1.) A.) Determine the set of parameters $p \in \mathbb{R}$ for which the origin is an asymptotically stable equilibrium of system (E) $\dot{y} = x + py$, $\dot{x} = -5x + (2 + 6p)y$. B.) Can we obtain a stable focus C.) or an unstable focus and, if yes, determine the set of the corresponding parameter values!

2.) Consider the system of linear differential equations

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 3 & 0 & 5 \\ 0 & a & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} , \qquad a \in \mathbb{R} .$$

Find the general solution and, separating the various cases according to the values of parameter a, sketch the various phase portraits!

3.) Given the system of linear differential equations (E) $\dot{x} = -5x - 22y$, $\dot{y} = x + 4y$, determine the three parameters in function $V(x, y) = \alpha x^2 + \beta xy + \gamma y^2$ such that $\dot{V}_{(E)}(x, y) = -x^2 - y^2 < 0$ for each $\binom{x}{y} \neq \binom{0}{0}$. What are the consequences for the stability of the origin?

4.) Linearize system (N) $\dot{x} = -3x + 7y$, $\dot{y} = -\sin(x) + e^{2y} - \cos(y)$ about the origin and sketch the local phase portrait of (N) near the origin.

5.) A.) Find the equilibria of system $\dot{x} = 1 - y^2$, $\dot{y} = 2xy$ and sketch the local phase portrait in their small neighborhoods! B.) Piece together the local figures leading up to the global phase portrait (indicating the overall behavior of all trajectories). (*HINT:* First clarify the symmetry properties! Then determine the dynamics on the axis of equation $y = 0$.)

6.) Let $\mu = 1 - \varepsilon$ where $0 < \varepsilon \ll 1$. A.) Starting from small figures portraying the dynamics in the vicinity of each equilibrium point, try to guess and draw the global phase portrait of system $\dot{x} = -xy$, $\dot{y} = x^2 - y - 1 + \mu(y - y^3)$. B.) Consider now the 'opposite' case $\mu = 1 + \varepsilon$ where $0 < \varepsilon \ll 1$ and draw the global phase portrait of system $\dot{x} = -xy$, $\dot{y} = x^2 - y - 1 + \mu(y - y^3)$ for these parameter values. C.) Describe the transition between these two types of the global phase portrait by Your own words when parameter μ passes from $\mu = 1 - \varepsilon$ to $\mu = 1 + \varepsilon$ in an increasing way!

7.) A.) Is linearization a suitable method to investigate the stability of the origin as of an equilibrium of the nonlinear sytem (E) $\dot{y} = x - y^3$, $\dot{x} = -y + 2xy^2 - x^3$? B.) And by using the sign of the derivative of the auxiliary function $V(x, y) = \frac{1}{2}(x^2 + y^2)$ along the trajectories?[1] Do we need

[1] The derivative of function V along the trajectories of (E) is defined by formula $\dot{V}_{(E)}(x, y) = (\frac{d}{dt} V(x(t), y(t)))|_{t=0}$. Is it true that $\dot{V}_{(E)}(x) = \langle \underline{grad} V(x), f(x) \rangle$? How do we call this formula? What is the definition of $\underline{grad} V(x)$? What is the differential equation here? What is the domain and the range of functions V and f, respectively? What is the smoothness class to work with?

to recall some properties of continuous functions? C.) Is the origin globally asymptotically stable?

8.) A.) Is linearization a suitable method to investigate the stability of the origin as of an equilibrium of the nonlinear sytem $\dot{x} = x^3 - y^3$, $\dot{y} = x^3 + y^3$? B.) What about using the auxiliary function $V(x, y) = \frac{1}{2}(x^2 + y^2)$! (*HINT:* Prove that all nonequilibrium trajectories approach the point at infinity?) C.) Point out that the origin is a globally asymptotically stable equilibrium of the nonlinear system $\dot{x} = -x^3 + y^3$, $\dot{y} = -x^3 - y^3$!

9.) A.) What can be shown about the stability of the origin as of an equilibrium of system $\dot{x} = -y - 3x^3 + xy^2 + x^5$, $\dot{y} = x + x^2y - y^3$ by linearization? B.) And with the help of the auxiliary function $V(x, y) = \frac{1}{2}(x^2 + y^2)$? (*HINT:* Determine the radius of a disc which is centered about the origin and belongs to its region of attraction!) C.) Is the origin globally asymptotically stable? (*HINT:* Find a half plane of starting points through which the trajectories tend to the point at infinity!)

10.) Sketch the dynamics induced by the the differential equation $\dot{r} = r(1 - r^2)$, $\dot{\varphi} = sin^2\left(\frac{\varphi}{2}\right)$ in polar coordinates! What can be said about stability and attraction properties of the equilibrium point given by polar coordinates $r = 1$, $\varphi = 0$?

11.) Is it true or not: "Equilibrium point $P = (\frac{3}{2}, 0)$ of the ecosystem $\dot{x} = x(3 - 2x - y)$, $\dot{y} = y(1 - 2y - x)$ is globally asymptotically stable with respect to the positive orthant, i.e., with respect to the interior of the first closed quadrant of the plain."

12.) Which of the planar Lotka–Volterra systems $\dot{x} = x(5 - x + y)$, $\dot{y} = y(4 - 2y + x)$ and $\dot{x} = x(4 - x + 2y)$, $\dot{y} = y(5 - y + x)$ is modeling a permanent symbiotic ecosystem?

13.) Find condition on parameters K and β (belonging to the second species of the ecosystem) $\dot{x} = x(3 - 2x - y)$, $\dot{y} = y(1 - \frac{y}{K} - \beta x)$ to let the first species die out!

14.) A.) Sketch the (one–dimensional) phase portraits of the differential equation $\dot{x} = \mu x - x^3$ for the three different cases $\mu < 0$, $\mu = 0$, $\mu > 0$ of the bifurcation parameter $\mu \in \mathbb{R}$! B.) In comparison to the solution of part A.), do the same for each of the parametrized families of differential equations $\dot{x} = -\mu x - x^3$, $\dot{x} = \mu x + x^3$, and $\dot{x} = -\mu x + x^3$.

15.) It is well–known that *"unstable"* $:\Leftrightarrow$ *"not stable"*. A.) Formulate the definition of instability for an equilibrium $x_0 \in \mathbb{R}^n$ of a dynamical system $\Phi : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ by using $\varepsilon > 0$ and $\delta > 0$! B.) Do the same for the negation of asymptotic stability! C.) This is pure logic than differential equations: but still make a pair of good quality figures that visualize the negations above!