A simple dynamic model of the primary circuit in VVER plants for controller design purposes

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Abstract

A simple low dimensional nonlinear dynamic model of the primary circuit in a VVER-type nuclear power plant is developed from first engineering principles that is able to capture the most important dynamics of the system in normal operating modes. The model includes the description of the main control loops in the system, too.

The model has been verified and validated by using measured data from three VVER-440 units of Paks Nuclear Power Plant in Hungary, and a good fit has been obtained. This qualifies the model to be the basis for the integrated re-design of the primary control loops.

1 Introduction

The increasing demands related to performance, operating costs and energy, environmental issues etc. in complex plants often bring up the requirement for the re-tuning or even complete re-design of their major control loops. This work is supported by two important factors: firstly, the huge development in modern systems and control theory (Isidori, 1995; van der Schaft, 2000) and secondly, the improving quantity and quality of dynamic measurement data providing the necessary amount of information to implement complex diagnostic and control systems using the available relatively cheap computing power. However, the most modern controller design methods require a special system structure for describing the dynamic behavior in the form of a (preferably low order) set of nonlinear ordinary differential equations with an appropriate classification of the parameters and variables. The aim of this paper is to give a dynamic model of this kind for the primary circuit of VVER plants.

One of the main motivations of the present work is the successful modeling, identification (Varga et al., 2006), controller design (Szabó et al., 2005) and implementation of the pressure control loop in the primary circuits of units 1, 3 and 4 of the Paks Nuclear Power Plant. Using this model-based design, the precise stabilization of the primary loop pressure (together with other significant safety and instrumentation developments) largely contributed to be able to safely increase the average thermal power of the units by an average of 1-2%. We would like to extend this modeling and parameter estimation approach to the whole primary circuit dynamics in this paper.

There are good dynamic models available for pressurized water reactors (PWRs and VVERs) for equipment design, safety and risk assessment and/or operator training (Fletcher and Schultz, 1995). The thermal-hydraulic part alone can be well modeled by using e.g. APROS (APROS, 2003), but usually coupled neutron kinetic/thermal codes are used for a dynamic analysis or simulation study (Mittag et al., 2001; Vanttola et al., 2005). These models and the computer codes behind them are, however, too much detailed for control studies because they contain too many state variables (they are of too high degree) and their structure does not allow us to design model-based feedbacks directly based on them. Of course, these high-fidelity codes are indispensable when the designed control loops are later fine-tuned and tested.

If one aims to study the dynamics of the system in order to design individual controllers or study their interactions, then a minimal dynamic model is needed. Such a model can be obtained in two basically different ways: either by reducing (simplifying) existing detailed models, or by constructing composite models from minimal elements. The second approach is chosen here because the model obtained in this way is more transparent and more easy to understand.

There are a few papers in the literature that report on developing simple dynamic models for boiling water or pressurized water reactors for various purposes. A simple model was developed by (Karve et al., 1997) for the thermal-hydraulics part of a BWR reactor that is used for stability analysis of the reactor under different operating conditions. A relatively simple dynamic model used in a training course for simulation purposes is reported in (PWR-sim, 2003). There are also a few simple dynamic models available for the individual operating units in the primary circuit. The modeling and identification of a drum boiler in a boiling water reactor is reported by (Aström and Bell, 2000).

Having constructed a model, one should verify it against engineering expectations, estimate its missing or uncertain parameters and validate it against measured data on the real plant. The model verification, parameter estimation and validation steps are also described in the second part of this paper using measured data from three VVER-440 units of the Paks Nuclear Power Plant in Hungary.

2 Modeling goal and assumptions

It is well known and intuitively evident, that the modeling goal plays a fundamental role in any model development. If one aims at studying the dynamic behavior of an open loop or controlled system or wants to design controllers he/she needs a low order lumped (concentrated parameter) model that captures the dynamic input-output behavior of the system. At the same time it is advantageous if the variables and parameters of this model have clear physical meaning because such a model is more transparent to the operating personnel and it is more easy to use engineering judgement during its verification.

In order to construct a simple dynamic model of the primary circuit a systematic modeling procedure suggested for constructing process models will be followed (Hangos and Cameron, 2001). The basis of this approach is to construct the model based on conservation balances for conserved extensive quantities such as overall mass, internal energy, component masses, number of neutrons with given energy etc. supplemented with algebraic constitutive equations. The procedure includes the explicit specification of the modeling goal, the identification of balance volumes, the specification of modeling assumptions and the evaluation of available data before constructing the model equations. The procedure also requires to perform model verification, model parameter estimation and model validation afterwards but before attempting to use the model for the intended modeling goal.

2.1 Overall modeling assumptions

In order to obtain a low dimensional dynamic model, the simplest possible set of operating units is considered in their simplest functional form. Part of the primary circuit with clear functionality is considered as an operating unit (like the pressurizer). An operating unit may contain more than one physical units (pipes, containers, valves, etc.) but it is then regarded as a primary balance volume over which conservation balances can be constructed. The overall modeling assumptions specify the considered operating units and their general properties.

G1 *The set of operating units*
considered in the simple dynamic model includes the reactor, the water in the primary circuit, the pressurizer and the steam generator.

G2 *The dynamic model of the operating units*
is derived from simplified mass, energy and neutron balances constructed for a single balance volume that corresponds to the individual unit.

G3 *The considered controllers*
in the simplified model are the pressure controller, the level controller of the pressurizer and the power controller of the reactor. All the other controllers (including the level controller in the steam generator, and the controller of the turbines, main circulating pumps and other compressors and valves in the system) are assumed to be ideal, that is, they keep their reference values ideally, without any dynamics or delays.

G4 *The domain of the model*
includes the dynamic behavior in normal operating mode together with the load changes between the day and night periods. In other words, failures and faulty mode transitions cannot be described by this simplified model.

2.2 The simplified operating units of the primary circuit

Figure 1 shows the operating units and their connections that are taken into account in the simplified model of the primary circuit. The sensors that provide on-line measurements are also indicated in the figure by small full rectangles. The controllers are denoted by double rectangles, their input and output signals are shown by dashed lines.
The steady-state values of the system variables in the normal 100 % power operating point are also indicated in Figure 1.

`This is the approximate location of Figure 1`

From the viewpoint of their dynamics and the type of their dependence on other operating units, the units of the simplified dynamic model are classified into three groups:

- *The reactor* which has a fast dynamics compared to the other operating units while its dynamics depends directly only on the temperature of the water in the primary circuit that is neglected.

- *The water in the primary circuit and the steam generator* which are the units that transport the energy generated by the reactor to the secondary circuit.

- *The pressurizer* that supplies a constant regulated pressure for the primary circuit.

3 Simplified dynamic models of the operating units

This section includes the derivation of the simplified dynamic models for each of the considered operating units. Because of convenience, an unique identifier is used for each of the operating units in the subscript of their related variables and parameters, as well as in all related modeling items, such as assumptions as follows:

R	reactor
PC	liquid in the primary circuit
PR	pressurizer
SG	steam generator(s)

The model equations of the operating units in the simplified model are derived from dynamic conservation balances that are supplemented with algebraic constitutive equations. The model of each operating unit in the simplified model is then described in terms of the applied modeling assumptions, its conservation balances and constitutive equations.

3.1 The reactor (R)

The reactor is the main operating unit in the primary circuit that acts primarily as an energy source in our model.

3.1.1 Modeling assumptions

In order to have a low order dynamic model of the reactor the following simplification assumptions are made.

R1 The reactor is regarded as a spatially homogeneous concentrated parameter (lumped) system with only a single balance volume.

R2 The time-dependent version of the single-group neutron diffusion equation (Kessler, 1983) is applied, that is, we only consider neutrons at the same energy level.

R3 Only a single type of delayed neutron emitting nuclei is considered with an average β total fraction of delayed neutrons and an average λ half-life of the delayed neutron emitting nuclei.

R4 The dependence of the nuclear physical mechanisms on the temperature is neglected. This includes the dependence of the reactivity on the coolant and core temperatures.

R5 The effect of the control rod position on the reactivity is approximated by a quadratic function.

R6 A quasi steady-state approximation is used for the concentration of the delayed neutron emitting nuclei.

R7 The reactor power is assumed to be a homogeneous linear function of the neutron flux.

R8 The reactor power controller is assumed to operate in its "N" mode providing a simple static feedback from the flux to the control rod position.

Note that from a physical viewpoint, assumption R4 seems to be the most restrictive. The main reason for this assumption is to obtain a dynamic model with the simplest possible algebraic structure and a minimum number of parameters to be estimated, because a more complex model might unnecesarily complicate the process of nonlinear model analysis and controller design. It is expected that this approximation will cause a small difference between the measured and the model predicted neutronflux and primary circuit water temperature values, but this is still an acceptable simplification of reality in the investigated operating region, as it is clearly visible from the results of section 5 (see also assumption G4 in section 2.1).

3.1.2 Conservation balances

The differential equations of the reactor model originate from the conservation balances for the concentration of the neutrons (with the neutron flux, N) and the delayed neutron emitting nuclei C in the following form:

$$\frac{dN}{dt} = \frac{\rho(v) - \beta}{\Lambda} N + \lambda C + S \tag{1}$$

$$\frac{dC}{dt} = \frac{\beta}{\Lambda} N - \lambda C \tag{2}$$

where ρ is the reactivity depending on the control rod position v, Λ is the generation time, β is the total fraction of delayed neutrons, λ is half-life of the delayed neutron emitting nuclei and S is the flux of a constant neutron source.

