Theory of nonlinear dynamic systems Practice 2

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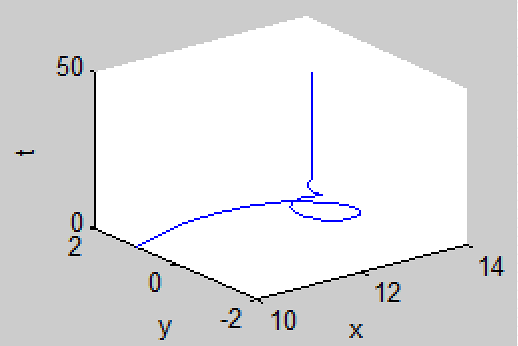
Damped pendulum equation

- Examine the following equation:

$$\ddot{x} + b\dot{x} + \sin x = 0$$



- Energy is decreasing, the system relaxes
- b: defines the speed of relaxation (damping factor)

Visualisations

- Trajectories:
  - From 1/ many initial conditions
- Energy level curves:
  - From the energy function (not known for most real life systems)
- Vector field:
  - Gradient vectors in each point
- Direction field:
  - Normalised vector field

Analyse the system: Show the equilibrium points!

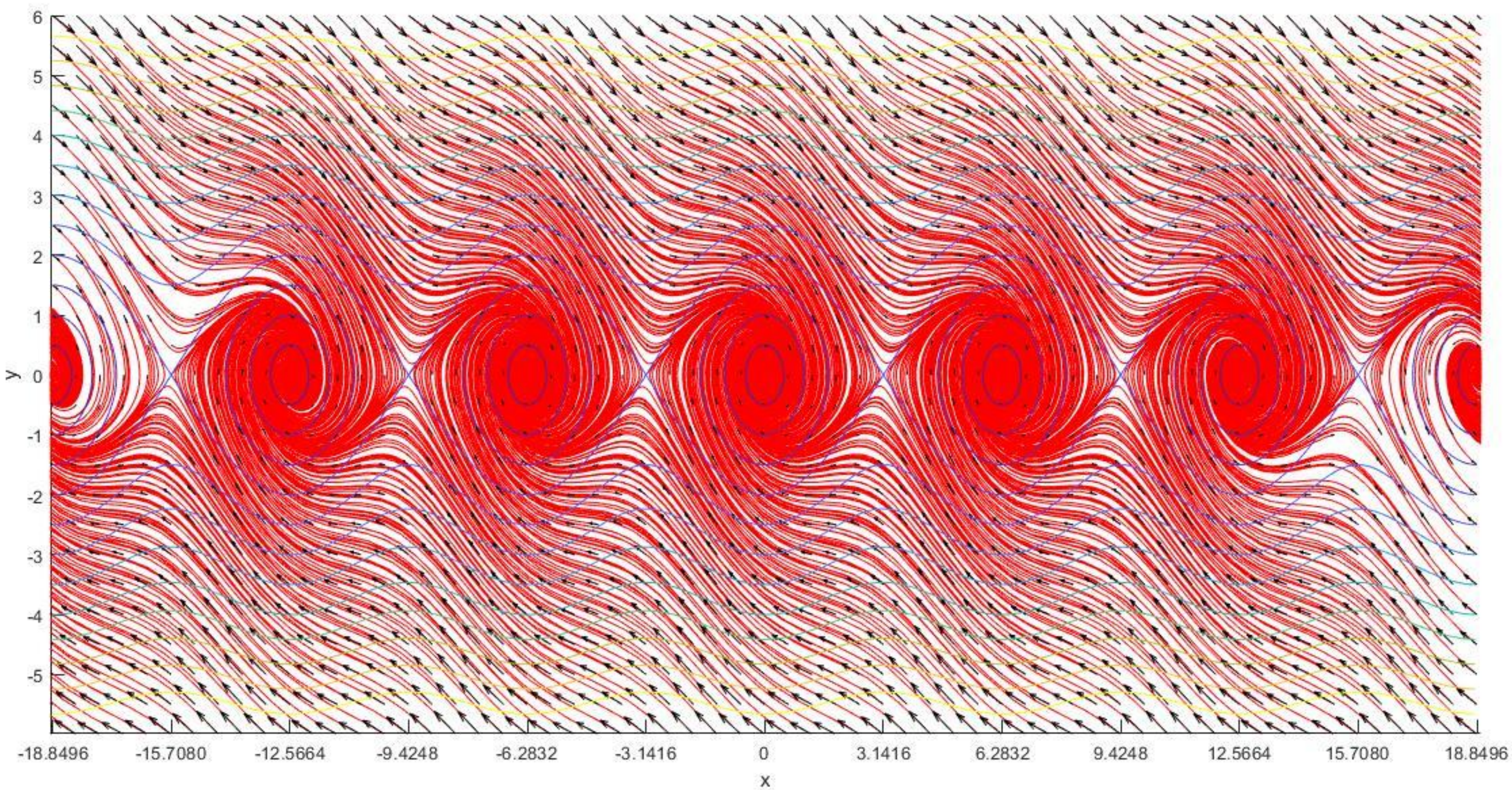
- What kinds of fixed points they are?
  - Try to visualise the stable (stable spiral, sink) and the instable (saddle point) fixed points!
  - Which fixed points are easy to show?
    - Determine the separation line (separatrix) between two stable fixed points!
- What is difference in cases of different damping parameter (see the shapes of the trajectories)?

