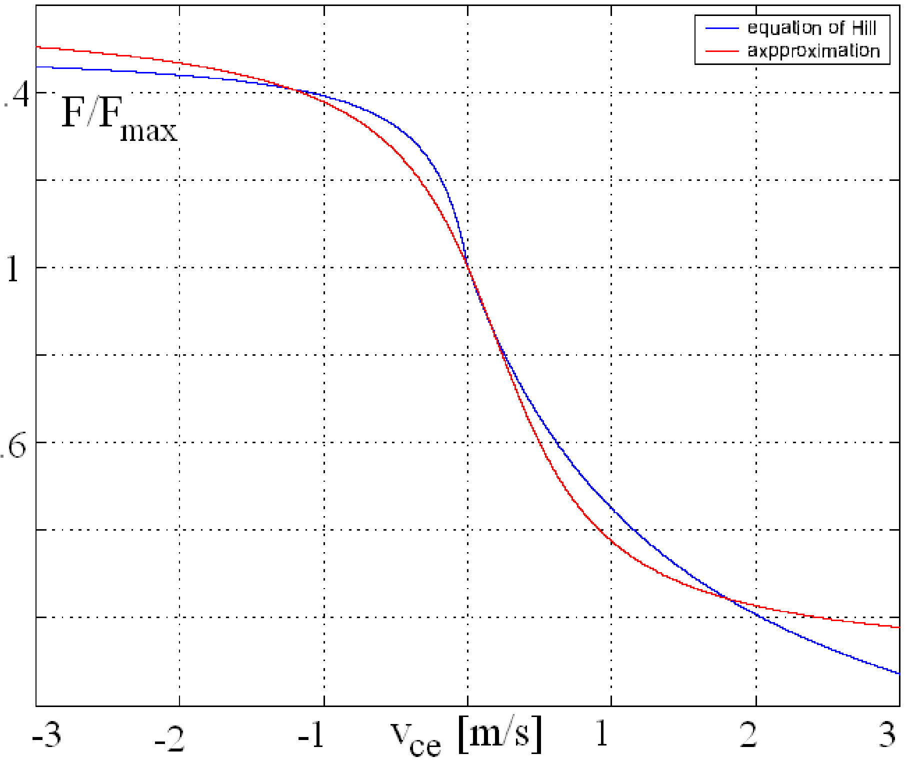
$$0 \leq u \leq 1$$



Nonlinearities

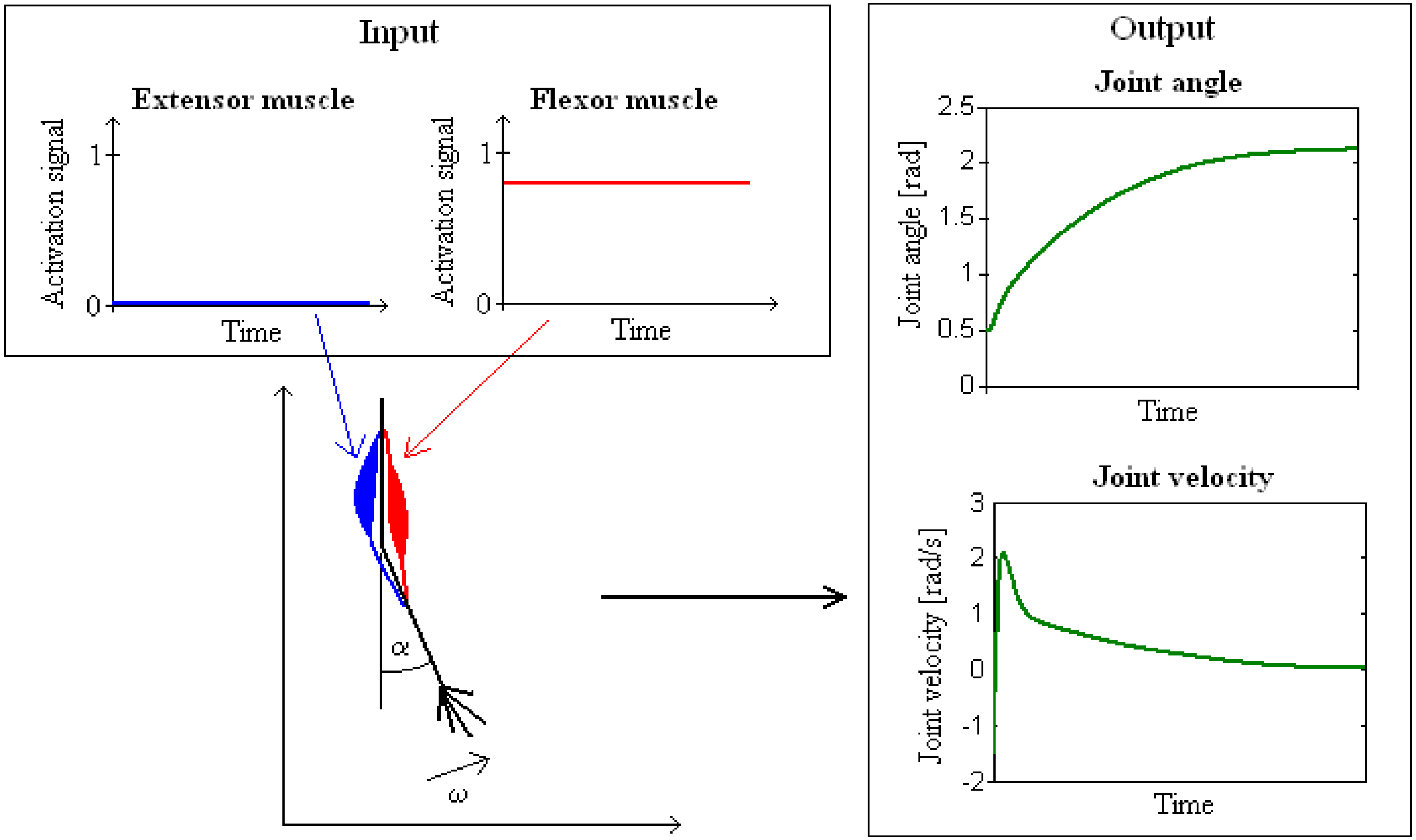
$$F = F^{max}\left(FL\left(l^{CE}\right)FV\left(v^{CE}\right)q + F_{PE}^{max}F^{PE}\right)$$





Input-output

- Inputs are the **activation signals** of the muscles
- Output is the **movement pattern**: joint angles time.



Model's Equations

$$
\begin{aligned}
\dot{x_1} &= -\left(\frac{1}{\tau_{act}}(\beta + [1-\beta]u_1(t))\right)x_1 + \frac{1}{\tau_{act}}u_1(t) \\
\dot{x_2} &= -\left(\frac{1}{\tau_{act}}(\beta + [1-\beta]u_2(t))\right)x_2 + \frac{1}{\tau_{act}}u_2(t) \\
\dot{x_3} &= x_4 \\
\dot{x_4} &= \frac{1}{\Theta + ml_{COM}^2}(M(\mathbf{x}) + ml_{COM}cos(x_3 - \pi/2)g_y) \\
\dot{x_5} &= x_7 \\
\dot{x_6} &= x_8 \\
\dot{x_7} &= -\frac{k_t(l_{T1} - l_t^{slack}) + s_T x_7 - F_{flexor}(x_1, x_3, x_4, x_5, x_7)}{z_T} \\
\dot{x_8} &= -\frac{k_t(l_{T2} - l_t^{slack}) + s_T x_8 - F_{extensor}(x_2, x_3, x_4, x_6, x_8)}{z_T} \\
y &= x_3
\end{aligned}
$$

Controllers and SISO form

- Controllers
  - Linear pole-placement servo designed for locally linearized model.
  - Nonlinear controller based on asymptotic output tracking.
- Convertion into SISO model
  - Applied nonlinear control methods require a SISO model.
  - Model was divided into two parts.
  - One of them contained an active flexor muscle and an inactive extensor muscle, while the other contained an inactive flexor muscle and an active extensor muscle
  - Control input was designed separately for these two parts and switching between them was controlled by rules.