We can further simplify the above equations if assumption R6 is considered, that is we assume that $\frac{dC}{dt} \approx 0$. Then a constant ratio of N and C is obtained from Eq. (2):

$$C = \frac{\beta N}{\lambda \Lambda}$$

that can be substituted to Eq. (1) to obtain:

$$\frac{dN}{dt} = \frac{\rho(v)}{\Lambda} N + S \tag{3}$$

3.1.3 Constitutive equations

Reactor power equation This algebraic equation is used to relate the neutron flux N to the reactor power W_R which is assumed to be homogeneous linear (see assumption R7) in the form:

$$\Psi(N) = c_\Psi N \tag{4}$$

with c_Ψ being a known constant.

The effect of the control rod position on the reactivity In the operating region from where we have plant data, it is enough to use a quadratic nonlinear function to model the dependence of the reactivity on the control rod position, i.e.

$$\rho(v) = p_1 v^2 + p_2 v + p_3 \tag{5}$$

where p_1, p_2, p_3 are scalar parameters to be estimated. The form (5) is advantageous from the point of view of parameter estimation because it contains a minimal number of unknown parameters and it is linear in them.

3.1.4 The reactor power controller

The reactor power controller has two operating modes:

1. "N" mode, when the value of the neutron flux is fed back to adjust the rod position to keep the neutron flux constant or to follow a reference trajectory,
2. "T" mode, when the pressure of the steam in the secondary circuit generated by the steam generator is used for the feedback.

The "N" mode of the reactor power controller is considered in our model with a static state feedback and with a constraint on the control rod velocity.

3.2 The liquid in the primary circuit (PC)

The liquid in the tubes of the primary circuit including the liquid in the reactor, in the primary side tubes of the six steam generators and that in the pressurizer are considered together to form a simple concentrated parameter balance volume for the liquid in the primary circuit.

3.2.1 Modeling assumptions

The following simplifying modeling assumptions are considered.

PC1 There is only a single concentrated parameter balance volume for the total liquid amount in the primary circuit that is assumed to be in liquid phase and assumed to be pure water (the amount of boron is regarded to be negligible).

PC2 The density of the water (φ) is assumed to depend on the temperature following a second order polynomial, and its dependence on the pressure is neglected.

PC3 The specific heat of the water ($c_{p,PC}$) is assumed to be a constant value (its dependence on the temperature and pressure is neglected).

PC4 The effect of the heating in the pressurizer that is applied to regulate the pressure is neglected in the energy balance for the water in the primary circuit.

PC5 It is assumed that the flow rate in the primary circuit is regulated in such a way that the temperature increase of the water in the reactor is approximately 30^oC, thus the temperature difference between the hot leg and cold leg temperatures is $T_{PC,HL} - T_{PC,CL} \approx 30^oC$. The average temperature of the water in the primary circuit is then

$$T_{PC} = \frac{T_{PC,HL} + T_{PC,CL}}{2}$$

3.2.2 Conservation balances

The overall mass balance of the water is in the form

$$\frac{dM_{PC}}{dt} = m_{in} - m_{out} \tag{6}$$

where M_{PC} is the water mass, m_{in} is the inlet mass flow rate, and m_{out} is the purge mass flow rate of the primary circuit.

The energy balance for the internal energy U_{PC} takes into account the energy generated by the reactor in unit time W_R, the energy transferred to the secondary circuit $(6 \cdot W_{SG})$ through the six steam generators, the energy effect of the mass inlet and purge (the first and second term) and the energy loss to the environment $W_{loss,PC}$:

$$\frac{dU_{PC}}{dt} = c_{p,PC} m_{in} T_{PC,I} - c_{p,PC} m_{out} T_{PC,CL} + W_R - 6 \cdot W_{SG} - W_{loss,PC} \tag{7}$$

where $c_{p,PC}$ is the specific heat of the water and $T_{PC,I}$ is the inlet water temperature.

3.2.3 Constitutive equations

The constitutive equations relate internal energy to temperature, the temperatures in the primary circuit to each other taking into account assumption PC5 and relate the energy transferred to the secondary circuit to the temperatures as follows.

$$U_{PC} = c_{p,PC} M_{PC} T_{PC}, \tag{8}$$

$$T_{PC,HL} = T_{PC} + 15 \quad , \quad T_{PC,CL} = T_{PC} - 15 \tag{9}$$

$$W_{SG} = K_{T,SG}(T_{PC} - T_{SG}) \tag{10}$$

where $K_{T,SG}$ is the heat transfer coefficient and T_{SG} is the (averaged) secondary circuit liquid temperature of the steam generator(s).

3.3 The pressurizer (PR)

The aim of the pressurizer as an operating unit is twofold: it regulates the pressure in the primary circuit by heating its water content (by a heating power $W_{heat,PR}$) and also serves as an indicator for the primary circuit inventory controller by its water level ℓ_{PR}.

3.3.1 Modeling assumptions

The liquid in the pressurizer is part of the primary circuit water, therefore these two operating units, and the assumptions imposed on their models are closely related.

PR1 The liquid in the pressurizer is assumed to be pure water (the amount of boron is regarded to be negligible) and it is assumed to be part of the water in the primary circuit, therefore no separate mass balance is constructed for the liquid phase. The water mass in the pressurizer is computed as an excess to a nominal mass M_{PC}^0 in the primary circuit.

PR2 The density of the water (φ) is assumed to depend on the temperature following a second order polynomial, and its dependence on the pressure is neglected (same as assumption PC2).

PR3 The specific heat of the water ($c_{p,PR}$) is assumed to be a constant value (its dependence on the temperature and pressure is neglected).

PR4 The vapor in the pressurizer is assumed to be saturated, and the vapor mass is assumed to be negligible compared to that of the liquid. Therefore no balances are constructed for the vapor in the pressurizer.

PR5 The pressure of the saturated vapor is assumed to depend linearly on the temperature in the pressurizer T_{PR} following a known function p_*^T

3.3.2 Conservation balances

Energy balance is constructed for the liquid in the pressurizer taking into account the mass in/out flow from the primary circuit m_{PR}, the heat loss $W_{loss,PR}$ and the heating $W_{heat,PR}$

$$\frac{dU_{PR}}{dt} = \chi_{m_{PR}>0} c_{p,PC} m_{PR} T_{PC,HL} + \chi_{m_{PR}<0} c_{p,PR} m_{PR} T_{PR} - W_{loss,PR} + W_{heat,PR} \tag{11}$$

with $\chi_{condition}$ is the indicator function of *condition* that is 1 when the condition is fulfilled and zero otherwise.

3.3.3 Constitutive equations

Physico-chemical property relations are taken into account to describe the relationships between internal energy, pressure and saturated pressure as functions of the temperature in the pressurizer vessel as follows.

$$U_{PR} = c_{p,PR} M_{PR} T_{PR}, \tag{12}$$

$$p_{PR} = p_*^T(T_{PR}) \tag{13}$$

where $c_{p,PR}$ is the specific heat, M_{PR} is the liquid mass, p_{PR} is the pressure, and p_*^T is the saturated vapor pressure.

Saturated vapor pressure The pressure of a saturated vapor depends only on the temperature (see Eq. (13)) where the function p_*^T is in the following form (ThermExcel, 2006):

$$p_*^T(\widetilde{T}) = 28884.78 - 258.01\widetilde{T} + 0.63\widetilde{T}^2 \tag{14}$$

with $\widetilde{T}$ being the temperature measured in ^oC and the pressure is obtained in kPa.

Density-temperature function If a second order polynomial type dependence of the density on the temperature is assumed (see assumption PC2) then one can approximate the density function by a quadratic function

$$\varphi(\widetilde{T}) = c_{\varphi,0} + c_{\varphi,1}\widetilde{T} + c_{\varphi,2}(\widetilde{T})^2 \tag{15}$$

with the coefficients

$$c_{\varphi,0} = 581.2, \quad c_{\varphi,1} = 2.98, \quad c_{\varphi,2} = -0.00848 \tag{16}$$

where $\widetilde{T}$ is the temperature measured in ^oC.

Pressurizer water level A set of constitutive equations describes the effect of the variation in the primary circuit water mass M_{PC} on the level of the pressurizer ℓ_{PR}

$$\begin{aligned} V_{PC} &= \frac{M_{PC}}{\varphi(T_{PC})} \quad , \quad V_{PR} = V_{PC} - V_{PC}^0 && (17) \\ \ell_{PR} &= \frac{V_{PR}}{A_{PR}}, && (18) \end{aligned}$$

where V_{PC} is the overall water volume in the primary circuit, V_{PC}^0 is its nominal value (a constant), $\varphi(T_{PC})$ is the density, V_{PR} is the liquid volume and A_{PR} is the cross-section of the pressurizer.