Analyse the system: Show the equilibrium points!

- Energy level curves
- Vector field, direction field
  - Which is the more informative/useful? (vector field or direction field)
- Trajectories:
  - 1 trajectory
  - Many trajectories

Show the equilibrium points!

- **Stable fixed points (sink, stable spiral)** are easy to see



Show the equilibrium points!

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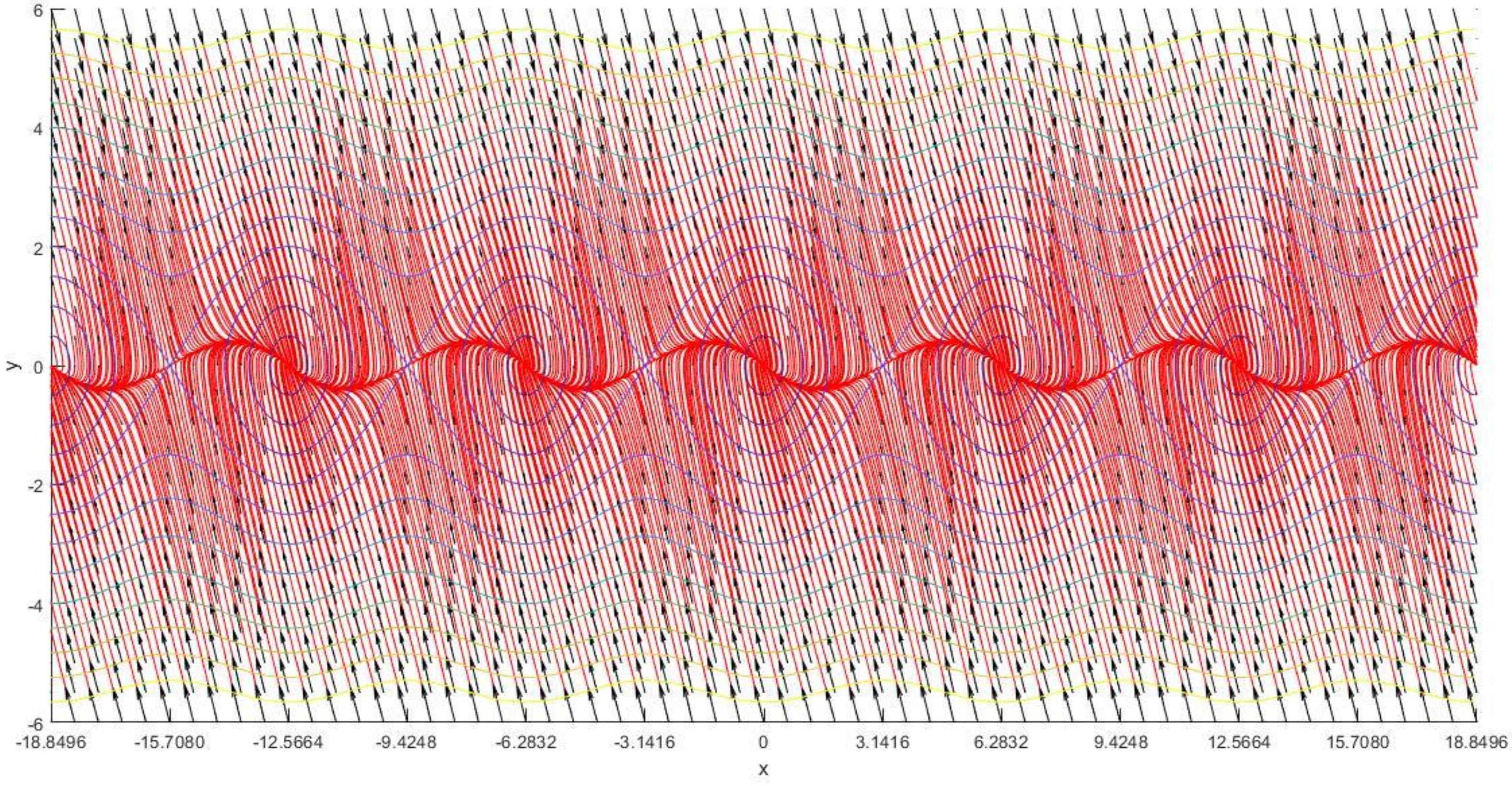
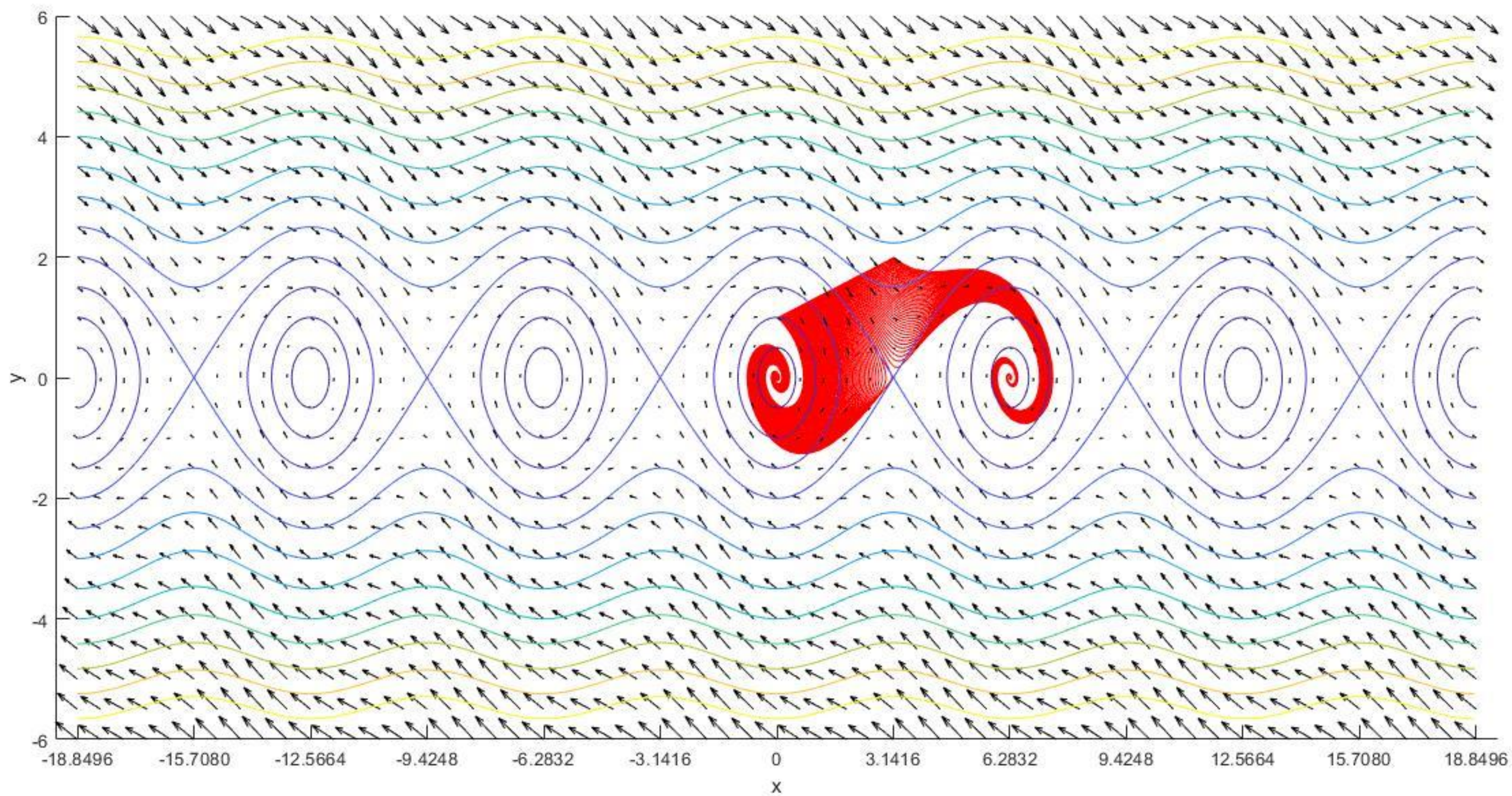