Linear PP

- Linearize around the required steady state:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

- Extended with an integrator:

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0^{n \times 1} \\ 1 \end{bmatrix} y_R$$

- Using a full state feedback

$$u = -K \begin{bmatrix} x \\ z \end{bmatrix}, \quad K = [K_x \quad K_z]$$

- Gain vector is design applying pole-placement technique

Asymptotic Output Tracking

- Nonlinear model is

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned}$$

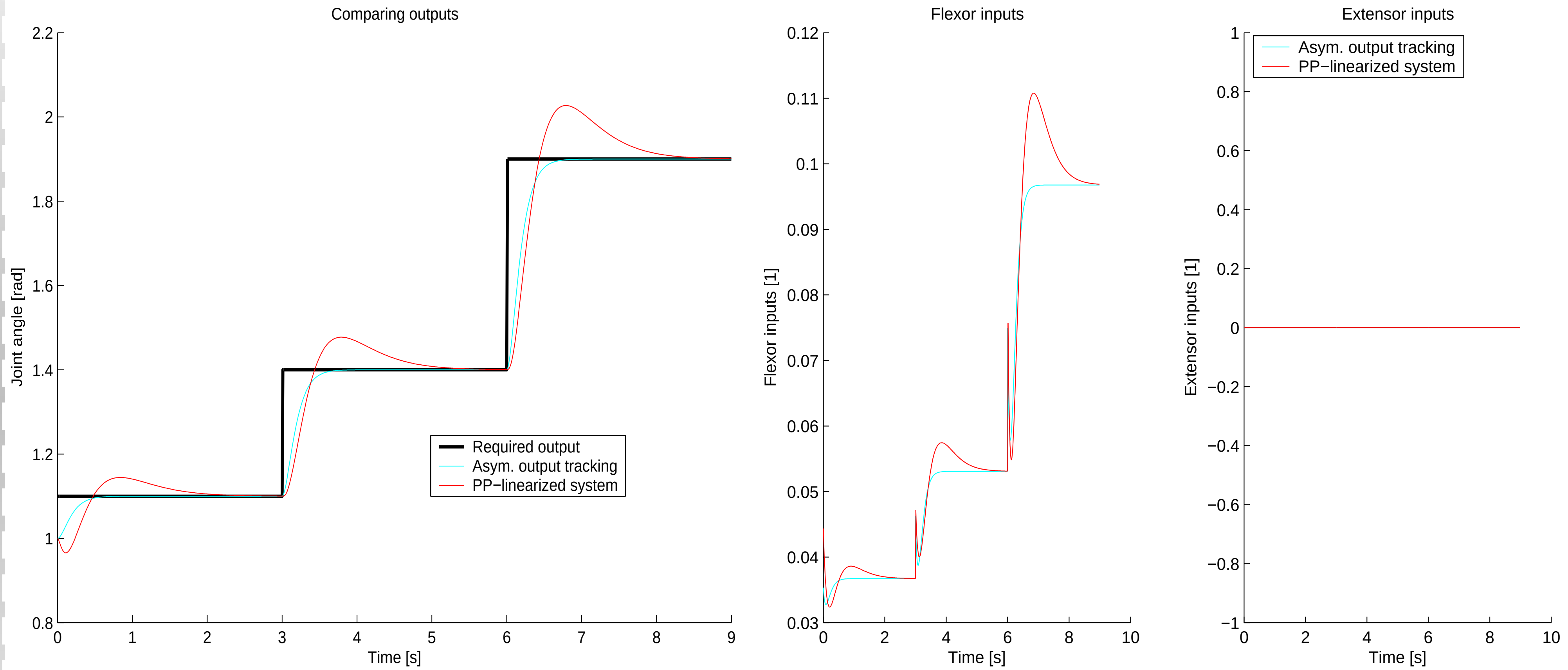
- Its relative degree is 3
- Controller generated input of the model is

$$\begin{aligned} u &= \frac{1}{L_g L_f^{r-1} h(x)} \Big(-L_f^r h(x) + y_R^{(r)} - \\ &\quad - \sum_{i=1}^{r} c_{i-1} (L_f^{(i-1)} h(x) - y_R^{(i-1)}) \Big) \end{aligned}$$

