Hosszú-tranziensű metastabil periodikus oszcillációk

Motivation

- one-dimensional, circular, standard CNN with a three segment piecewise linear activation
- two-sided, nonsymmetric, cooperative (positive) interactions
- due to theoretical considerations [1]-[5]: the generic solution converges toward an asymptotically stable equilibrium point in the long run
- however in simulation we can observe long-lasting oscillations before the CNN converges

Video 1

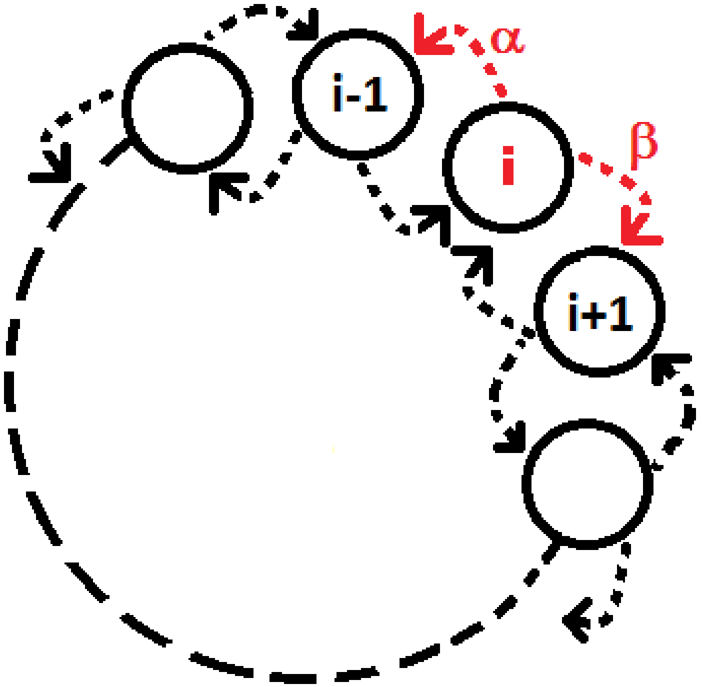
Cooperative CNN Rings

$$\tau \dot{x}_i = -x_i + \alpha g(x_{i-1}) + \beta g(x_{i+1}); \qquad i = 1,2,\ldots,N$$

$$\tau > 0; \qquad \alpha, \beta > 0$$

$$g(\rho) = \frac{1}{2}(|\rho+1| - |\rho-1|)$$

$$A = \begin{pmatrix} 0 & \beta & 0 & 0 & \ldots & \alpha \\ \alpha & 0 & \beta & 0 & \ldots & 0 \\ 0 & \alpha & 0 & \beta & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & \alpha & 0 & \beta \\ \beta & \ldots & 0 & 0 & \alpha & 0 \end{pmatrix}$$



- Initial conditions: `+++---’
- Dominant Floquet multiplier of the periodic solution induced

N=6, λ_1=2.5883

N=8, λ_1=1.0985

N=10, λ_1=1.00917

N=12, λ_1=1.00089

N=14, λ_1=1.00014

N=16, λ_1=1.000097

N=18, λ_1=1.000044

Patterns within Floquet eigenvectors -- the dominant eigenvector $\mathbf{s}_1$

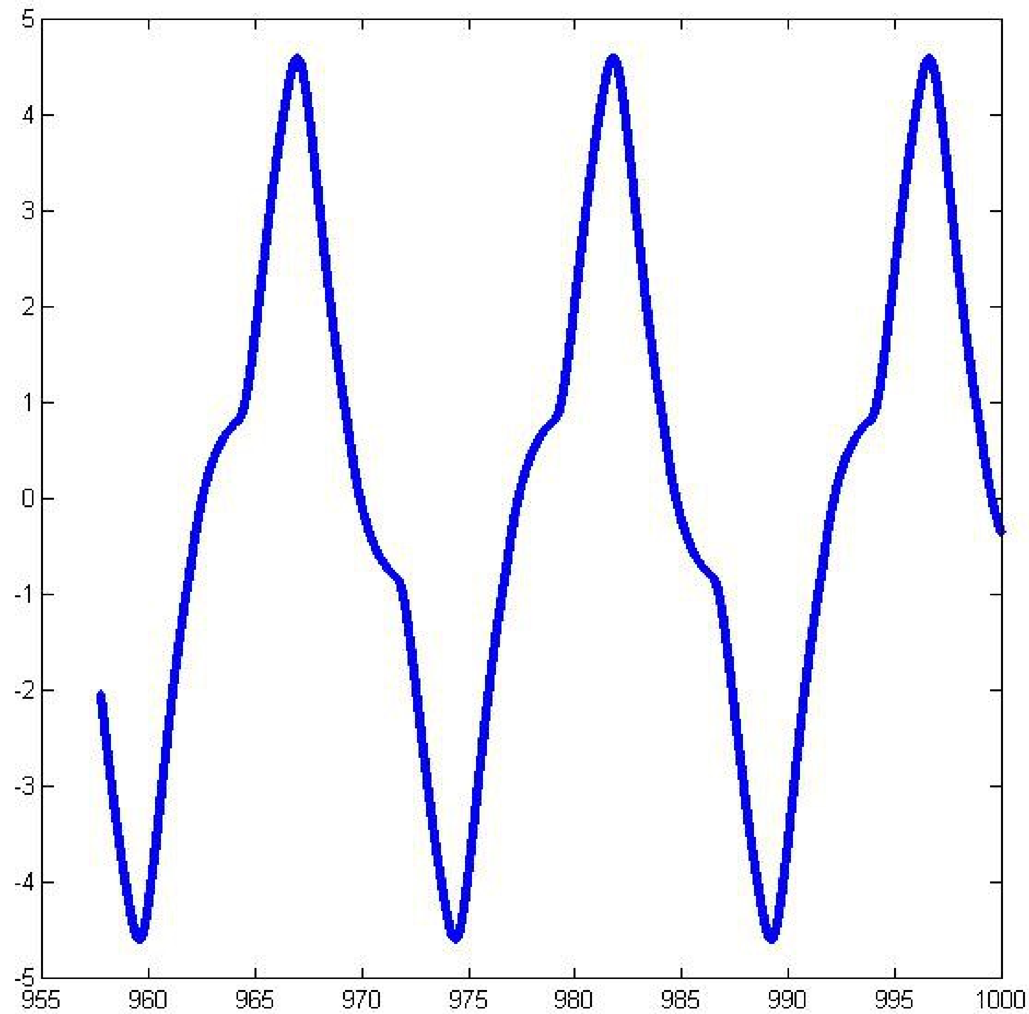
N = 8	N = 10	N = 12	N = 14	N = 16
				0.01447
			0.01822	0.00000
		-0.00360	0.00000	0.00000
	0.00754	0.00000	0.00000	0.00000
0.01030	-0.00007	0.00000	0.00000	0.00000
0.00095	-0.00001	0.00000	0.00000	0.00000
-0.74413	0.73979	-0.74072	-0.74554	0.73833
-0.30155 →	0.31105 →	-0.30914 →	-0.29732 →	0.31382
-0.55499	0.55250	-0.55302	-0.55554	0.55167
-0.21722	0.22404	-0.22267	-0.21415	0.22604
-0.00258	0.01734	-0.01723	-0.01657	0.01749
	0.01191	-0.00133	-0.00128	0.00135
		-0.00515	-0.00009	0.00010
			0.02550	0.00000
				0.02026

Patterns within Floquet eigenvectors -- the second eigenvector $\mathbf{s}_2$

N = 8	N = 10	N = 12	N = 14	N = 16
				-0.41098
			0.41105	-0.00000
		-0.41103	0.00000	-0.00000
	-0.41101	0.00000	0.00003	-0.00000
0.41095	-0.00000	0.00000	0.00000	-0.00000
-0.00000	0.00000	0.00000	0.00000	-0.00000
-0.57544	0.57537	0.57534	-0.57534	0.57540
0.00000 →	0.00001 →	0.00002 →	-0.00001 →	0.00000
-0.41101	0.41098	0.41095	-0.41099	0.41101
0.00000	0.00001	0.00001	-0.00001	0.00000
0.57536	0.00000	-0.00000	-0.00002	0.00000
	-0.57542	-0.00000	-0.00000	0.00000
		-0.57545	-0.00000	0.00000
			0.57541	0.00000
				-0.57537

A ``metastable" rotating wave

- **N** = 6, `+++---´
- **λ_1= 2.58**;
  unstable under small perturbations
- MATLAB : stable (EE, IE, RK4, ODE45)
- C++ : dies after approx. 40 periods



