Theory of nonlinear dynamic systems
Practice 5

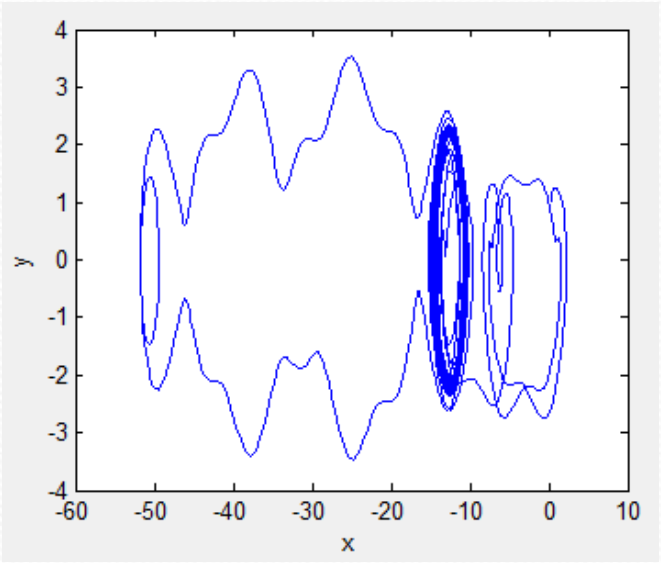
Juhász János
juhasz.janos@.itk.ppke.hu

2019.12.04.

Oscillation with excitation and damping

- Examine the following equation:

$$\ddot{x} + b\dot{x} + \sin x = \cos t$$



- 2D system with time dependent periodic excitation
- Oscillation with excitation and damping
- Transient chaos
- Sensitive dependence from the initial conditions

Logistic mapping

where $f(x) = \mu x(1-x)$

and $0 \leq \mu \leq 4$

$0 \leq x \leq 1$

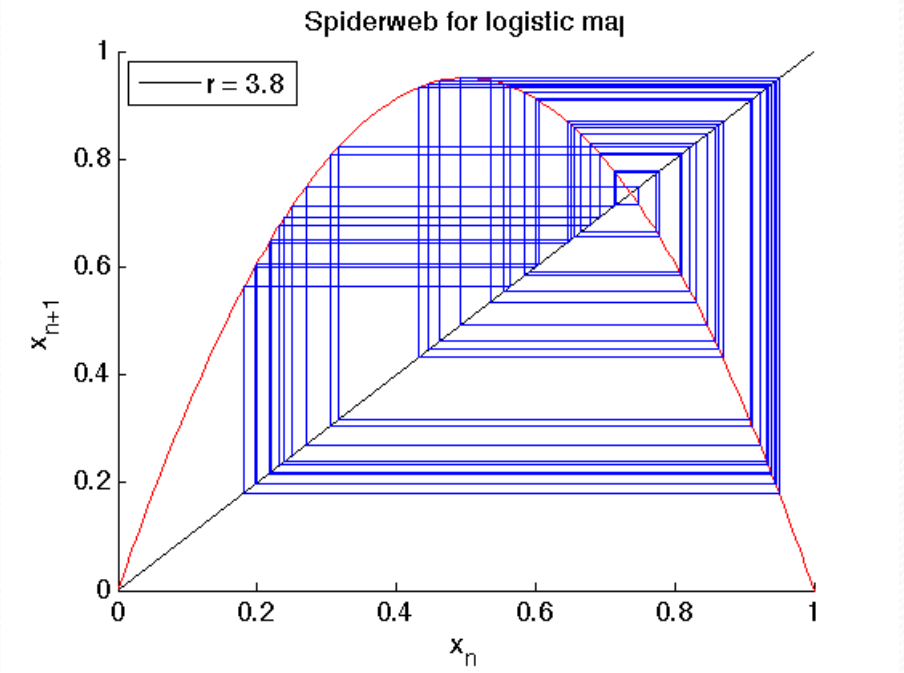
- Biological meaning:

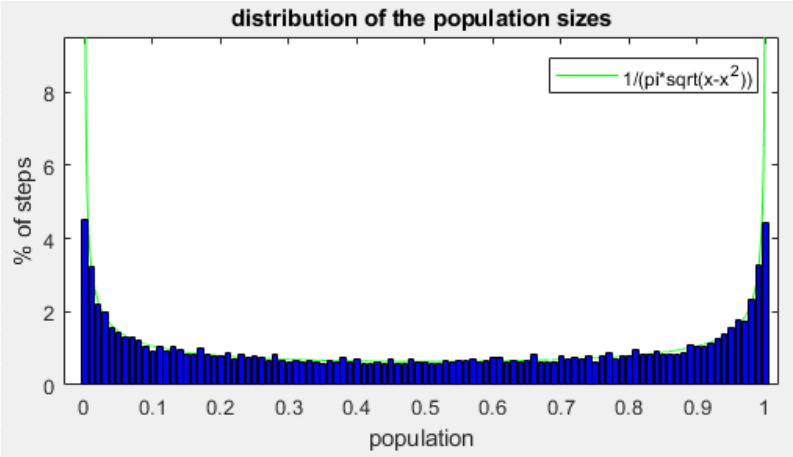
$$\dot{x} = bx - cx^2$$

where, b is the birth rate and cx is the death rate in a population.