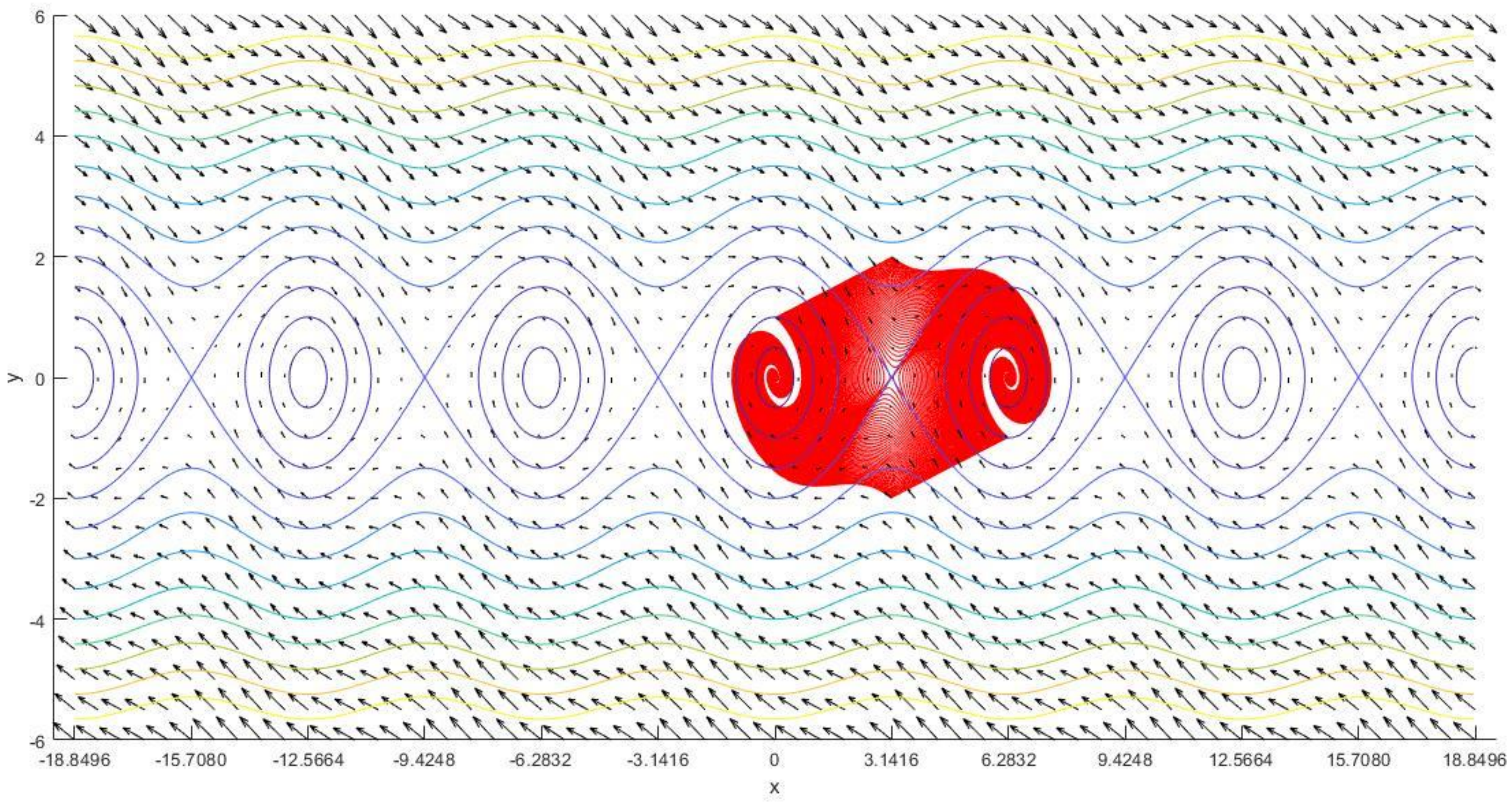
Show the equilibrium points!