Cooperative CNN Rings

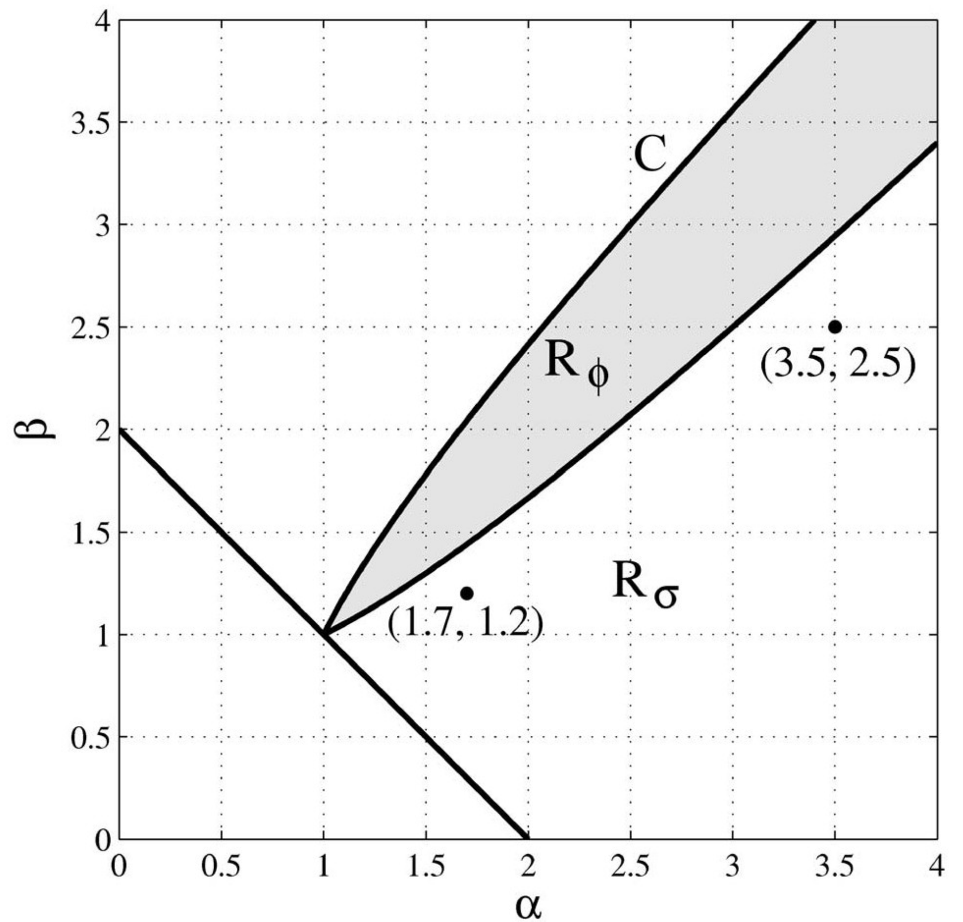
Parameter space:

$$C=\{(\alpha,\beta):(\alpha+\alpha\beta-\beta^2-1)(\beta+\alpha\beta-\alpha^2-1)=0\}$$

$$\alpha,\beta>0, \qquad \alpha+\beta>2$$

$$R_\phi=\left\{\begin{array}{l}(\alpha,\beta):\alpha>\dfrac{\beta^2+1}{\beta+1};\\ \qquad\qquad \beta>\dfrac{\alpha^2+1}{\alpha+1}\end{array}\right\}$$

$$R_\sigma=\{(\alpha,\beta):\alpha+\beta\geq 2\}\setminus closure(R_\phi)$$



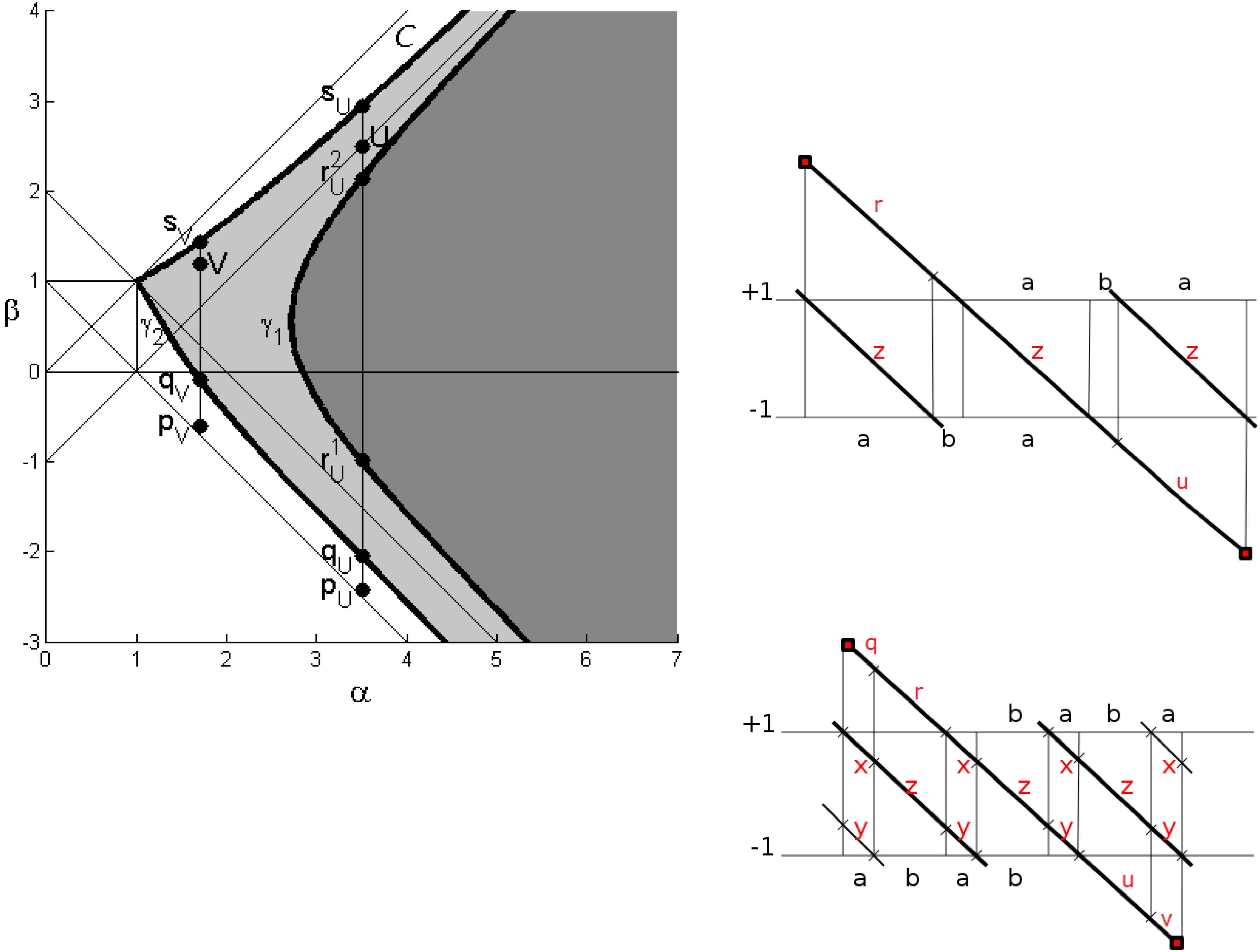
Number of equilibrium points:

R_σ : 3

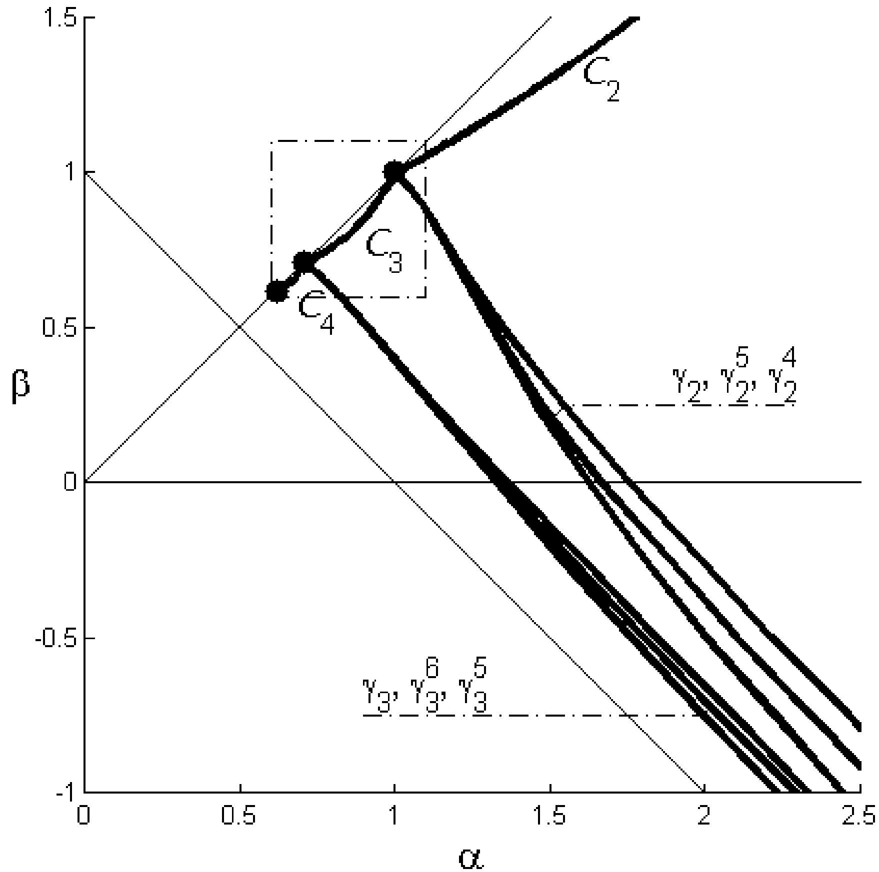
R_ϕ : several

Long – lasting oscillations:

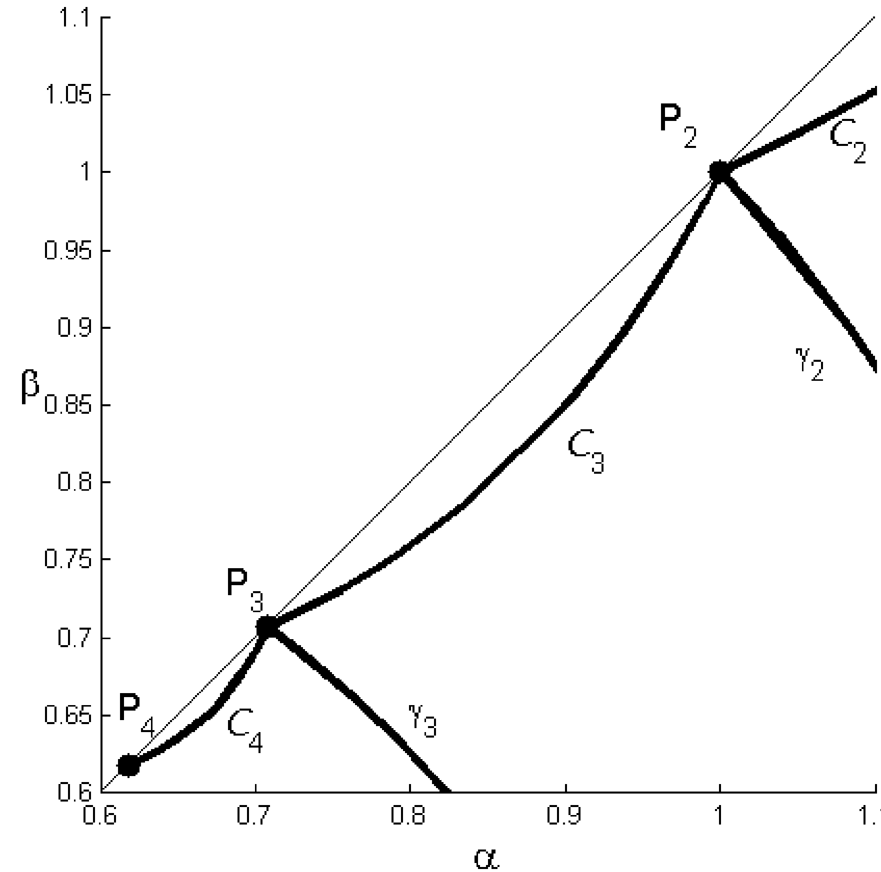
due to the presence of metastable rotating waves, region of existence: R_σ



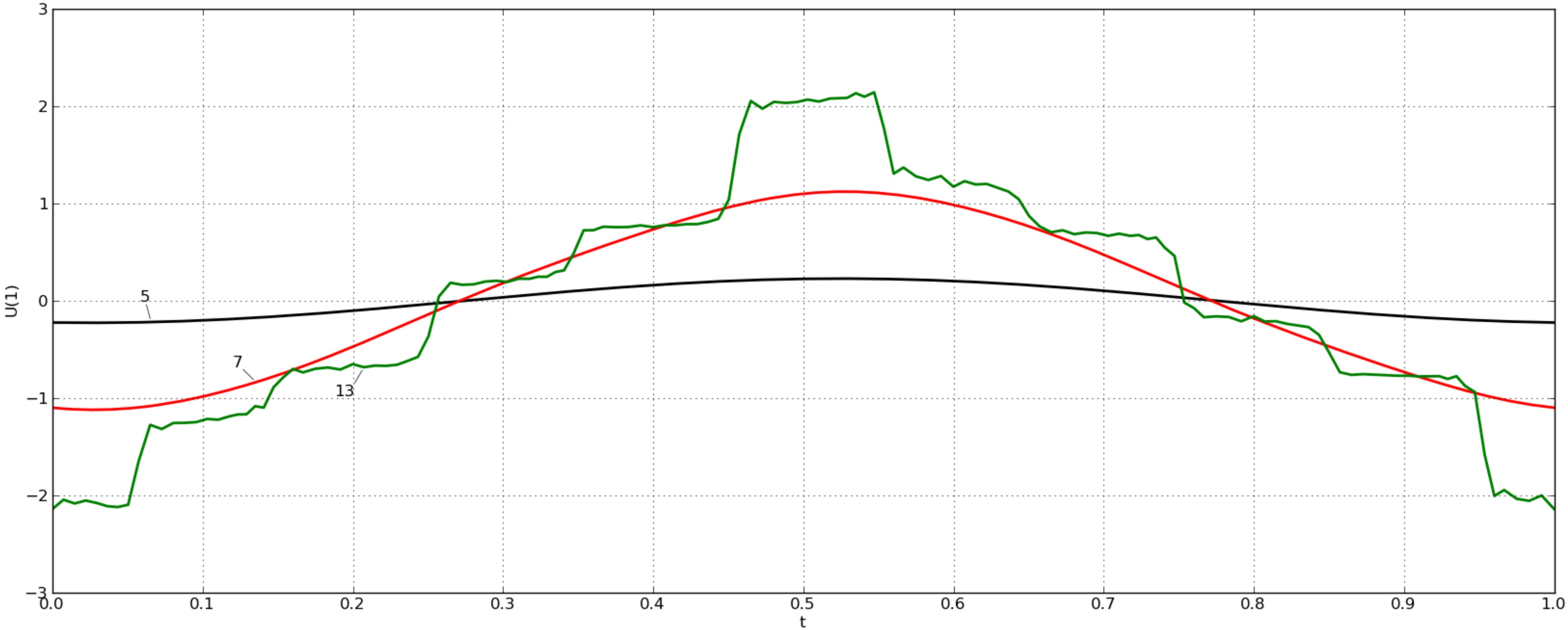
β
α
C
s_U
U
r_U^2
s_V
V
γ_1
γ_2
q_V
p_V
r_U^1
q_U
p_U
0 1 2 3 4 5 6 7
-3 -2 -1 0 1 2 3 4
r
a b a
+1
z z z
-1
a b a
u
q
r
b a b a
+1
x z x z x z x
y y y y y
-1
a b a b
u
v



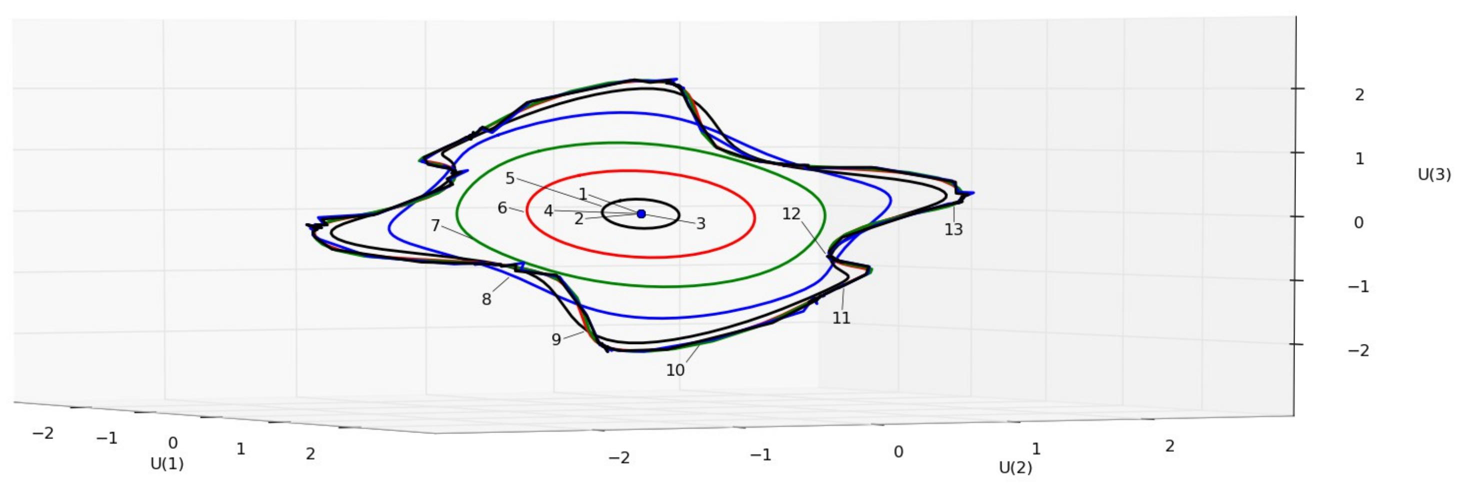
β
α
C_2
C_3
C_4
$\gamma_2, \gamma_2^5, \gamma_2^4$
$\gamma_3, \gamma_3^6, \gamma_3^5$