The water mass in the pressurizer is computed from the overall mass M_{PC} in the primary circuit according to assumption PR1 as

$$M_{PR} = M_{PC} - M_{PC}^0 = M_{PC} - \varphi(T_{PC}) V_{PC}^0 \tag{19}$$

The above equation can be used to compute the mass in/outflow from the primary circuit to the pressurizer as follows:

$$\begin{aligned} m_{PR} &= \frac{dM_{PR}}{dt} = \frac{dM_{PC}}{dt} - V_{PC}^0 \frac{\partial \varphi(T_{PC})}{\partial T_{PC}} \frac{dT_{PC}}{dt} \\ m_{PR} &= m_{in} - m_{out} - V_{PC}^0 \left(c_{\varphi,1} + 2c_{\varphi,2} T_{PC}\right) \frac{dT_{PC}}{dt} && (20) \end{aligned}$$

with a second order approximation of the density-temperature function (Eq. 15).

3.3.4 Inventory controller for the primary circuit

The inventory controller of the primary circuit uses the measured value of the average temperature T_{PC} and the water level in the pressurizer ℓ_{PR} to set the value of the inlet mass flow rate m_{in} to keep the mass in the primary circuit M_{PC} to its reference value M_{PC}^{ref} according to the following simple linear equations

$$\hat{\ell}_{PR} = a_1 \ell_{PR} - a_2 T_{PC} - a_3 M_{PC}^{ref} + I_{elt} \tag{21}$$

$$m_{in} = a_4 \hat{\ell}_{PR} \tag{22}$$

with a_1, a_2, a_3, a_4 and I_{elt} being known constants.

3.3.5 Pressure controller

The pressure controller is a linear discrete-time, dynamic inversion based feedback of the form (Szabó et al., 2005):

$$x(k+1) = Ax(k) + B \begin{bmatrix} T_{ref}(k) \\ d(k) \end{bmatrix} \tag{23}$$

$$W_{heat,PR}(k) = Cx(k) + D \begin{bmatrix} T_{ref}(k) \\ d(k) \end{bmatrix}, \tag{24}$$

where the matrices A, B, C, D of appropriate dimensions contain the known controller parameters, x represents the two-dimensional state of the controller with $x_1(k)$ corresponding to the pressurizer water temperature T_{PR}, T_{ref} is the temperature reference and the elements of the vector d are the measurable disturbances.

3.4 The steam generators (SG)

The steam generators connect the primary and secondary circuit and transfer the energy generated by the reactor to the secondary steam flow. There are six steam generators in a reactor unit but we model them as a single operating unit.

3.4.1 Modeling assumptions

Because the focus of our model is the primary circuit and its controllers, the following simplifying assumptions are made for the steam generators.

SG1 The dynamics of the primary side of the steam generators is very quick compared that of the secondary side, therefore it is assumed to be in a quasi steady state and no conservation balances are constructed for it.

SG2 The dynamics of the secondary side vapor phase in the steam generators is also assumed to be very quick compared that of the secondary side liquid, an equilibrium is assumed between the water and the vapor phases.

SG3 Constant physical properties are assumed for the secondary side of the steam generators.

SG4 All the controllers acting on the secondary side (including the liquid level controller and the secondary steam pressure controller) are assumed to be ideal.

3.4.2 Conservation balances

There is only a single balance volume in the steam generators, the liquid of the secondary side, where the overall mass balance is simplified to an algebraic equation, because the inlet secondary water mass flow rate $m_{SG,SW}$ and the outlet secondary steam mass flow rate $m_{SG,SS}$ is kept to be equal by the ideal water level controller of the steam generators

$$m_{SG,SW} = m_{SG,SS} = m_{SG}$$

Then the energy balance for the secondary water in the steam generators is in the form

$$\begin{aligned} \frac{dU_{SG}}{dt} &= c^L_{p,SG} m_{SG} T_{SG,SW} - c^V_{p,SG} m_{SG} T_{SG} - m_{SG} E_{evap,SG} + \\ &+ K_{T,SG}(T_{PC} - T_{SG}) - W_{loss,SG} \end{aligned} \tag{25}$$

where U_{SG} is the internal energy, $c^L_{p,SG}$ is the water specific heat, $c^V_{p,SG}$ is the vapor specific heat, $T_{SG,SW}$ is the inlet temperature, T_{SG} is the temperature, $E_{evap,SG}$ is the evaporation energy, and $W_{loss,SG}$ is the heat loss.

3.4.3 Constitutive equations

The algebraic constitutive equations describe the relationships between physical properties and temperature:

$$U_{SG} = c^L_{p,SG} M_{SG} T_{SG} \tag{26}$$

$$p_{SG} = p^T_*(T_{SG}) \tag{27}$$

where p_{SG} is the pressure, and p^T_* is the same quadratic function as in Eq. (14) for the pressurizer.

4 The state-space model of the system

In order to perform model verification, parameter estimation and model validation, one needs to transform the above developed engineering model into its state-space model form. From system theoretic point of view, the developed model falls into the concentrated parameter (i.e. lumped) nonlinear class, that has the following general state-space model form:

$$\frac{dx(t)}{dt} = F(x(t), u(t), d(t)) \quad , \quad y(t) = H(x(t), u(t), d(t)) \tag{28}$$

where $x(t)$ is the state, $u(t)$ is the input, $d(t)$ is the disturbance and $y(t)$ is the output variable. The differential equations with the nonlinear function F form the state equations, and the algebraic ones with the nonlinear function H constitute the output equations.

It is clear from the derivation of the engineering model, that the conservation balances will be transformed to the state equations by substituting all algebraic constitutive equations into the differential ones, if this is possible.

4.1 Intensive form of the balance equations

The first step of obtaining a state-space model consists of deriving the intensive form of the energy balance equations to obtain differential equations for the measurable temperature $T_{\bullet}$ instead of its related internal energy $U_{\bullet}$ in a balance volume identified by $\bullet$ with mass $M_{\bullet}$. For this purpose the energy-temperature relationship is used that is in the form

$$U_{\bullet} = c_{p,\bullet} M_{\bullet} T_{\bullet}$$

where $c_{p,\bullet}$ is the specific heat. If one differentiates the above equation with respect to time and assumes constant specific heat, then

$$\frac{dU_{\bullet}}{dt} = c_{p,\bullet} M_{\bullet} \frac{dT_{\bullet}}{dt} + c_{p,\bullet} T_{\bullet} \frac{dM_{\bullet}}{dt}$$

and the factor $\frac{dM_{\bullet}}{dt}$ in the last term can be substituted to this expression from the mass balance for the same balance volume.
Following the above general procedure, the intensive forms of the energy balance equation for the balance volumes PC, SG and PR are:

$$\begin{aligned} \frac{dT_{PC}}{dt} &= \frac{1}{c_{p,PC} M_{PC}} \left(c_{p,PC} m_{in} \left(T_{PC,I} - T_{PC}\right) + \right. \\ &\quad \left. + c_{p,PC} m_{out} \left(T_{PC} - T_{PC} + 15\right) + W_R - 6 \cdot W_{SG} - W_{loss,PC}\right) \end{aligned} \tag{29}$$

$$\begin{aligned} \frac{dT_{SG}}{dt} &= \frac{1}{c^L_{p,SG} M_{SG}} \left(c^L_{p,SG} m_{SG} T_{SG,SW} - c^V_{p,SG} m_{SG} T_{SG} - m_{SG} E_{evap,SG} + \right. \\ &\quad \left. + K_{T,SG}(T_{PC} - T_{SG}) - W_{loss,SG}\right) \end{aligned} \tag{30}$$

$$\begin{aligned} \frac{dT_{PR}}{dt} &= \frac{1}{c_{p,PR} M_{PR}} \left(\chi_{m_{PR}>0} c_{p,PC} m_{PR} T_{PC,HL} + \chi_{m_{PR}<0} c_{p,PR} m_{PR} T_{PR} - c_{p,PR} m_{PR} T_{PR} - \right. \\ &\quad \left. - W_{loss,PR} + W_{heat,PR}\right) \end{aligned} \tag{31}$$

4.2 State, input and output variables

It follows from the general structure of a nonlinear state-space model (28) that the time-dependent variables (see the list of all variables in Table 6) that appear in the differential equations can be classified as follows:

- *State variables*: differential variables in the differential equations
  N, M_{PC}, T_{PC}, T_{PR}, T_{SG}
- *Input variables*: manipulable variables affected by the considered controllers
  v, m_{in}, $W_{heat,PR}$
- *Disturbances*: all other possibly time-dependent variables appearing on the right-hand side of the differential equations
  m_{out}, m_{SG}, M_{SG}, $T_{SG,SW}$, $T_{PC,I}$
- *Output variables*: measurable variables that are regulated by the considered controllers
  N (W_R), p_{SG}, ℓ_{PR} (M_{PC}), p_{PR}

Majority of the system variables above can be directly (or indirectly) measured on the units of the Paks Nuclear Power Plant, see Table 1 for the details.