- **Instable fixed points (saddle points** are harder to catch)
  - Trajectories will not stay there, only get close to them
- 1. start many trajectories from a line (b=0.5)



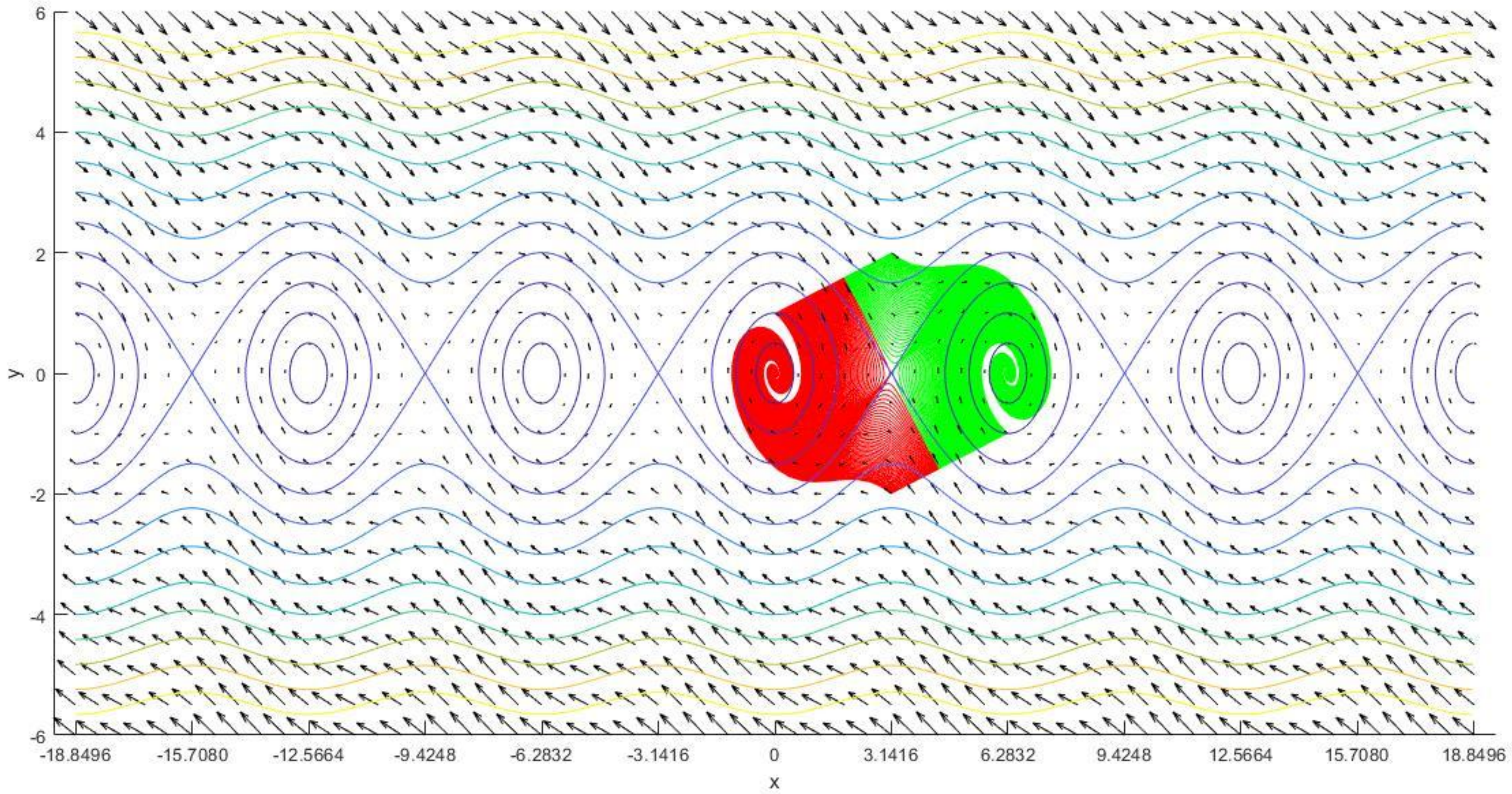
Show the equilibrium points!

