Theory of nonlinear dynamic systems
Practice 3

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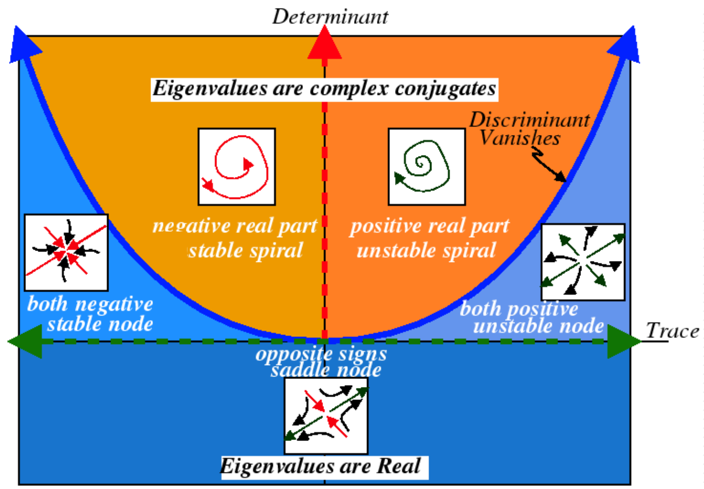
Trace-Determinant Diagram

$$\left.\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}\right\} \Rightarrow \quad p(\lambda) = \lambda^2 - T\lambda + D, \qquad T = a + d\ , \ D = ad - bc$$

- Unstable Focus, Spiral Source $\leftrightarrow T > 0 \ \& \ D > \frac{T^2}{4}$
- Unstable Node, Source $\leftrightarrow T > 0 \ \& \ 0 < D < \frac{T^2}{4}$
- Saddle Point $\leftrightarrow$ D< 0
- Stable Node, Sink $\leftrightarrow T < 0 \ \& \ 0 < D < \frac{T^2}{4}$
- Stable Focus, Spiral Sink $\leftrightarrow T < 0 \ \& \ D > \frac{T^2}{4}$

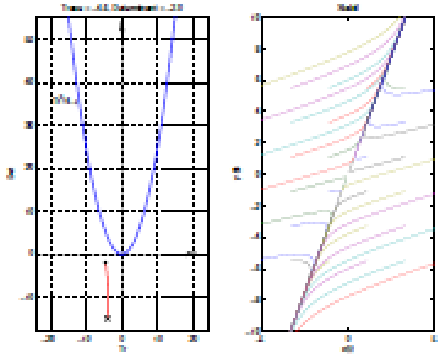


The most important of transient case

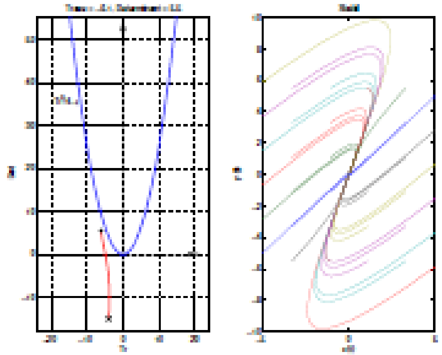
- Center $\leftrightarrow$ T = 0 & D > 0 − stability without attraction

Asymptotic stability ($\leftrightarrow$ stability and attraction)

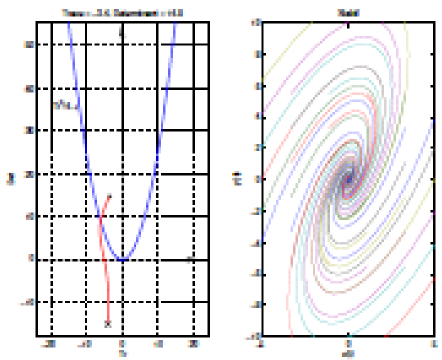
- Stable Node or Stable Focus $\leftrightarrow$ T < 0 & D > 0
- In other words: $p(\lambda) = \lambda^2 + a_1\lambda + a_0,$ *where* $a_1 > 0 \ \& \ a_0 > 0$