β
α
P_2
P_3
P_4
C_2
C_3
C_4
γ_2
γ_3



U(1)
t
5
7
13



U(1)
U(2)
U(3)
1
2
3
4
5
6
7
8
9
10
11
12
13

Cooperative CNN Rings

Theoretical results [6]:

For $(\alpha, \beta) \in R_\sigma$ and N = 2M ≥ 6

exponential estimates for the Floquet eigenvalues

$$\lambda_1 > \lambda_0 = 1 > |\lambda_2| \geq |\lambda_3| \geq \ldots \geq |\lambda_{N-1}|$$

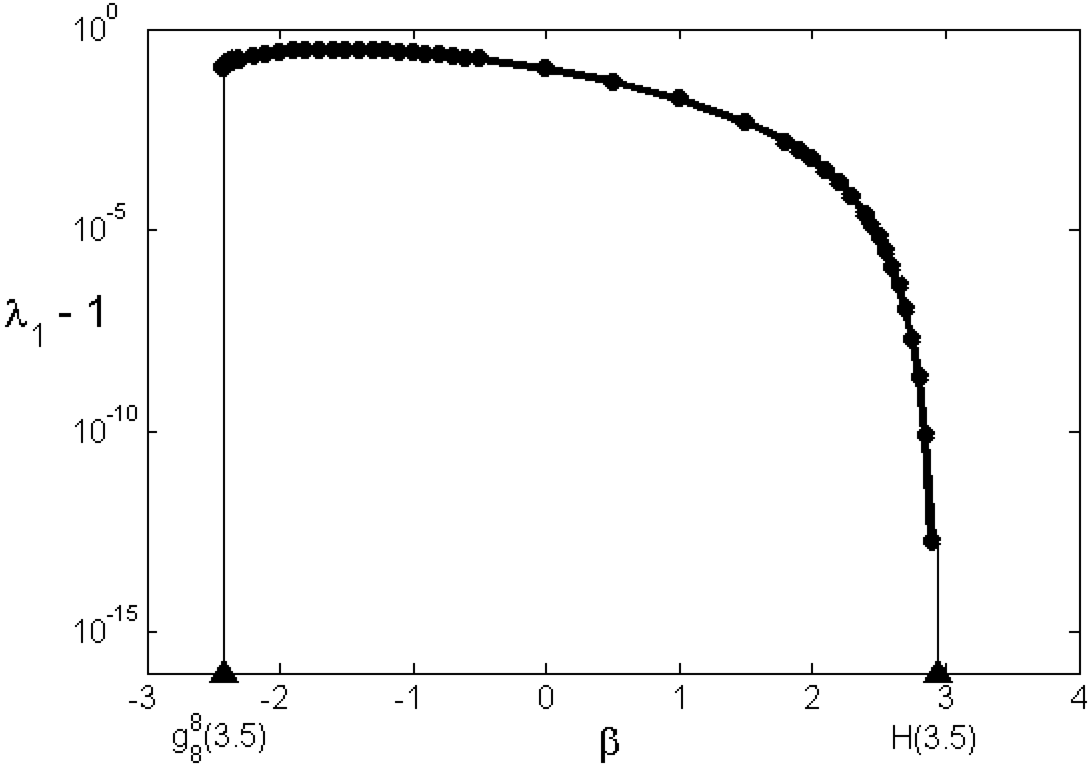
$$\lambda_1 < 1 + c_1 \frac{1}{(1+c_2)^N}$$