- **Instable fixed points (saddle points** are harder to catch)
- 2. start many trajectories from 2 lines (b=0.5)



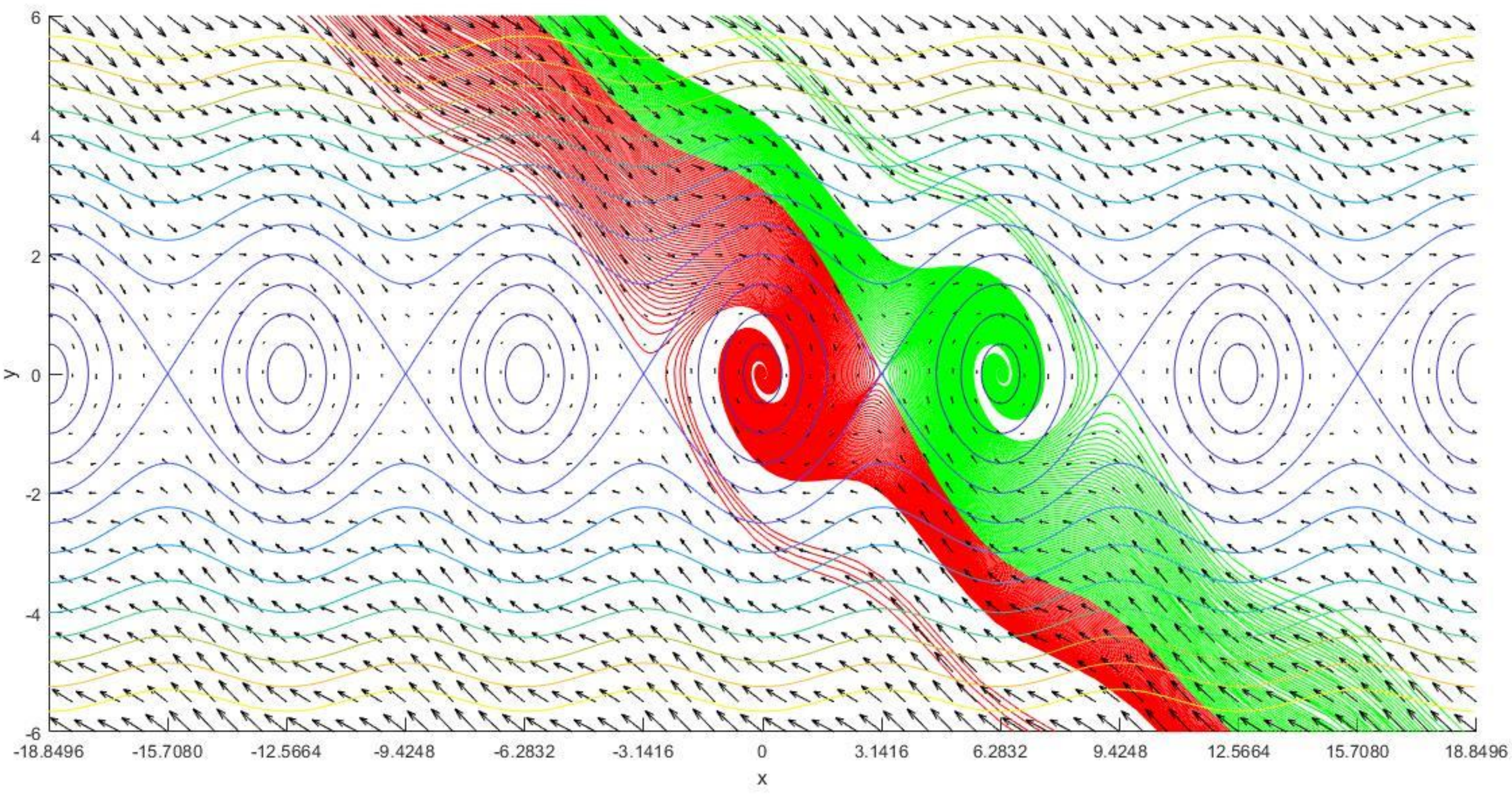
Show the equilibrium points!

- **Instable fixed points (saddle points** are harder to catch)
- 3. start many trajectories from 2 lines (b=0.5)
- Color the trajectories based on there attractor



Show the equilibrium points!

- **Instable fixed points (saddle points** are harder to catch)
- 4. start many trajectories from 2 lines (b=0.5)
- Color the trajectories based on there attractor
- Turn back time (calculate not only f(x), but also -f(x)) to see the entire separatrix



Show the equilibrium points!