4.3 State and output equations

It is easy to see that one can substitute all of the algebraic constitutive equations into the differential ones (into the overall mass balances and to the intensive form of the energy balances), thus the following set of state equations is obtained:

$$\begin{aligned}
\frac{dN}{dt} &= \frac{1}{\Lambda}\left(p_1 v^2 + p_2 v + p_3\right) N + S && (32)\\
\frac{dM_{PC}}{dt} &= m_{in} - m_{out} && (33)\\
\frac{dT_{PC}}{dt} &= \frac{1}{c_{p,PC} M_{PC}} \left[c_{p,PC} m_{in} \left(T_{PC,I} - T_{PC}\right) + c_{p,PC} m_{out} \left(15\right) + \right. \\
&\left. + c_{\Psi 1} N - 6 \cdot K_{T,SG}(T_{PC} - T_{SG}) - W_{loss,PC}\right] && (34)\\
\frac{dT_{PR}}{dt} &= \frac{1}{c_{p,PR} M_{PR}} \left(\chi_{m_{PR}>0} c_{p,PC} m_{PR} T_{PC,HL} + \chi_{m_{PR}<0} c_{p,PR} m_{PR} T_{PR} - c_{p,PR} m_{PR} T_{PR} - \right. \\
&\left. - W_{loss,PR} + W_{heat,PR}\right) && (35)\\
\frac{dT_{SG}}{dt} &= \frac{1}{c^L_{p,SG} M_{SG}} \left(c^L_{p,SG} m_{SG} T_{SG,SW} - c^V_{p,SG} m_{SG} T_{SG} - m_{SG} E_{evap,SG} + \right. \\
&\left. + K_{T,SG}(T_{PC} - T_{SG}) - W_{loss,SG}\right) && (36)
\end{aligned}$$

where Eq. (32) is obtained by combining Eqs. (3) and (5).

The output equations are as follows:

$$\begin{aligned}
W_R &= c_{\Psi 1} N && (37)\\
p_{SG} &= p^T_*(T_{SG}) && (38)\\
\ell_{PR} &= \frac{1}{A_{PR}} \left(\frac{M_{PC}}{\varphi_{PC}(T_{PC})} - V^0_{PC}\right) && (39)\\
p_{PR} &= p^T_*(T_{PR}) && (40)
\end{aligned}$$

4.4 Model parameters

The model parameters shown in Table 7 are the constants in the above state-space model equations. They can be classified according to the operating unit they belong to as follows:

`(R)`: $\frac{1}{\Lambda}$, S; parameters in $\rho(v)$ (p_1, p_2, p_3 in Eq. (5)), $c_{\Psi 1}$;
`(PC)`: $c_{p,PC}$, $K_{T,SG}$, $W_{loss,PC}$;
`(PR)`: $c_{p,PR}$, $W_{loss,PR}$; V_{PC}^0, A_{PR};
`(SG)`: $c_{p,SG}^L$, $c_{p,SG}^V$, $W_{loss,SG}$;
`(phys-chem)`: parameters in functions φ (Eq. (15)) and p_*^T (Eq. (14)).

5 Model verification, parameter estimation and validation

Model verification, parameter estimation and validation are the key necessary steps that one should carry on before using any newly developed model. The aim of model verification is to check if the model behaves in a reasonable way, that is, it matches at least qualitatively our engineering expectations. Thereafter, measured data collected from the real plant can be used to determine or refine any unknown or not exactly known model parameters which constitutes the parameter estimation step. Finally, a new set of measured data is used to see if the model fulfills the original modeling goal, that is, it is able to describe the behavior of the modeled system.

5.1 Model parameter estimation

The state-space model form in Eqs. (32)-(40) was used for parameter estimation purposes, that is highly nonlinear both in the time-dependent variables and in many of the parameters to be estimated.

5.1.1 Parameter estimation strategy and method

Because of the above mentioned nonlinearity, one needs a dynamic predictive model, measured data and an optimization-based estimation method to perform model parameter estimation.

Measured data from three of the VVER-440 units of Paks Nuclear Power Plant were collected for parameter estimation purposes. In order to span a relatively wide operating domain, transient data of increasing and decreasing the power of the units when shifting from day to night load conditions and back have been used.
Table 1 lists the available measured signals that can be used for model parameter estimation.

`This is the approximate location of Table 1`

Dynamic simulator A dynamic simulator implemented in MATLAB/SIMULINK (Matlab, 2000) has been used for the parameter estimation to generate the values of the output and state variables given the value of the input and disturbance variables and the model parameters according to the model equations (32)-(40).

Parameters to be estimated Some parameters present in the model were considered to be known in order to reduce the number of parameters to be estimated. Preliminary sensitivity analysis has been used together with the evaluation of the accuracy of the available parameter values to select those parameters that need to be included in the set to be estimated. Therefore, physico-chemical and equipment parameters were generally regarded as known.

Parameter estimation method Because of the nonlinearity of the model in its parameters, an optimization-based parameter estimation method, the Nelder-Mead simplex method (Nelder and Mead, 1965; Lagarias et al., 1998) available in MATLAB has been used that is effective only for a low number of parameters. Therefore, the parameter estimation has been carried out separately for each operating unit. The simplex method requires to have good initial values of the parameters, that we obtained from the literature (Perry and Green, 1999; ThermExcel, 2006) from plant design data and from operation experience.

5.1.2 Parameter estimation results

The results of the parameter estimation are given separately for each operating unit by describing the input, disturbance and output variables, the estimated and known parameters and the measure of fit in terms of the 2-norm between the measured and the model-predicted output signals, i.e.

$$e = \sqrt{\frac{\int_0^T (y_m(t) - y_p(t))^2 dt}{\int_0^T y_m^2(t) dt}} \tag{41}$$

where y_m is the measured output, y_p is the model-predicted (simulated) output signal and T denotes the time-span of the measurement/simulation. It is important to observe that e is a dimensionless quantity, independent of scaling.

Reactor

Input variable: v (control rod position)
Output variable: N (neutron flux)
Known parameters: Λ
Parameters to be estimated: p_1, p_2, p_3, S.

The estimated parameters and the measure of fit is given in Table 2 together with an example of the fit in the output signal shown in Fig. 2.

`This is the approximate location of Table 2`

`This is the approximate location of Figure 2`

Water in the primary circuit

Input and disturbance variables: m_{in}, $T_{PC,I}$, m_{out}, N, T_{SG}

Output variable: $T_{PC} = \frac{1}{2}(T_{PC,HL} + T_{PC,CL})$

Known parameter: c_{Φ}

Parameters to be estimated: $c_{p,PC}$, $K_{T,SG}$, $W_{loss,PC}$

The estimated parameters and the measure of fit is given in Table 3 together with an example of the fit in the output signal shown in Fig. 3.

`This is the approximate location of Table 3`

`This is the approximate location of Figure 3`

Steam generator

Input and disturbance variables: $m_{SG,SW}$, $T_{SG,SW}$, $m_{SG,SS}$, $T_{PC} = \frac{1}{2}(T_{PC,HL} + T_{PC,CL})$

Output variable: T_{SG}, p_{SG}

Known parameter: $K_{T,SG}$ (from the estimation of parameters of primary circuit)

Parameters to be estimated: M_{SG}, $W_{loss,SG}$, $c^{L}_{p,SG}$, $c^{V}_{p,SG}$

The estimated parameters and the measure of fit is given in Table 4 together with an example of the fit in the output signal shown in Fig. 4.

`This is the approximate location of Table 4`

`This is the approximate location of Figure 4`

Pressurizer

Input and disturbance variables: m_{in}, $W_{heat,PR}$, m_{out}

Output variables: ℓ_{PR}, p_{PR}, T_{PR}

Known parameters: coefficients in $\varphi(T)$

Parameters to be estimated: $c_{p,PR}$, $W_{loss,PR}$

Here the measured data from unit 2 were used where an old, on-off type pressure controller has been operating that provided sufficient excitation for the parameter identification. The estimated parameters and the measure of fit is given in Table 5 together with an example of the fit in the output signal shown in Fig. 5.

`This is the approximate location of Table 5`

`This is the approximate location of Figure 5`

5.2 Model integration and verification

In this final step, the previously separately identified subsystems were integrated into one model described by equations (32)-(40). The trajectories of the state variables were computed from the model, the measured data were only used to determine their initial values and to substitute the other necessary input and disturbance variable measurements (e.g. m_{in}, m_{out}) on the right hand side of the equations. The temperature data of the integrated model compared to the measurements for unit 1 are shown in Fig. 6

Model verification has been performed by using small step-like control rod position movements in both directions. The model for unit 3 with the identified parameters was used for the model verification tests, the initial state corresponded to its steady state corresponding to the measured data (e.g. neutronflux is 100.3 %).

The time variation of the state and output variables of the system as a response to control rod position changes (change is one per cent of value of steady state rod position) are shown in Figs 7-10.

It is seen from the figures that the model satisfies engineering expectations in each of the state and output variables and meets not only the qualitative but also the quantitative requirements in both the steady-state values and in the approximate time constants. The unit step responses are not exactly comparable to that observed experimentally in the transient data of increasing and decreasing the power of the units when shifting from day to night load conditions and back, because we have assumed ideal level controller for the primary circuit mass holdup (the mass in the primary circuit is kept constant) during the verification tests and, of course, no additional measurement noise has been added.

`This is the approximate location of Figure 6`

`This is the approximate location of Figure 7`

`This is the approximate location of Figure 8`

`This is the approximate location of Figure 9`

`This is the approximate location of Figure 10`

6 Conclusions

A low-dimensional model in the form of nonlinear ordinary differential equations has been proposed in this paper for describing the most important dynamic phenomena in the primary loop of VVER-type nuclear power plants. The primary circuit has been decomposed to subsystems based on a system and control theoretical point of view taking into consideration the present controller configuration, too. The model variables have been classified appropriately and the unknown model parameters have been estimated using a quadratic error function and a nonlinear optimization algorithm. The identified model shows excellent fit to the measured data and it will probably serve as a basis for the integrated re-design of the primary loop controllers in the near future.