$$|\lambda_2| < c_1 \frac{1}{(1+c_2)^N}$$

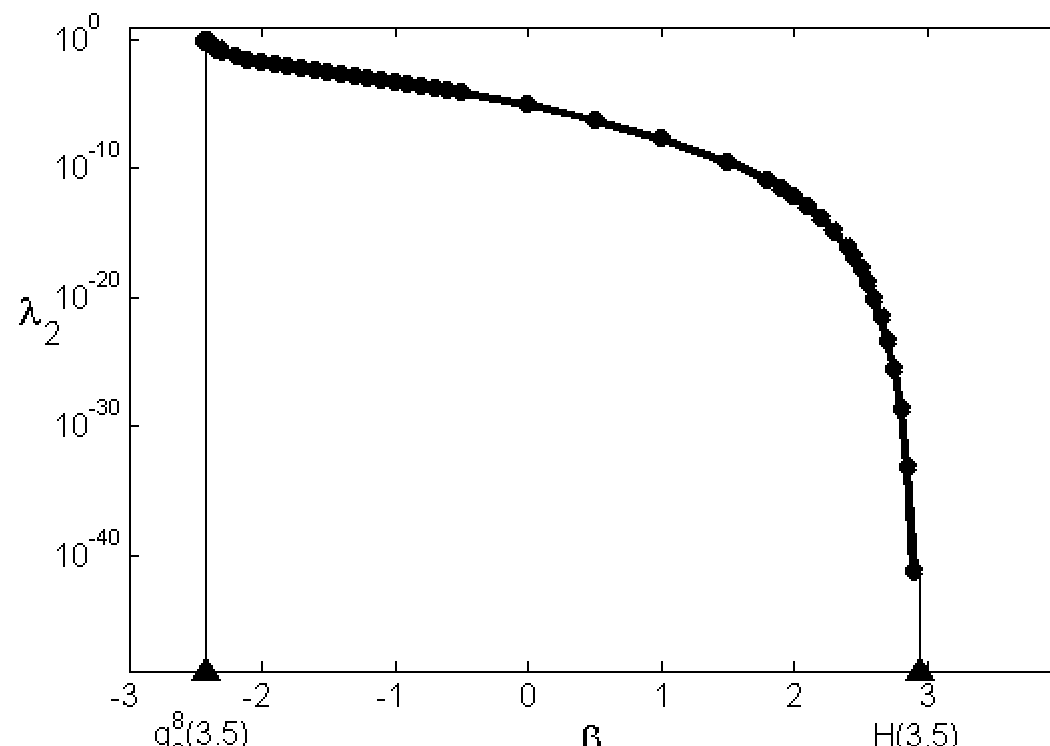
where

$$c_1 = c_1(\alpha, \beta), \quad c_2 = c_2(\alpha, \beta)$$

are positive constants, independent of N

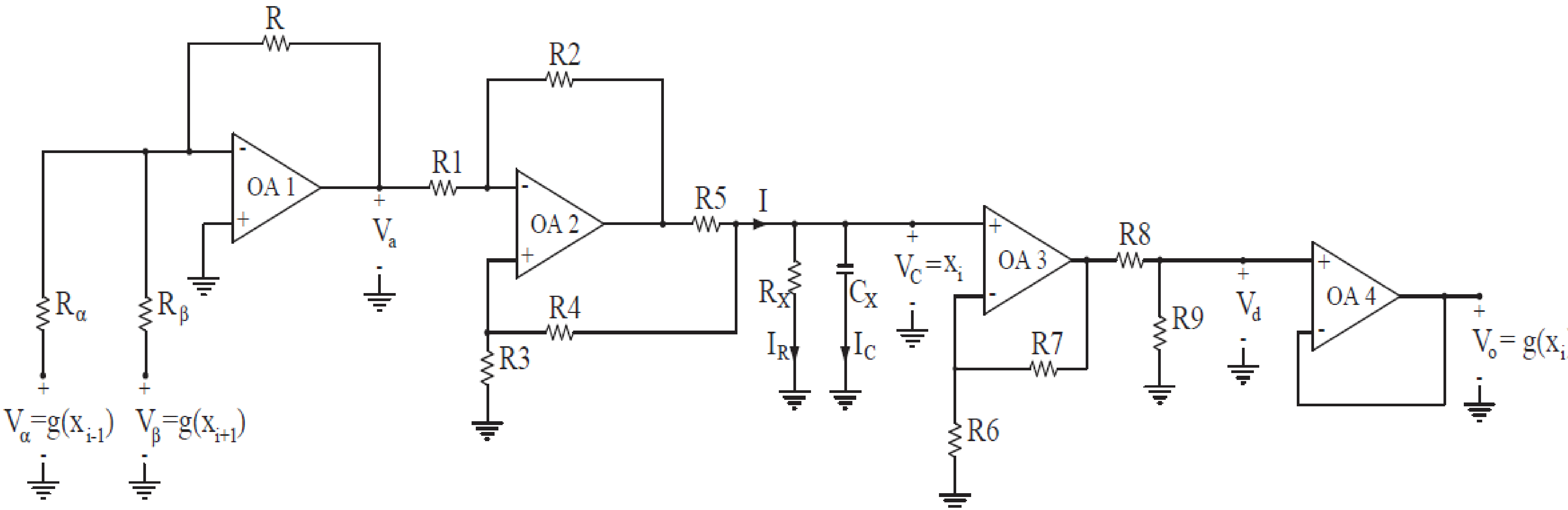


10^0
10^{-5}
10^{-10}
10^{-15}
$\lambda_1 - 1$
-3
-2
-1
0
1
2
3
4
$g_8^8(3.5)$
β
H(3.5)



10^0
10^{-10}
10^{-20}
10^{-30}
10^{-40}
λ_2
-3
-2
-1
0
1
2
3
4
$g_8^8(3.5)$
β
H(3.5)

Implementation of a CNN Cell



Slight modification of the circuit, originally proposed in [9]

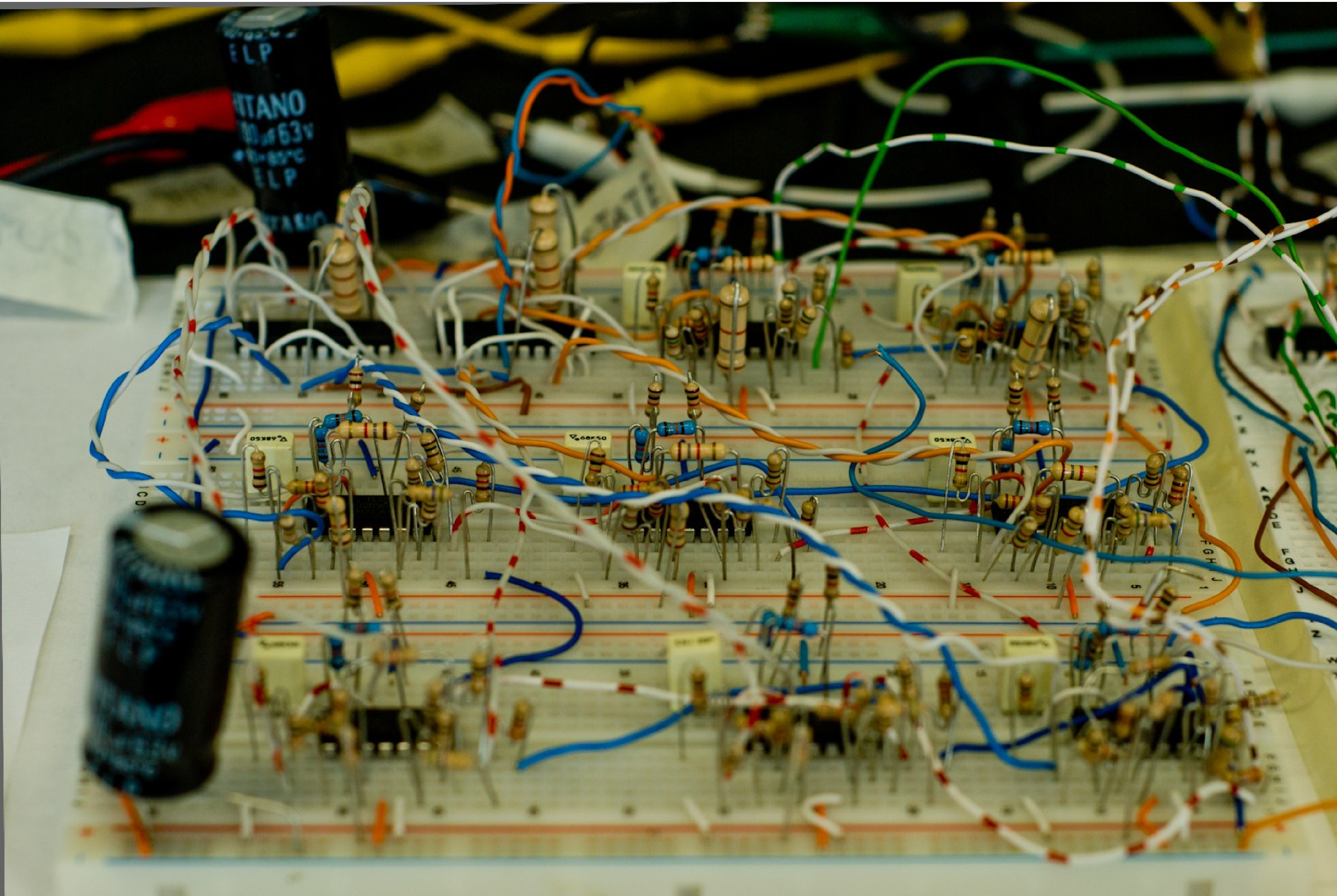
Four stages:

1st part implements the weighted sum of the inputs to the i-th neuron

2nd part is a voltage-controlled current source

3rd part realizes the inner state of the i-th neuron

4th part carries out the piecewise linear output-function


ELP
ELP

Implementation of a CNN Cell

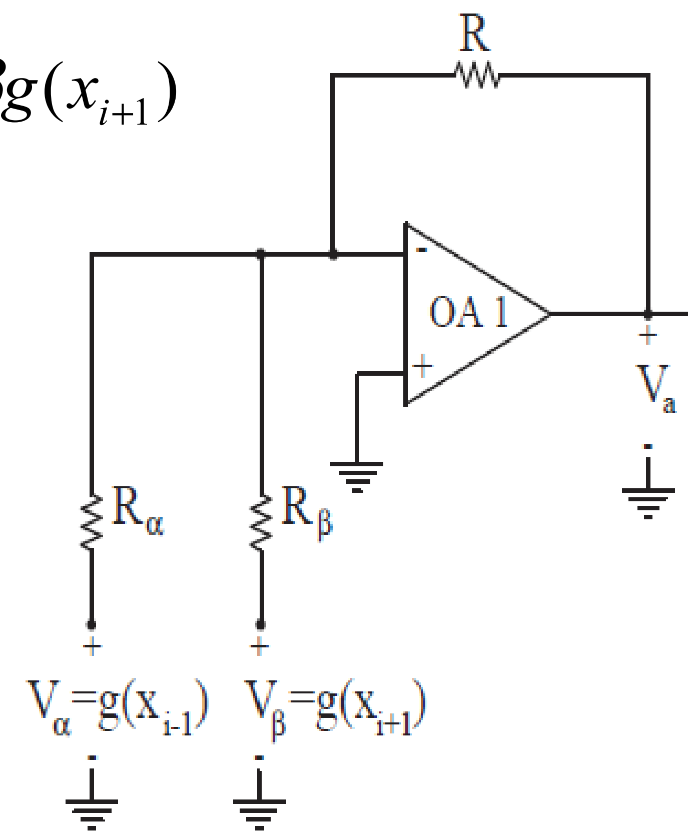
1st stage

$$V_a = -\frac{R}{R_\alpha} V_\alpha - \frac{R}{R_\beta} V_\beta = -\alpha g(x_{i-1}) - \beta g(x_{i+1})$$

$$\alpha = \frac{R}{R_\alpha}; \beta = \frac{R}{R_\beta}$$

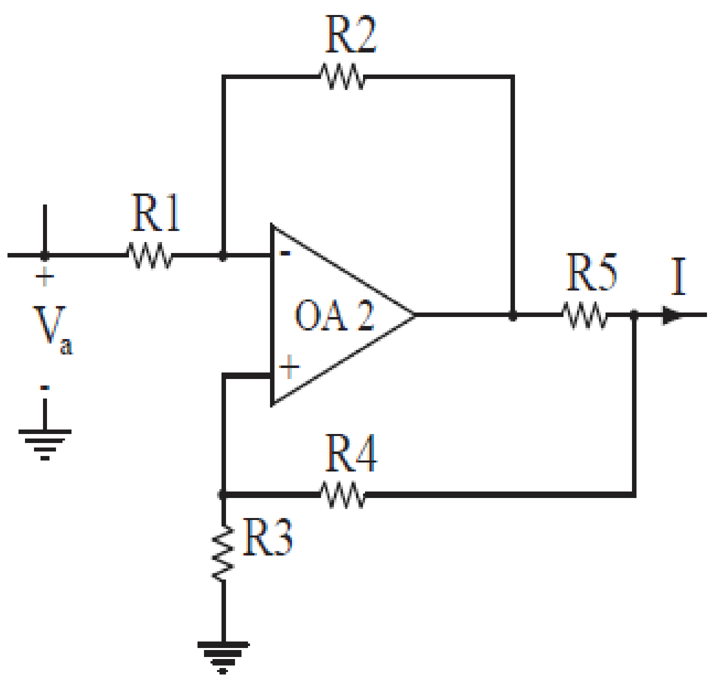
$$\text{as we have chosen}: R = 560\Omega$$

$$R_\alpha = \frac{R}{\alpha} = \frac{560}{\alpha}\Omega; R_\beta = \frac{R}{\beta} = \frac{560}{\beta}\Omega$$