- **Instable fixed points (saddle points** are harder to catch)
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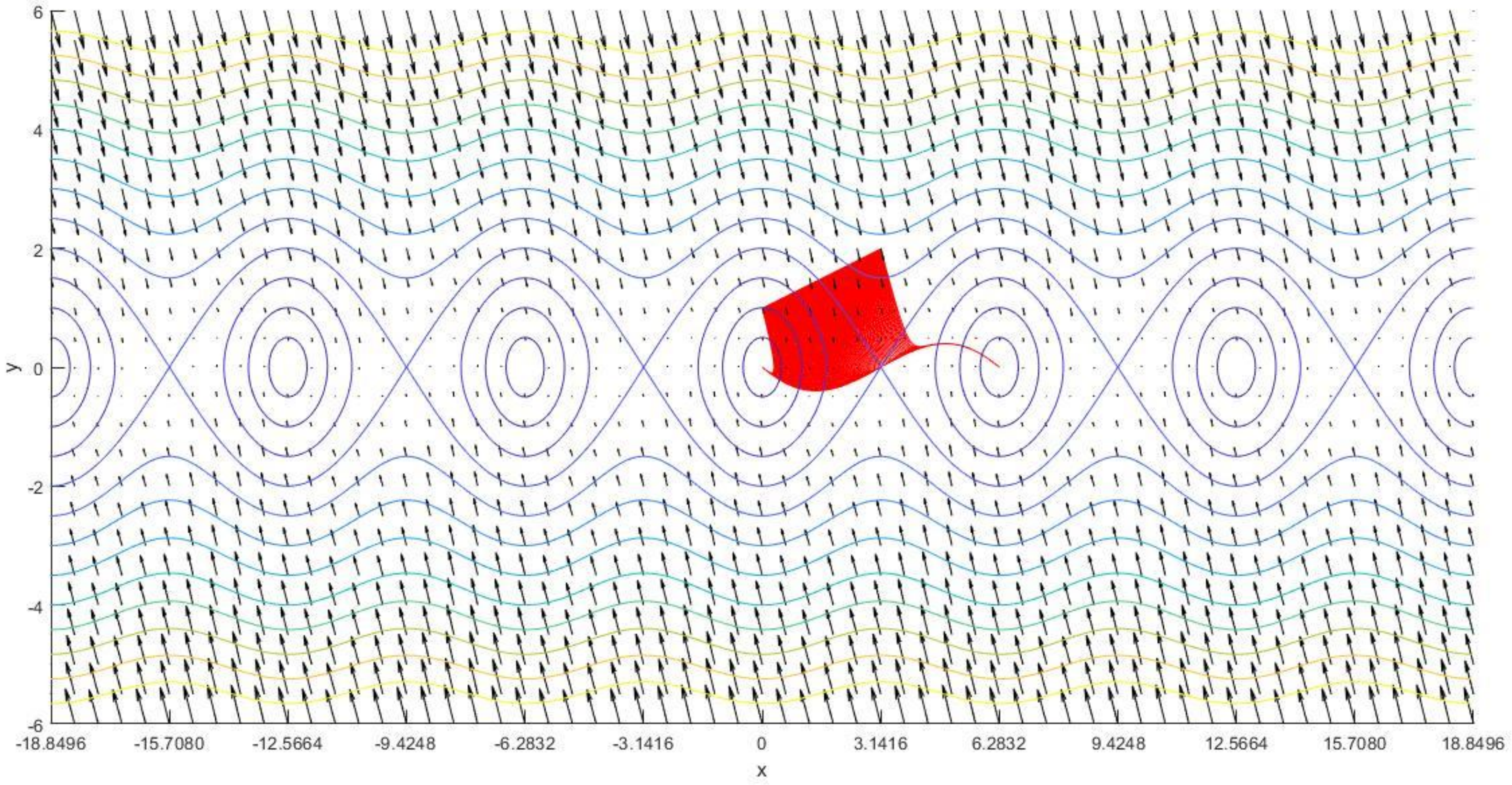
- 1. start many trajectories from a line (b=2.5)



Show the equilibrium points!