Spiderweb diagram of the logistic mapping

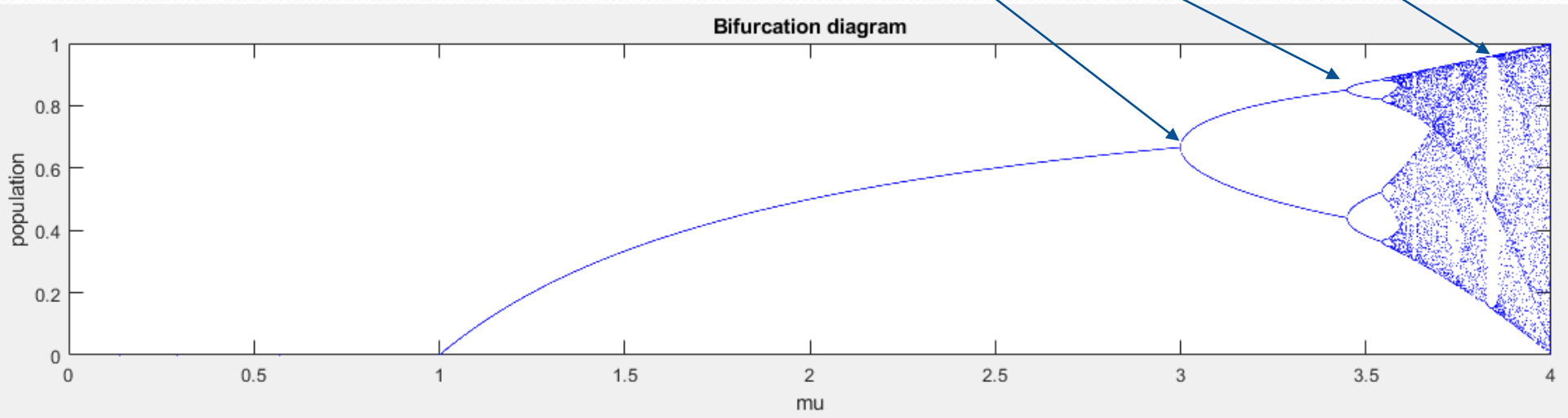
- Produces chaos with some μ parameters
- The chaos develops from period duplications
- The chaos can temporarily disappear
- There are more points near 0 and 1 than the middle region





Bifurcation diagram

- General shape
- Period duplications
- Temporary disappearence of the chaos



Lorenz system

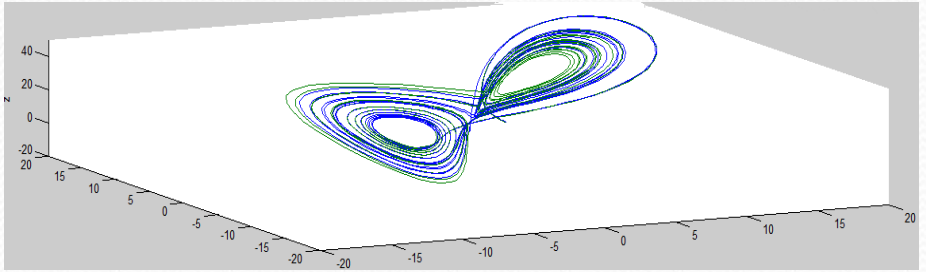
- The nonlinear and chaotic dynamics of a shallow fluid layer are investigated numerically using large-scale parallel numerical simulations
- Very implified flow model or atmospheric circulation

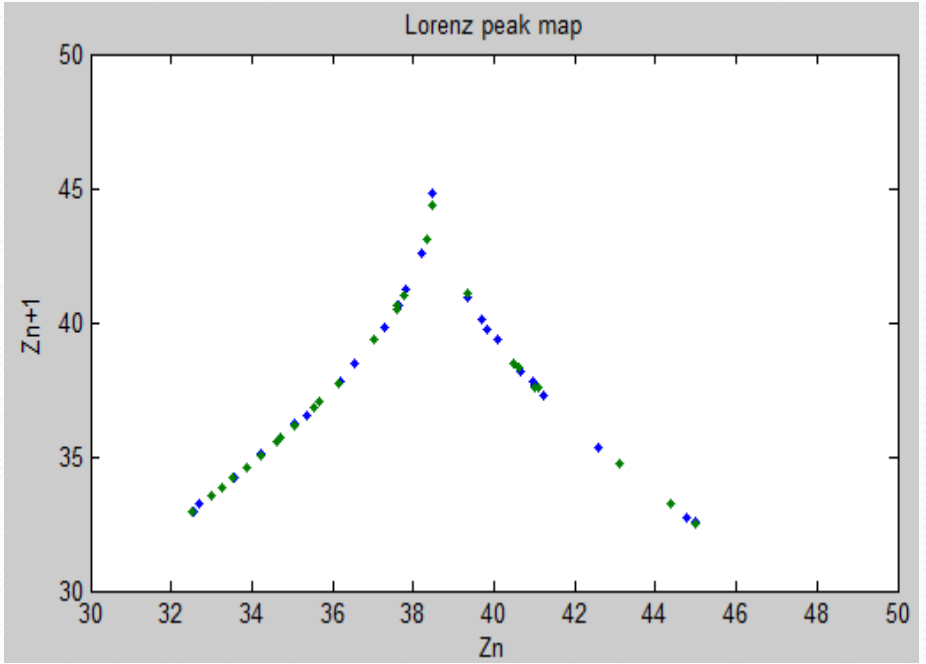
$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(\rho - z) - y$$
$$\dot{z} = xy - \beta z$$

- Under certain parameter settings behave chaotically
- We assume that the parameters are positive.

Lorenz system

- 3D autonomous system
- Different kinds of chaos
- Globally attracting attractor
- The chaos is within the attractor
- This chaos is permanent
- Lorenz peak map: simple demonstration of the **order in the chaos**





Thank you for your attention!