Implementation of a CNN Cell

2nd stage



$$\frac{R_2}{R_1} = \frac{R_4 + R_5}{R_3}$$

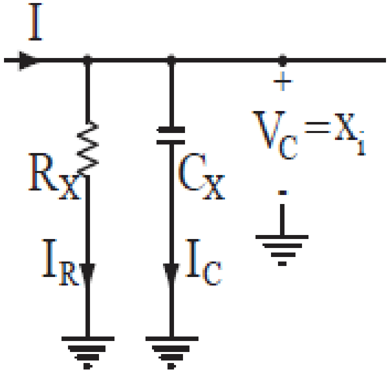
$$I = -\frac{R_2}{R_1 R_5} V_a$$

$$R_1 = 1.8k\Omega;\ R_2 = 2.7k\Omega;\ R_3 = 1.8k\Omega;\ R_4 = 1.2k\Omega;\ R_5 = 1.5k\Omega$$

$$I = -\frac{1}{1000} V_a = \frac{\alpha}{1000} g(x_{i-1}) + \frac{\beta}{1000} g(x_{i+1})$$

Implementation of a CNN Cell

3rd stage



$$\tau = R_x C_x$$

$$I = I_C + I_R = C_x \dot{V}_C + \frac{V_C}{R_x} = C_x \dot{x}_i + \frac{x_i}{R_x}$$

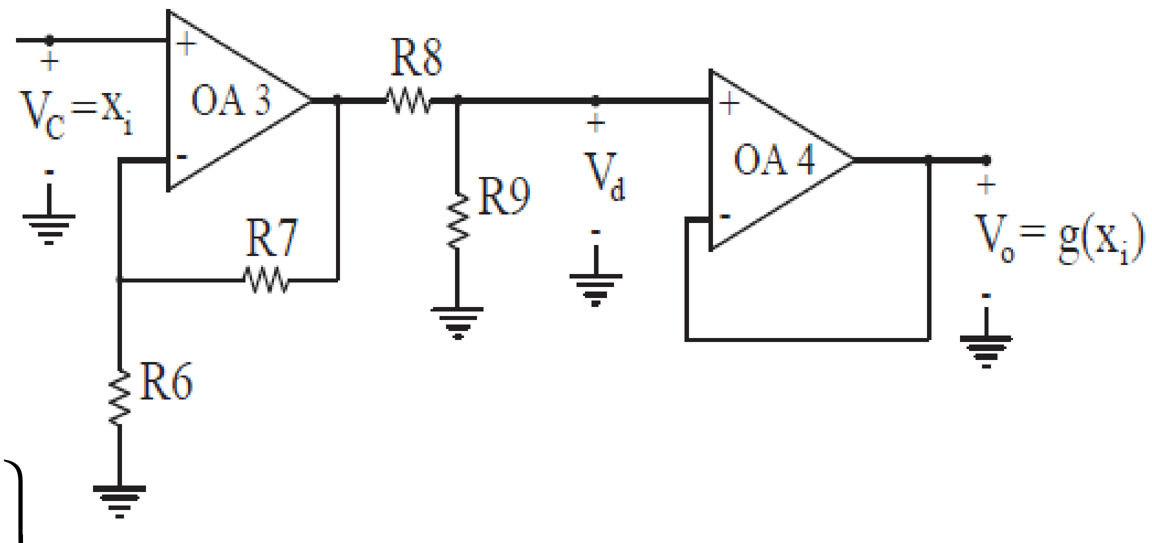
as we have chosen $R_x = 1k\Omega; C_x = 680nF$

$$680 \cdot 10^{-9} \dot{x}_i = -\frac{x_i}{1000} + \frac{\alpha}{1000} g(x_{i-1}) + \frac{\beta}{1000} g(x_{i+1})$$

$$\tau = 6.8 \cdot 10^{-4} \sec$$

Implementation of a CNN Cell

4th stage



$$\left.\begin{array}{c} \dfrac{R_6 + R_7}{R_6} \\ \dfrac{R_9}{R_8 + R_9} = \dfrac{R_6}{R_6 + R_7} \end{array}\right\} V_d = g(x_i)$$