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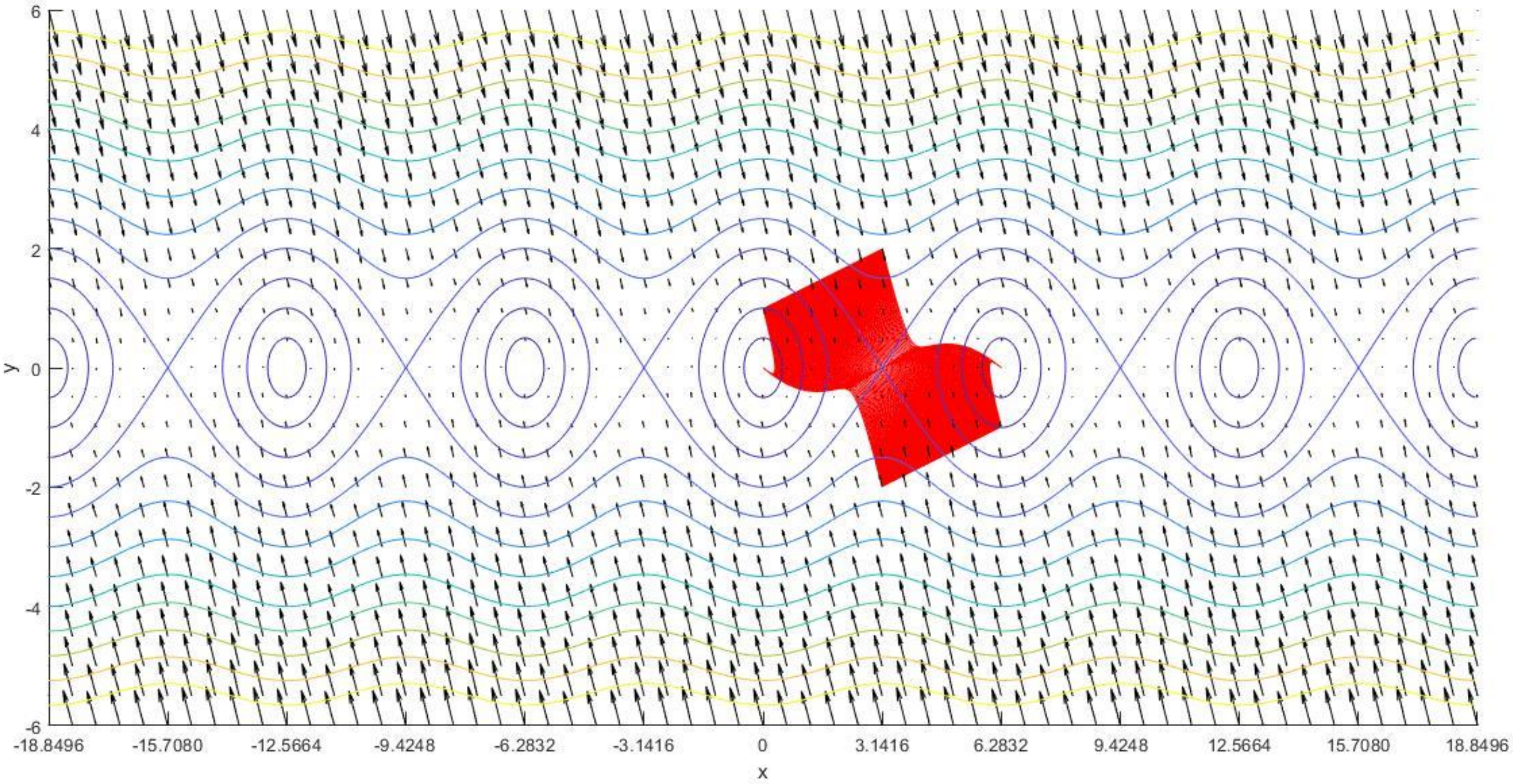

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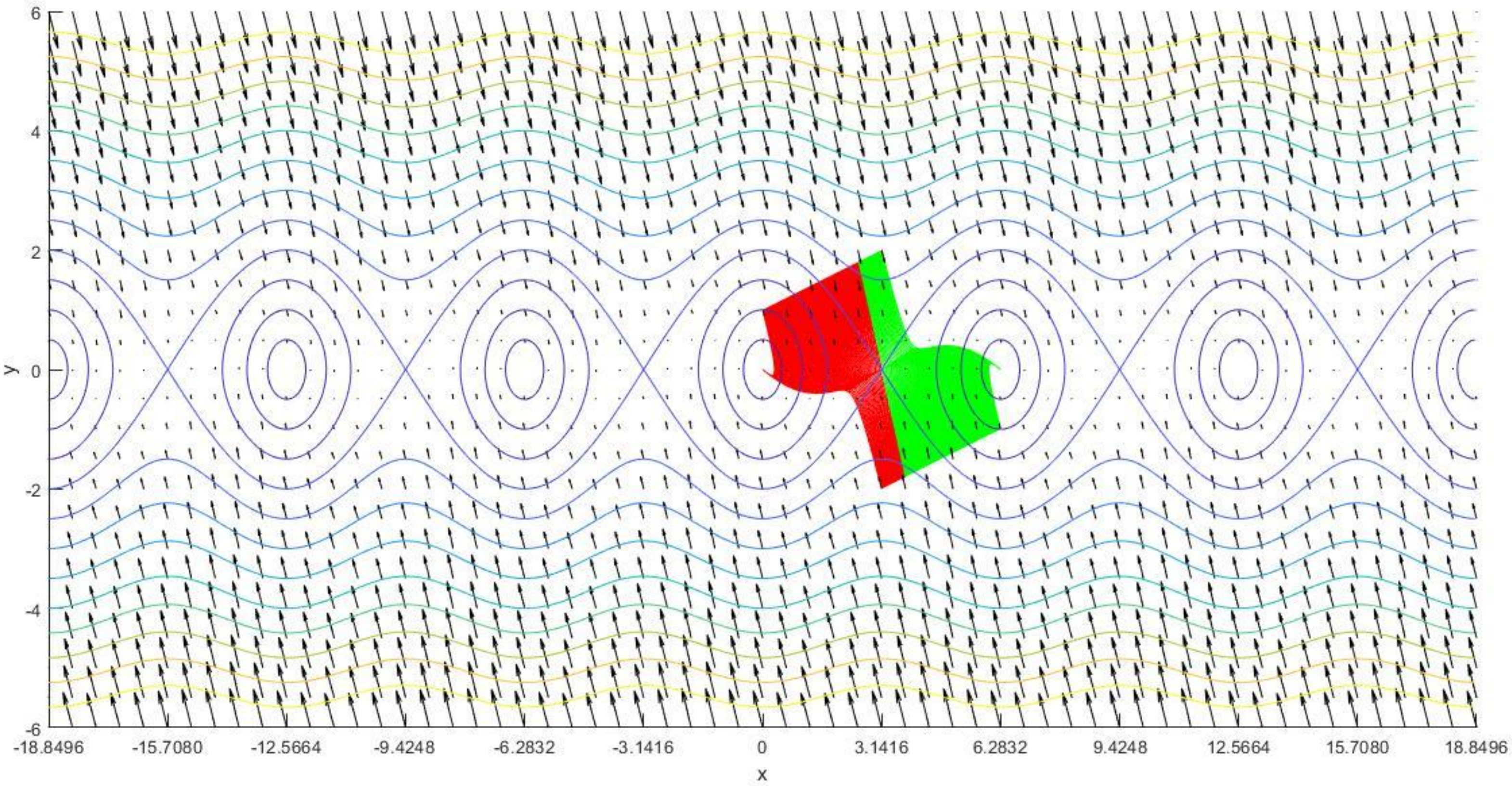