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Appendix: Notations

The variables and parameters are grouped according to their operating units. The subscripts in the variable names refer to the operating unit with $_R$ being the reactor, $_{PC}$ is the primary circuit, $_{PR}$ is the pressurizer, $_{SG}$ is the steam generator identifier.

`This is the approximate location of Table 6`

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Identifier	Variable	Type:(**s**tate, **i**nput, **o**tput, **d**isturbance)
N	R neutron flux	s
v	R control rod position	i
W_R	R reactor power	o
m_{in}	PC inlet mass flow rate	i
m_{out}	PC purge mass flow rate	d
$T_{PC,I}$	PC inlet temperature	d
$T_{PC,CL}$	PC cold leg temperature	(s)
$T_{PC,HL}$	PC hot leg temperature	(s)
p_{PR}	PR pressure	o,(s)
T_{PR}	PR temperature	s
ℓ_{PR}	PR water level	o,(s)
$W_{heat,PR}$	PR heating power	i
$m_{SG,SW}$	SG water mass flow rate	d
$m_{SG,SS}$	SG steam mass flow rate	d
$T_{SG,SW}$	SG inlet water temperature	d
p_{SG}	SG steam pressure	o

Table 1: Measured variables

Parameter	**Unit**	unit 1 Time span: 4.44h	unit 3 Time span: 2.5h	unit 4 Time span: 2.5h
p_1	$1/m^2$	$-1.322 \cdot 10^{-4}$	$-1.223 \cdot 10^{-4}$	$-1.286 \cdot 10^{-4}$
p_2	$1/m$	$-6.08 \cdot 10^{-5}$	$-5.502 \cdot 10^{-5}$	$-6.79 \cdot 10^{-5}$
p_3	1	$-2.85 \cdot 10^{-4}$	$-1.953 \cdot 10^{-4}$	$-2.887 \cdot 10^{-4}$
S	$\%/s$	2859	1938.9	2910.3
Error	1	$5.7678 \cdot 10^{-3}$	$7.8638 \cdot 10^{-3}$	$3.5002 \cdot 10^{-3}$

Table 2: Estimated reactor parameters

Parameter	**Unit**	unit 1 Time span: 4.44h	unit 3 Time span: 2.5h	unit 4 Time span: 2.5h
$c_{p,PC}$	$J/kg/K$	5415	5355	5197.6
$K_{T,SG}$	W/K	$8.4004 \cdot 10^6$	$9.5296 \cdot 10^6$	$7.554 \cdot 10^6$
$W_{loss,PC}$	W	$2.2469 \cdot 10^8$	$2.996 \cdot 10^7$	$3.2733 \cdot 10^8$
Error	1	$6.5248 \cdot 10^{-4}$	$9.0649 \cdot 10^{-4}$	$1.717 \cdot 10^{-3}$

Table 3: Estimated primary circuit parameters

Parameter	**Unit**	unit 1 Time span: 4.44h	unit 3 Time span: 2.5h	unit 4 Time span: 2.5h
M_{SG}	kg	35611	34920	34374
$W_{loss,SG}$	W	$1.9166 \cdot 10^5$	$1.8932 \cdot 10^7$	$1.324 \cdot 10^7$
$c^V_{p,SG}$	$J/kg/K$	3489	3635.6	3449.8
$c^L_{p,SG}$	$J/kg/K$	3871.3	3809.9	4314
Error	1	$2.6 \cdot 10^{-3}$	$1.4086 \cdot 10^{-4}$	$8.7336 \cdot 10^{-4}$

Table 4: Estimated steam generator parameters

Parameter	**Unit**	Old data Time span: 8.88 h
$c_{p,PR}$	$J/kg/K$	6873.1
$W_{loss,PR}$	W	$1.6823 \cdot 10^5$
Error	1	$6.2436 \cdot 10^{-4}$

Table 5: Estimated pressurizer parameters

Description	**Identifier**	**Unit**	**Nominal value**		
			unit 1	unit 3	unit 4
R neutronflux	$N(t)$	%	100.3	99.3	100.8
R control rod position relative to the nom. pos.	$v(t)$	cm		0	
PC water mass	$M_{PC}(t)$	kg		200000	
PC temperature (mean)	$T_{PC}(t)$	^oC	278.03	281.13	280.08
PC temperature (hot leg)	$T_{PC,HL}(t)$	^oC	293.08	296.13	295.21
PC temperature (cold leg)	$T_{PC,CL}(t)$	^oC	262.98	266.13	264.95
PC inlet temperature	$T_{PC,I}(t)$	^oC	246.1	258.85	258.23
PC purge mass flow rate	$m_{out}(t)$	kg/s	2.9722	2.11	2.29722
PC inlet mass flow rate	$m_{in}(t)$	kg/s	1.4222	1.4222	2.19444
PR water level	$\ell_{PR}(t)$	m		4.8000	
PR water mass	$M_{PR}(t)$	kg		19400	
PR temperature	$T_{PR}(t)$	^oC		326.57	
PR pressure	$p_{PR}(t)$	bar		123	
PR heating power	$W_{heat}(t)$	kW		168	
SG water mass	$M_{SG}(t)$	kg	35611	34920	34374
SG water level	$\ell_{SG}(t)$	m		1.850	
SG temperature	$T_{SG}(t)$	^oC	255.13	257.78	256.72
SG pressure	$p_{SG}(t)$	bar	43.3	45.3	44.5
SG secondary circ. steam mass flow rate	$m_{SG,SS}(t)$	kg/s	120.56	119.31	120.11
SG secondary circ. water mass flow rate	$m_{SG,SW}(t)$	kg/s	120.56	119.31	120.11
SG secondary circ. inlet temperature	$T_{SG,SW}(t)$	^oC	219.65	220.85	220.12
Reactor power	$W_R(t)$	W		$13.75 \cdot 10^8$	
Power transferred to the steam generators	$6 \cdot W_{SG}(t)$	W	$11.542 \cdot 10^8$	$13.351 \cdot 10^8$	$10.589 \cdot 10^8$

Table 6: Model variables

Description	**Identifier**	**Unit**	**Nominal value**		
			unit 1	unit 3	unit 4
R constant in the power equation	c_Ψ	$W/\%$	$13.75 \cdot 10^6$		
R total fraction of delayed neutrons	β		0.0064		
R generation time	Λ	sec	10^{-5}		
R average half-life	λ	sec^{-1}	0.1		
R rod reactivity coefficients	$p_1,\ p_2,\ p_3$		see Table 2		
R flux of the constant neutron source	S	%	2859	1938.9	2910.3
PC specific heat (on 282 ^oC)	$c_{p,PC}$	$J/kg/K$	5415	5355	5197.6
PC-SG Heat transfer coefficient	$K_{T,SG}$	$J/K/s$	$8.4004 \cdot 10^6$	$9.5296 \cdot 10^6$	$7.554 \cdot 10^6$
PC heat loss	$W_{loss,PC}$	J/s	$2.2469 \cdot 10^8$	$2.996 \cdot 10^7$	$3.2733 \cdot 10^8$
PC water nominal volume	V_{PC}^0	m^3	242		
PC water nominal mass	M_{PC}^0	kg	180600		
PR water specific heat (on 325 ^oC)	$c_{p,PR}$	$J/kg/K$	6873.1		
PR heat loss	$W_{loss,PR}$	J/s	$1.6823 \cdot 10^5$		
PR vessel cross section	A_{PR}	m^2	4.52		
PR vessel volume	$V_{PR,vessel}$	m^3	44		
SG secondary circ. water specific heat (at 260 ^oC)	$c_{p,SG}^L$	$J/kg/K$	3871.3	3809.9	4314
SG secondary circ. vapor specific heat (at 260 ^oC)	$c_{p,SG}^V$	$J/kg/K$	3489	3635.6	3449.8
SG heat loss	$W_{loss,SG}$	J/s	$1.9166 \cdot 10^5$	$1.8932 \cdot 10^7$	$1.324 \cdot 10^7$
SG evaporation energy (at 260 ^oC)	$E_{evap,SG}$	J/kg	$1.658 \cdot 10^6$		
Water density function	$\varphi(T)$	kg/m^3			
(coefficients)	$c_{\varphi,0}$	kg/m^3	581.2		
	$c_{\varphi,1}$	$kg/m^3/K$	2.98		
	$c_{\varphi,2}$	$kg/m^3/K^2$	-0.00848		
Saturated vapor function coefficients	p_*^T		see eq. (14)		