$$R_6 = 1k\Omega;\ R_7 = 18k\Omega;\ R_8 = 18k\Omega;\ R_9 = 1k\Omega$$

$$V_o = V_d$$

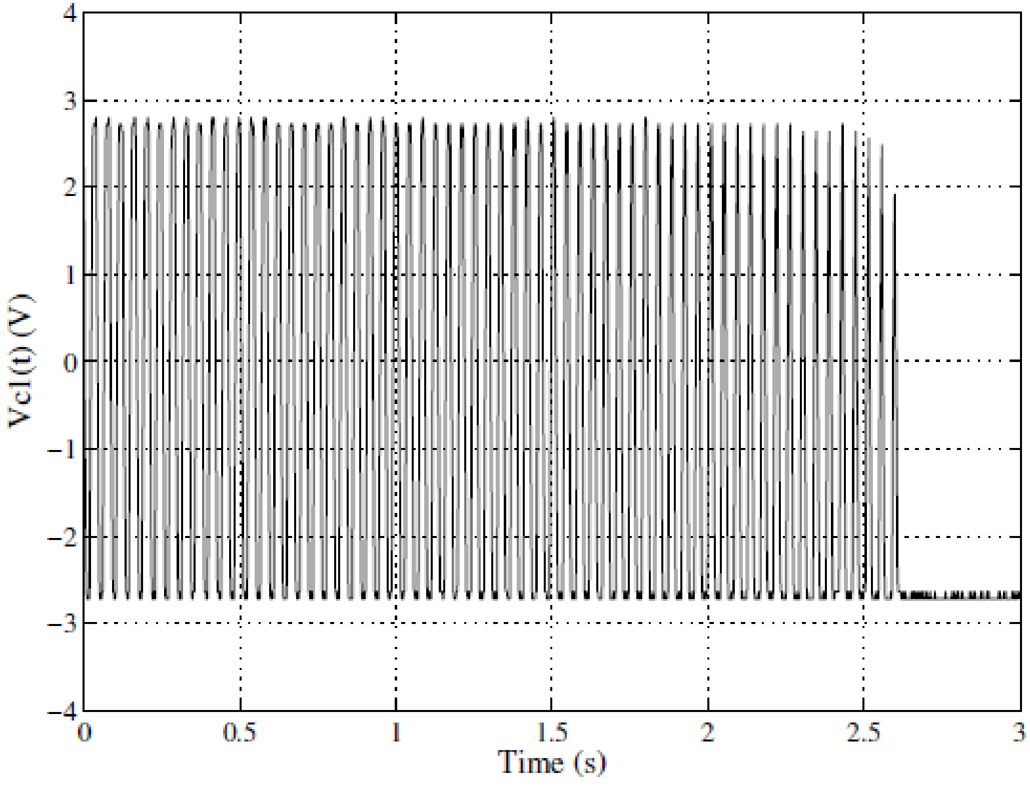
Experimental Results

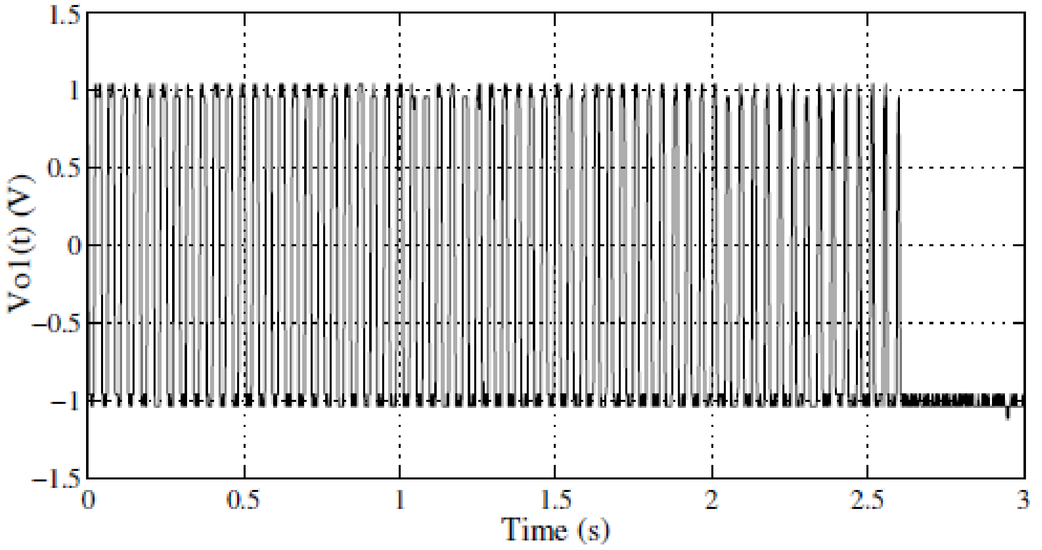
- three laboratory prototypes: N=4, 8, 16 neurons
- tolerances of the discrete components:
  resistors 5%; capacitors 10%
- operational amplifiers: TL084
- supply voltage to the op-amps: ± 20 V
- switches: MAX333

- in the case N=4, $(\alpha,\beta)\in R_\sigma$ no oscillations were observed
- in the case N=6, $(\alpha,\beta)\in R_\sigma$ we already observed oscillations
- the longer the ring, the longer the oscillations
  (up to N=16)

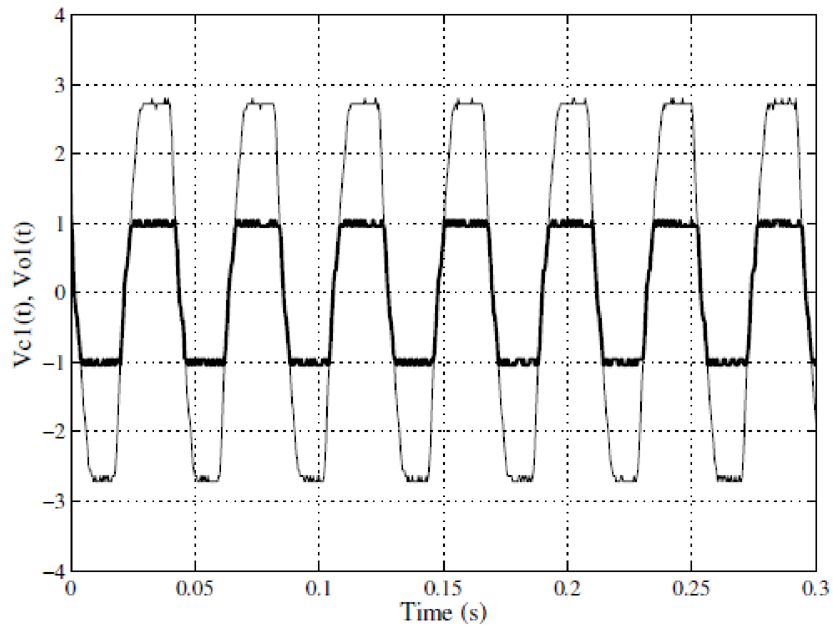
Experimental Results

- N=16
- $(\alpha, \beta) = (1.7, 1.2) \in R_\sigma$
- $x'_0 = 2.9\ (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ -1\ -1\ -1\ -1\ -1\ -1\ -1\ -1)'$