y_R is the reference output, r is the relative degree

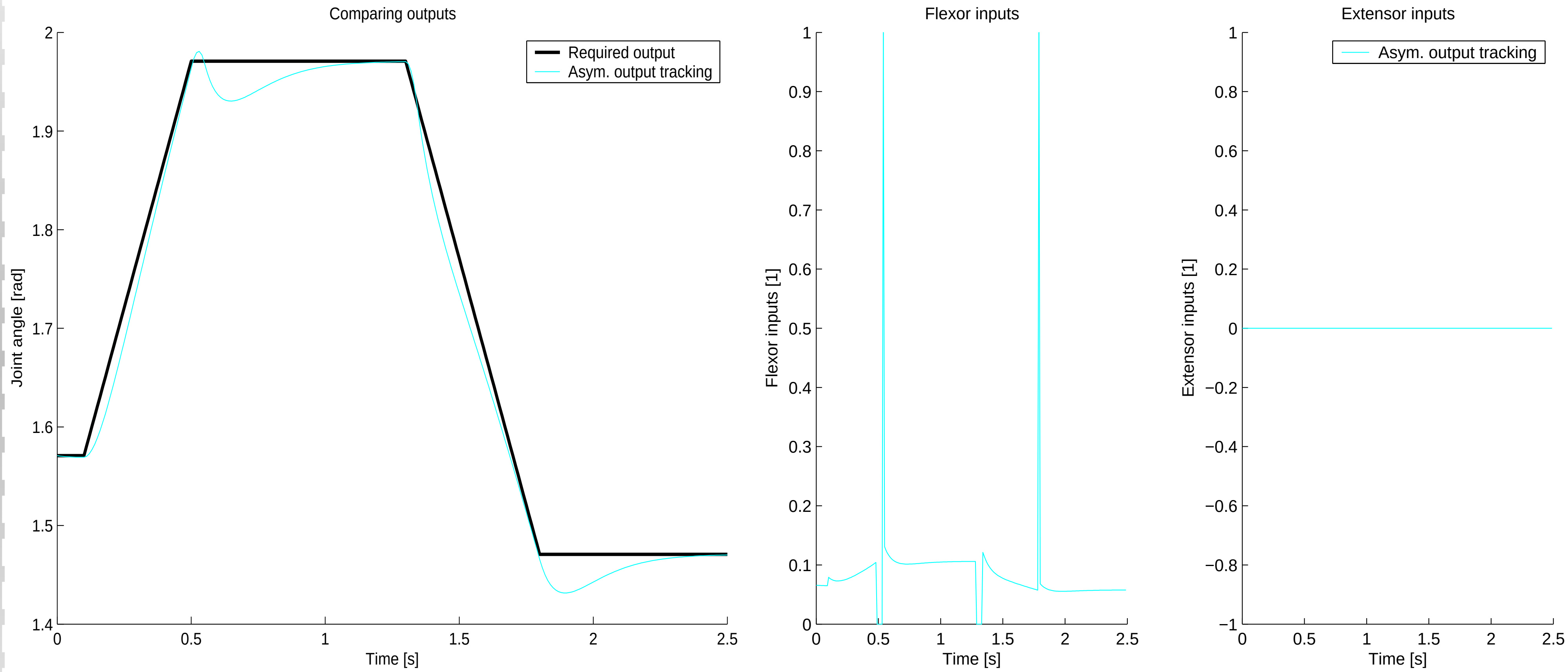
- Design: choose c_i

Results: Piecewise Const. Ref.



	Input norm	Error norm
Pole-placement servo	2.0508	2.948
Asy. output tracking	1.9536	2.0561

Results: Piecewise Lin. Ref.



Conclusion

- Nonlinear asymptotic output tracking control gave a better performance than the pole-placement servo with the nonlinear model.
- Nonlinear controller was much faster.
- Nonlinear controller was able to track a continuously changing reference output, with minimal overshooting.
- Computation time of asymptotic output tracking controller was higher.
- Linear control can be applied when the movement range is small or efficient computation is very important.
- Nonlinear control theory becomes necessary when the motion range is wide or the reference input is a more complex function