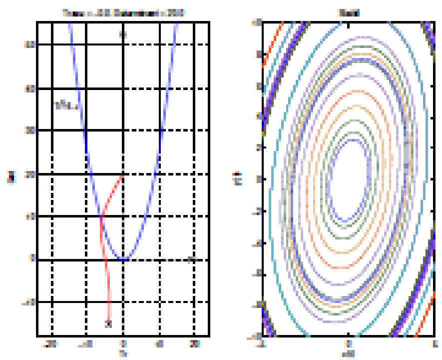
(a) Saddle Point



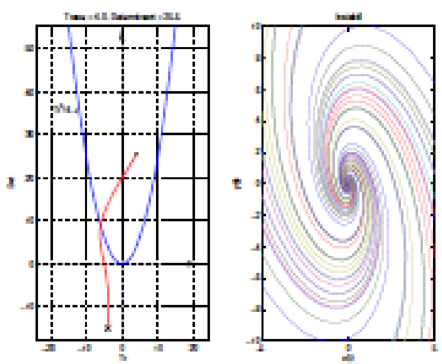
(b) Stable Node



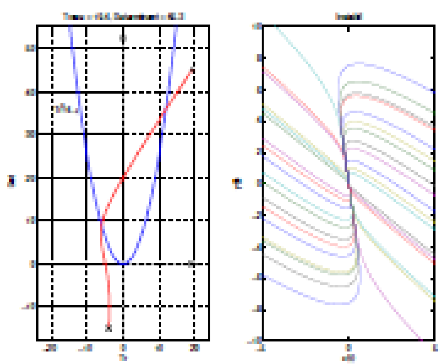
(c) Stable Spiral



(d) Center



(e) Unstable Spiral



(f) Unstable Node

Trace-Determinant Diagram

How the trajectories behave, if the linear system looks like this near the fixed point?

- 1.: $\dot{x} = -0.5x$
  $\dot{y} = -1.5x - 3y$
- 2.: $\dot{x} = -x - 0.5y$
  $\dot{y} = 2.5y$
- 3.: $\dot{x} = y$
  $\dot{y} = (-4 + a^2)x + 2ay$ $\quad -3 < a < 3$

Fixed points in inhomogeneous systems

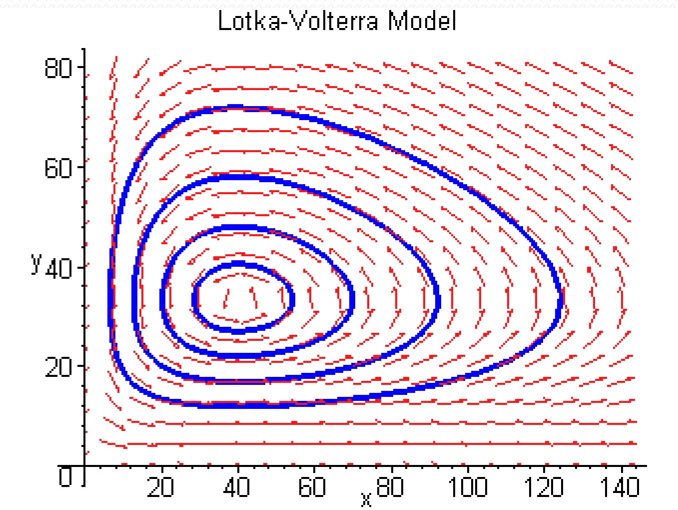
- What are the fixed points of the following homogenous system:
  - $xx'' + bx' + x = 0$ (damped string equ.2 from practice 1)
- How does external input ($\cos(\omega * t)$) change the system (inhomogenous system) and the stable point(s)?
  - $xx'' + bx' + x = \cos(\omega * t)$ (~ equ.6 from practice 1)
  - Hint: use small b (e.g.: $b=0.01$) and small resonant frequency (e.g.: $\omega = 0.1$)

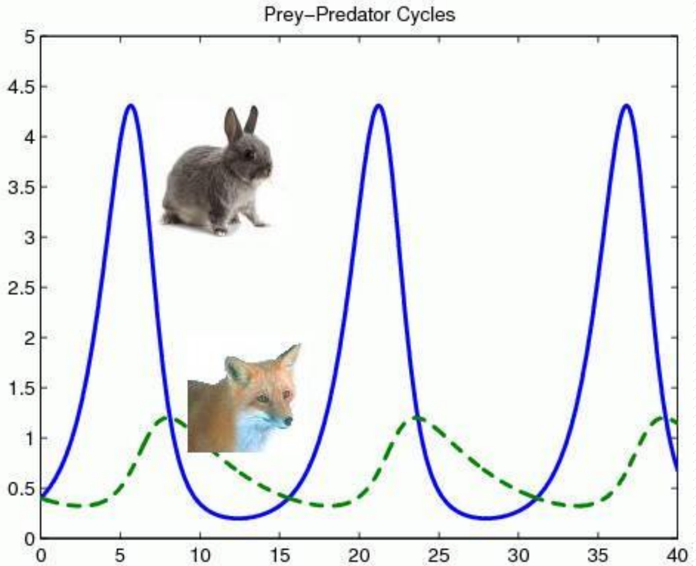
Competitive Lotka–Volterra equations

- Models of oscillating chemical reactions, or
- coexistence of Predator (y) and Prey (x) in an ecosystem
  - Only positive x y values are realistic
- dx=x*(α-β*y)
- dy=y*(-γ+δ*x)

Rate of natural increase
Natural death
Interactions between the species (here: predation)

- Initial value dependent stable oscillation emerges in the quantity of the two species.





Population dynamics example

- The size of species are x and y form the system
- Equations of their changes:
  - dx=x*(4-x-2y);
  - dy=y*(6-y-2x);

Rate of natural increase
Natural death
Interactions between the species (here: competition)

- The equations show us how the sizes of populations change from initial x and y
- „+-xy” parts define the type of the interactions
- Tasks:
  - Show the (stable and unstable) equilibrium points
  - What kinds of fixed point they are?
  - Show the separating curve of "life and death" (a curve that determines which species will remain

Thank you for your attention!