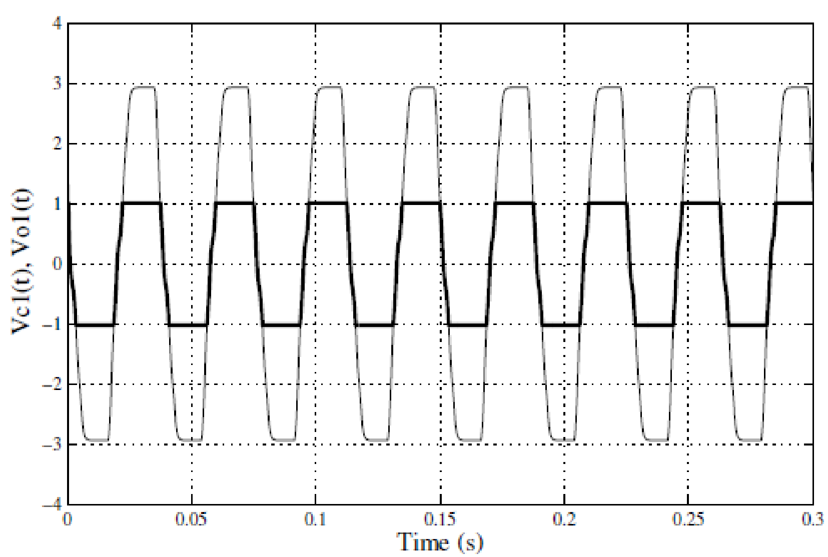




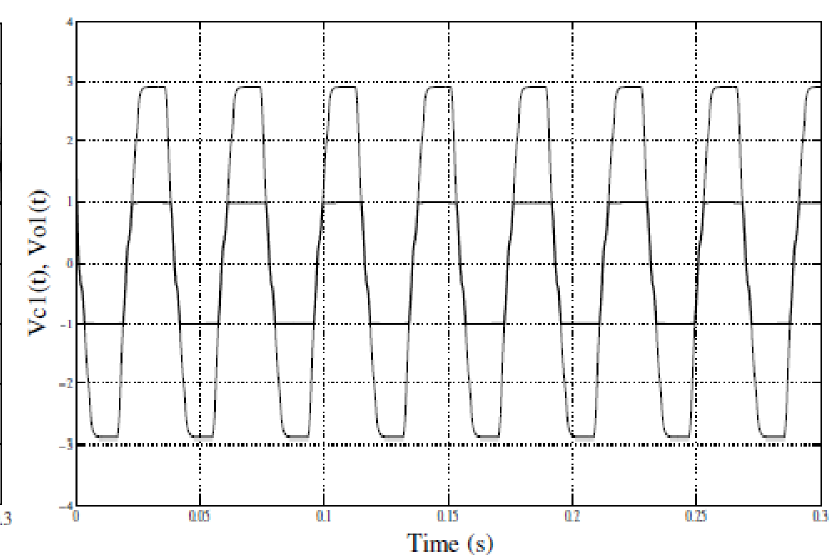
Experimental Results



measured with oscilloscope



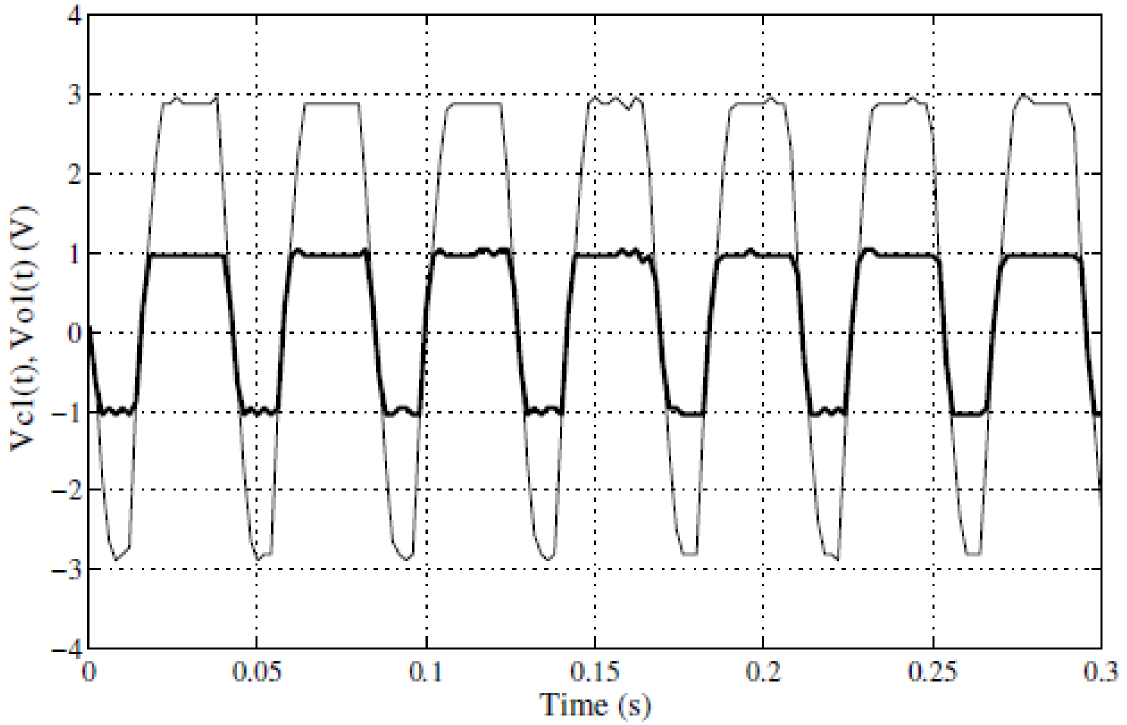
acquired from SPICE simulation



acquired from MATLAB simulation

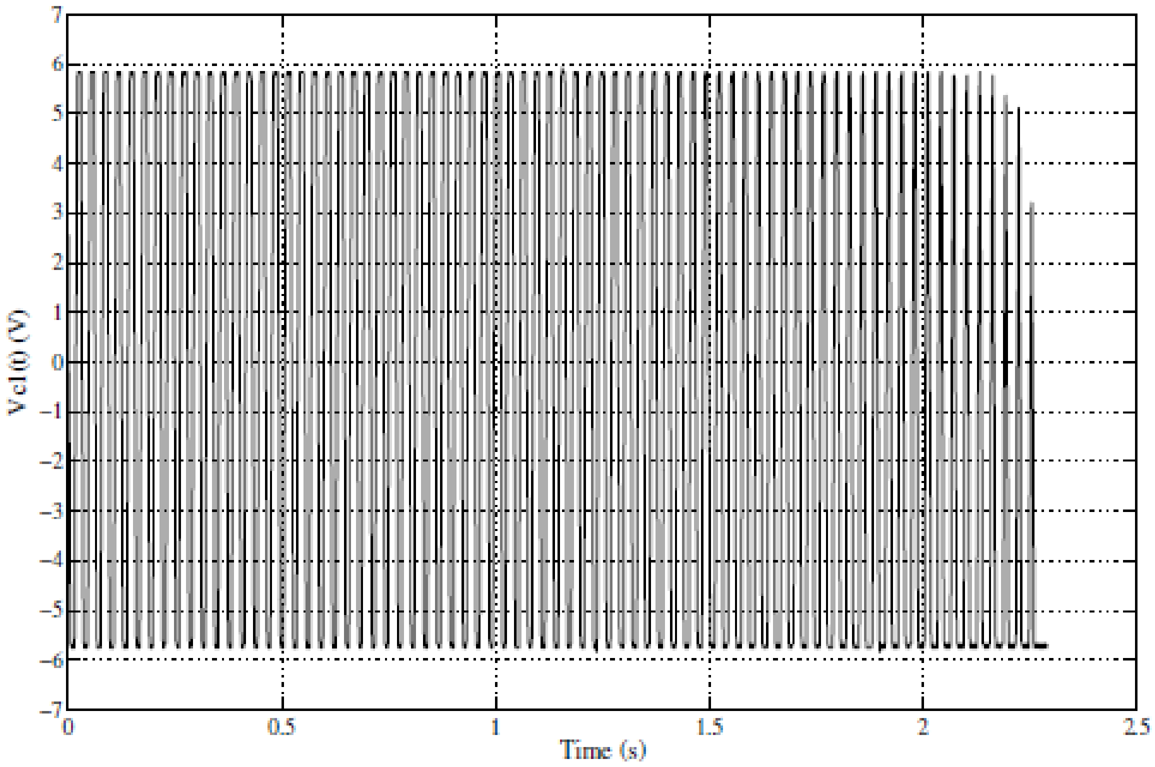
Experimental Results

- N=16
- $(\alpha, \beta) = (1.7, 1.2) \in R_\sigma$
- $x''_0 = 2.9\ (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ \text{-}1\ \text{-}1\ \text{-}1\ \text{-}1\ \text{-}1\ \text{-}1)'$



Experimental Results

- N=16
- $(\alpha, \beta) = (3.5, 2.5) \in R_\sigma$
- $x'_0 = 2.9\ (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ {-1}\ {-1}\ {-1}\ {-1}\ {-1}\ {-1}\ {-1}\ {-1})'$

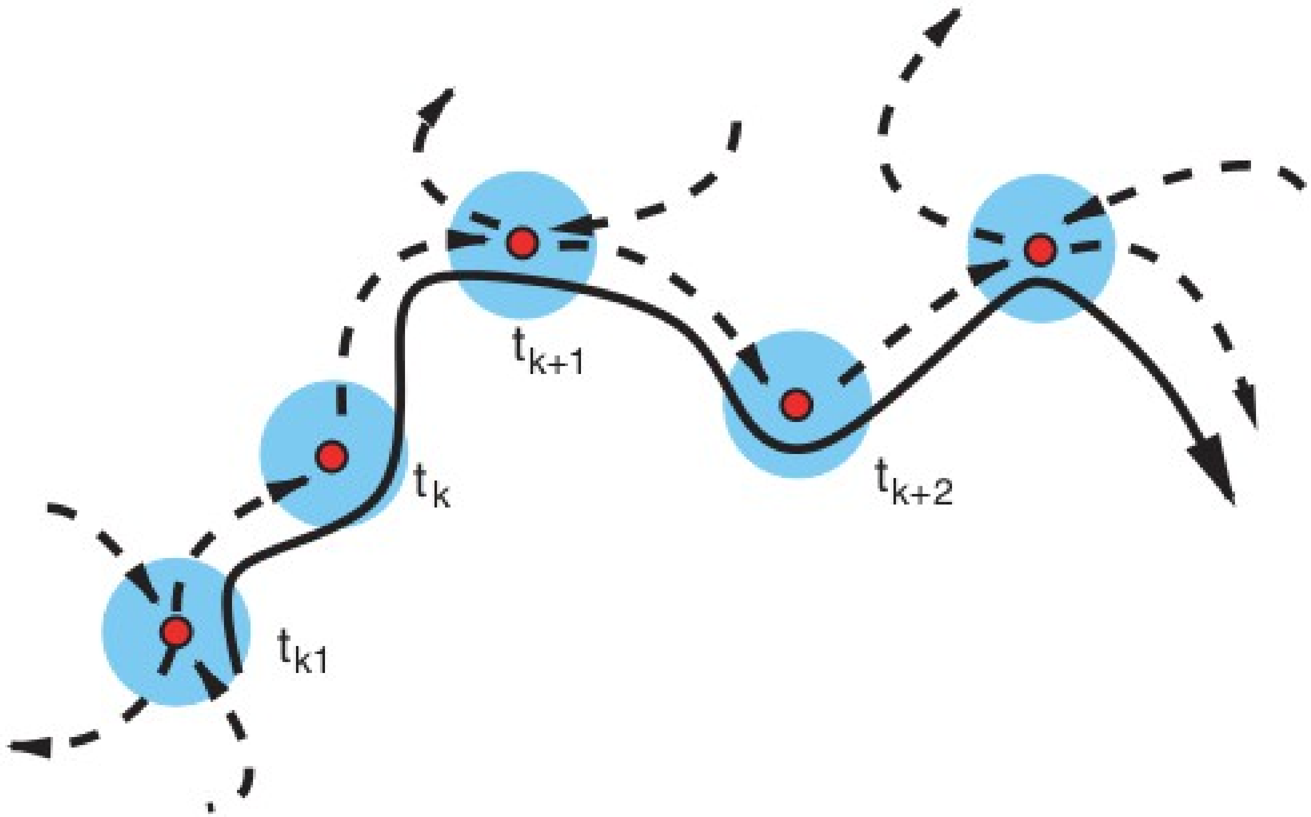


Conclusions

Video 2

- long transient oscillations are observed in a **wide range** of parameters ($(\alpha,\beta) \in R_\sigma$) and for **wide sets** of initial conditions
- the phenomenon is **physically robust** with respect to tolerances and other nonidealities in the electronic implementation
- a **theoretical analysis** for explaining the basic phenomena leading to the presence of the long transient oscillations is of crucial importance **for better understanding** the real-time processing capabilities of CNN arrays and neural network paradigms in general
- fundamental **theoretic results** already obtained **in [6]:** long oscillations are due to the presence of metastable rotating waves whose degree of instability is exponentially decreasing with the dimension of the CNN ring

Biológiai példák!



A kép forrása: Rabinovich-Huerta-Varona-Afraimovich

References

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[6] M. Forti, B. Garay, M. Koller, and L. Pancioni, “Floquet multipliers of a metastable rotating wave,” 2012, in preparation.

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