Table 7: Model parameters

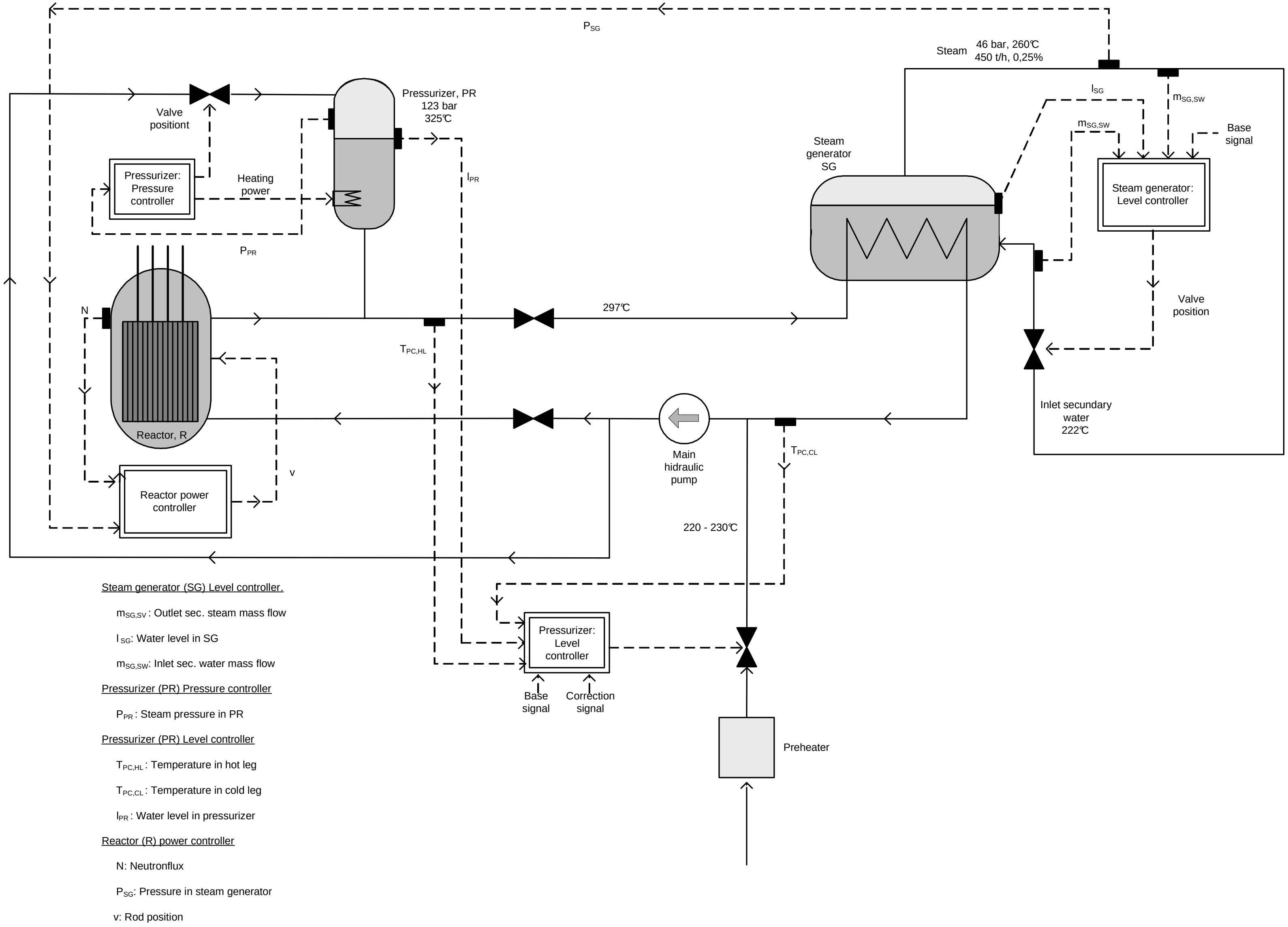
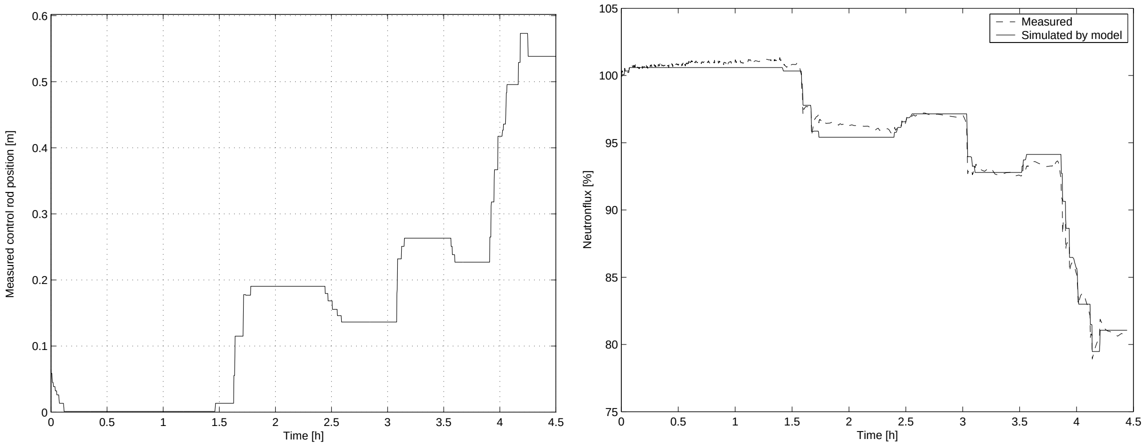


Figure 1: Process flowsheet with the operating units of the simplified model



a **b**

Figure 2: **a**. Measured control rod position in unit 1, **b**. The measured and model computed neutron flux in unit 1

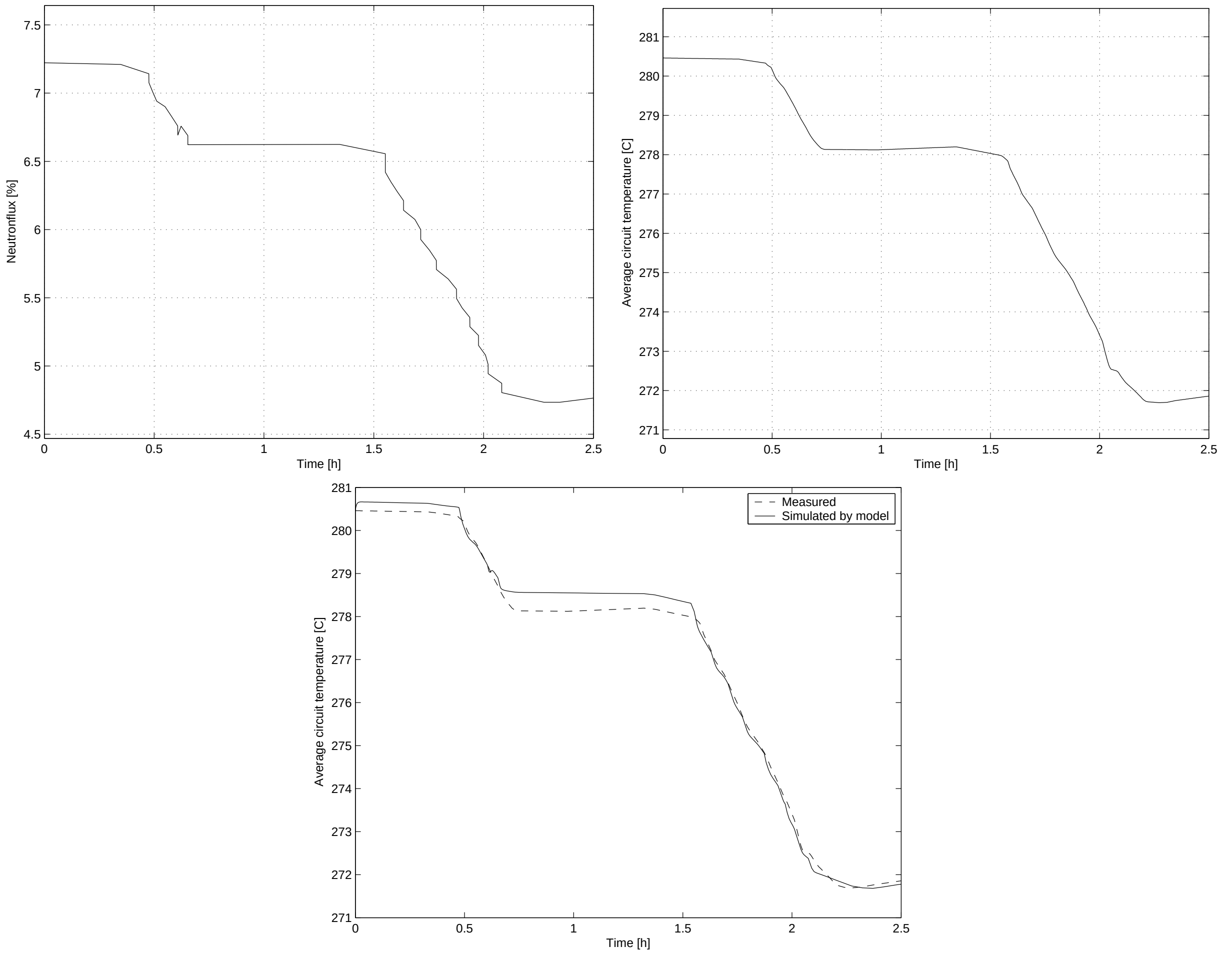


Figure 3: Measured neutron flux and average temperature and the fit, unit 3

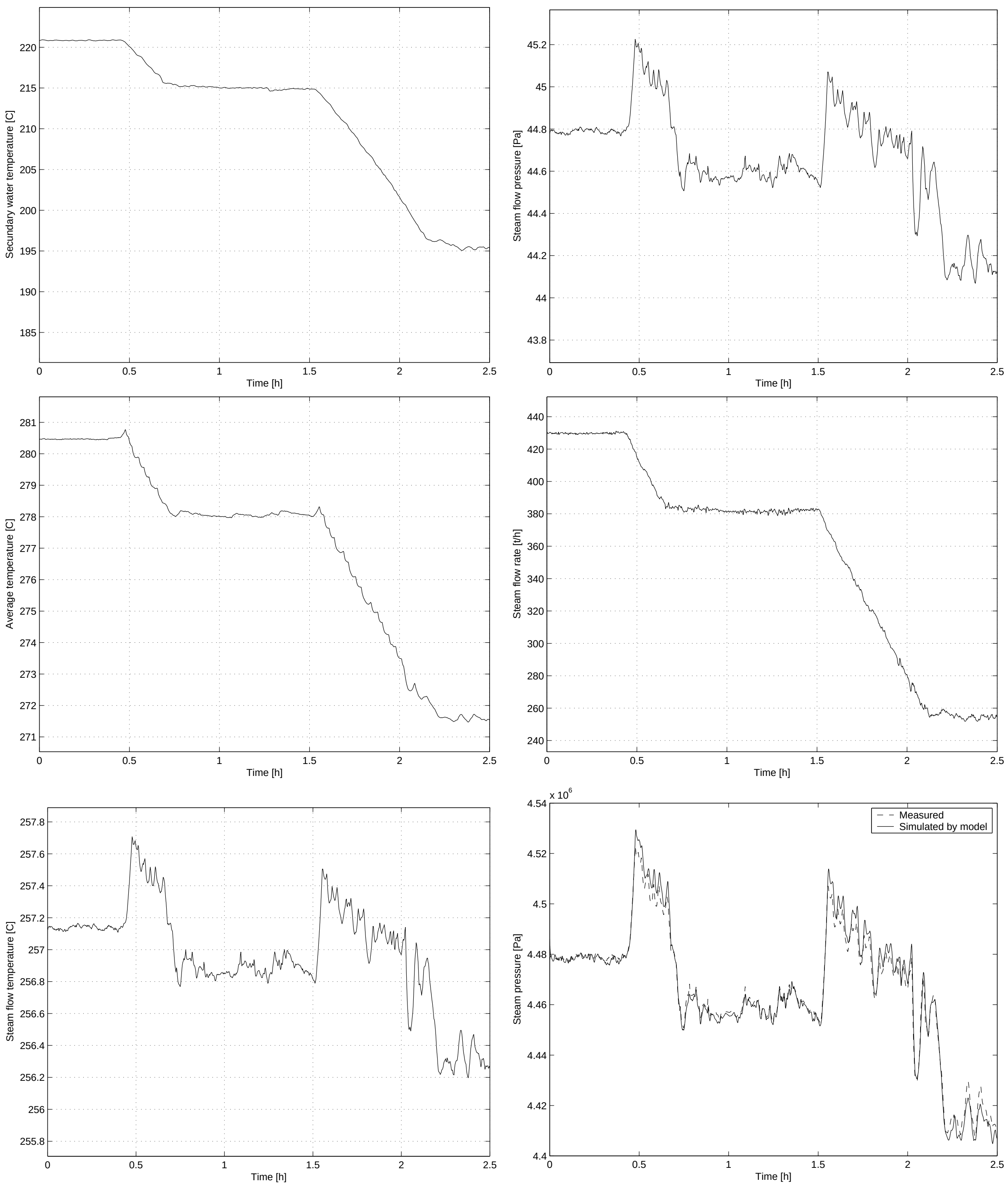


Figure 4: Measured secondary water temperature, pressure, average primary temperature, steam flow rate and temperature and the fit, unit 3

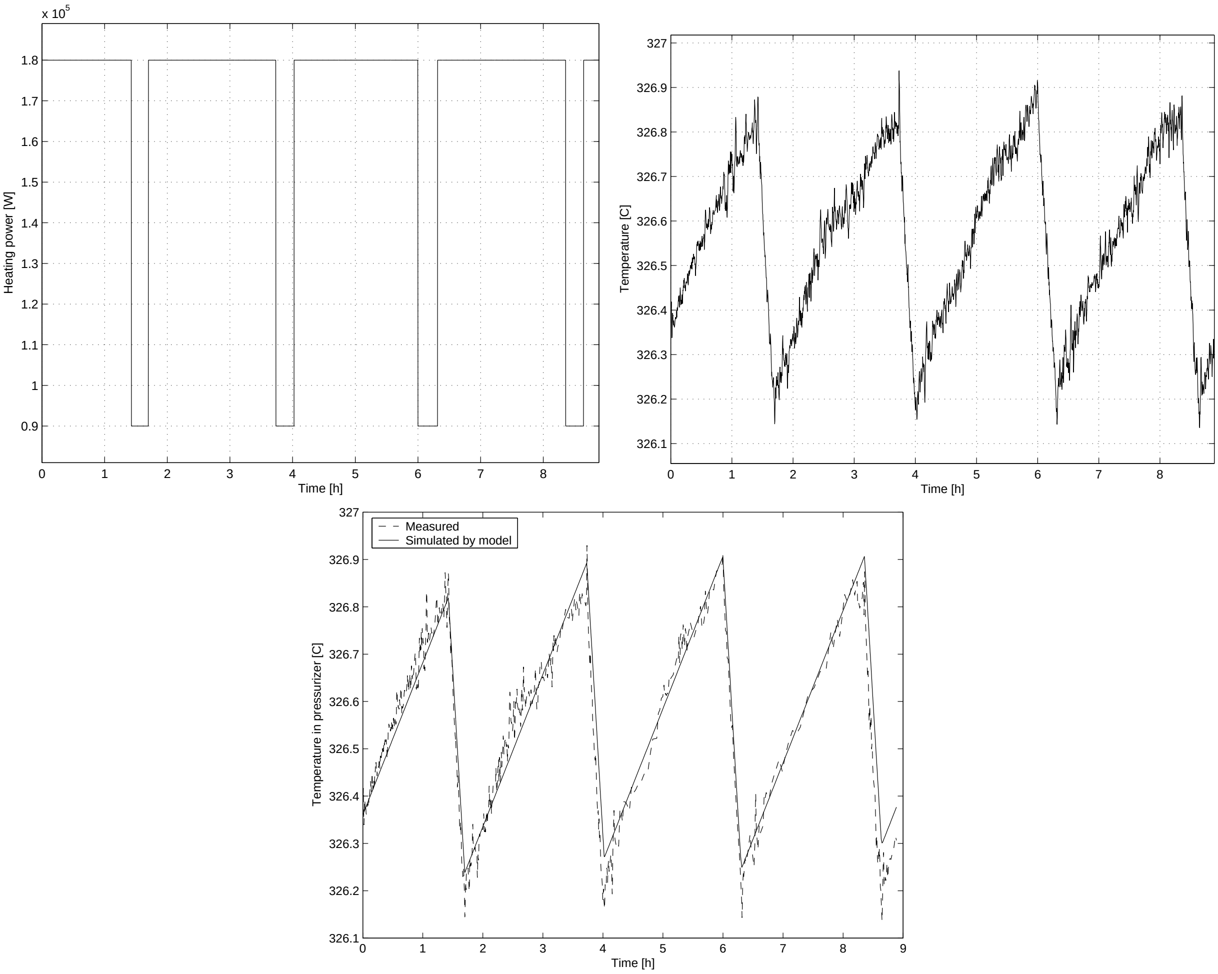


Figure 5: Measured pressurizer heating power and temperature, and the fit, unit 2

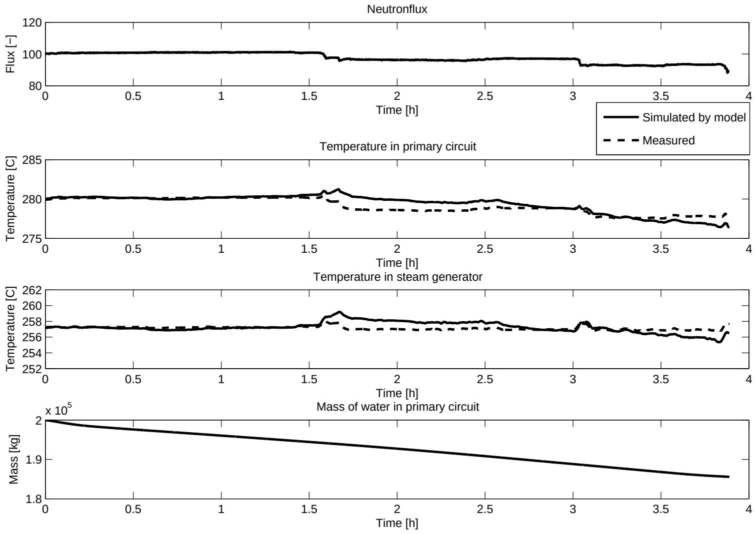


Figure 6: Measured and simulated temperatures after model integration for unit 1

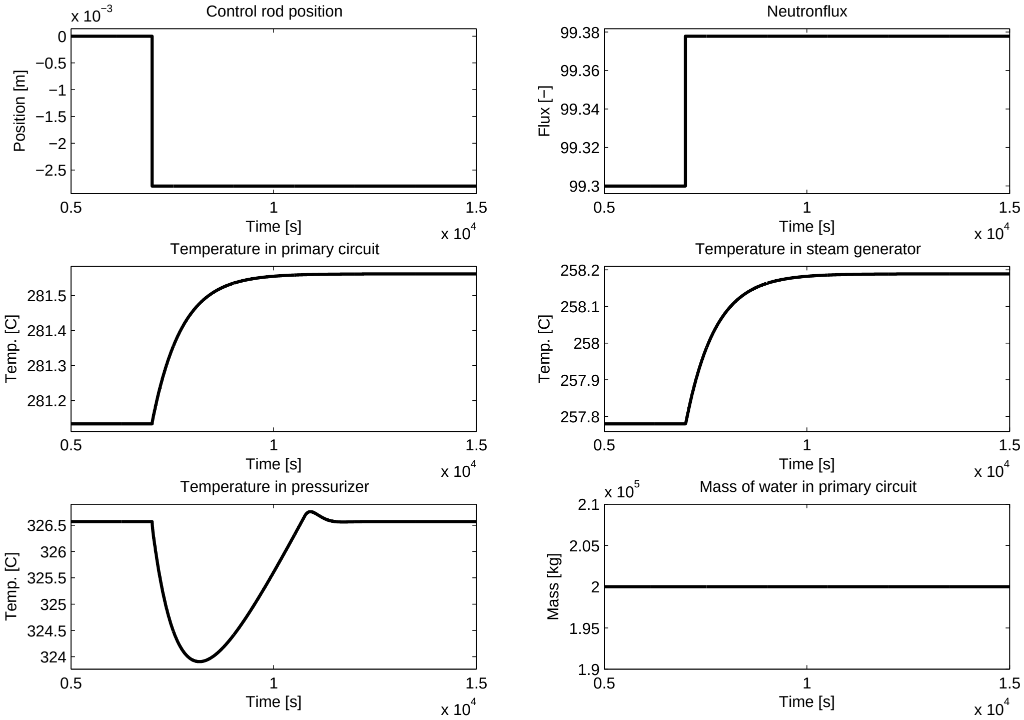


Figure 7: The effect of control rod withdrawal on the state variables

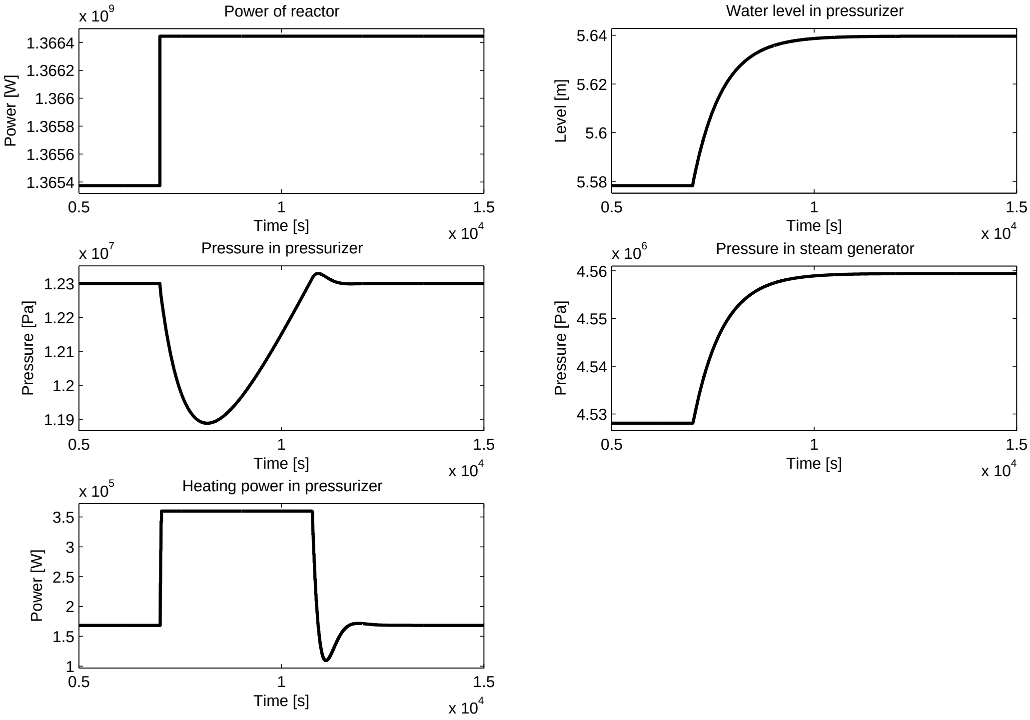


Figure 8: The effect of control rod withdrawal on the output variables

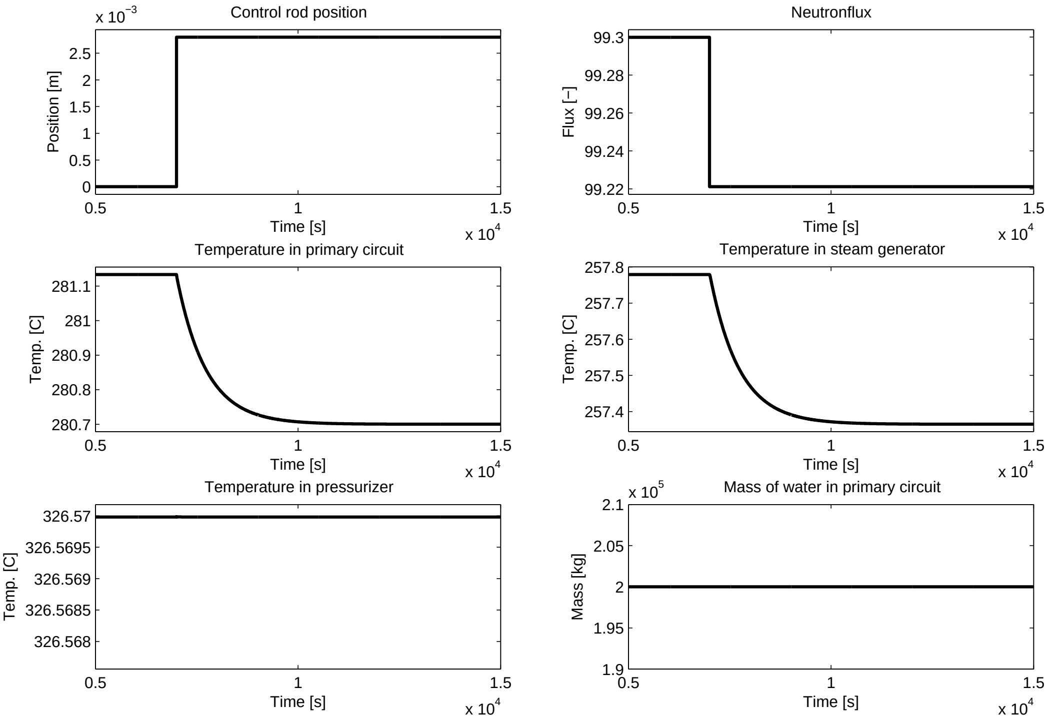


Figure 9: The effect of control rod insertion on the state variables

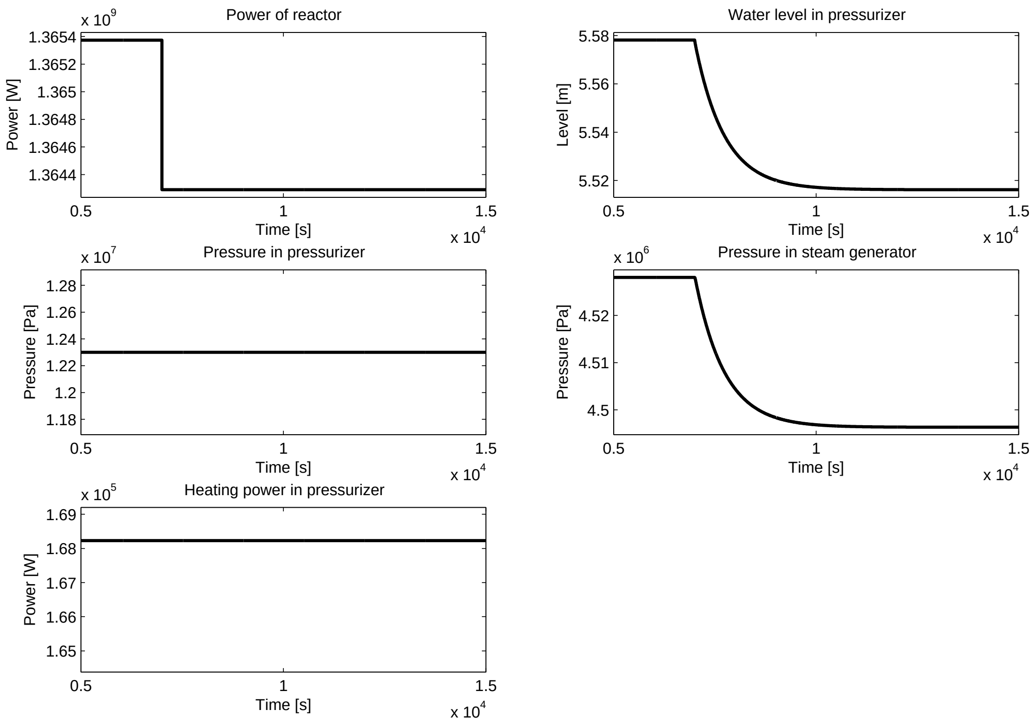


Figure 10: The effect of control rod insertion on the output variables