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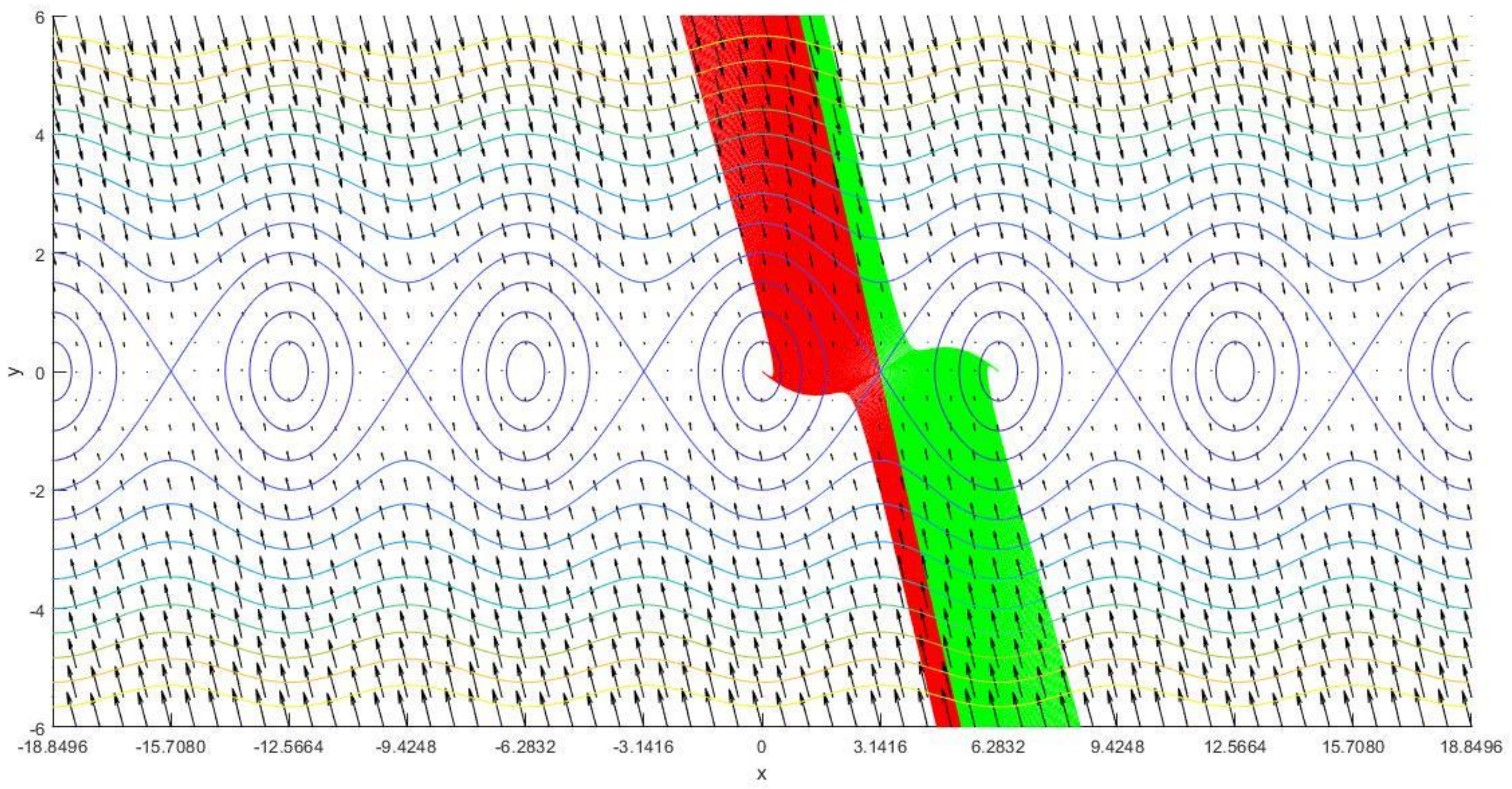
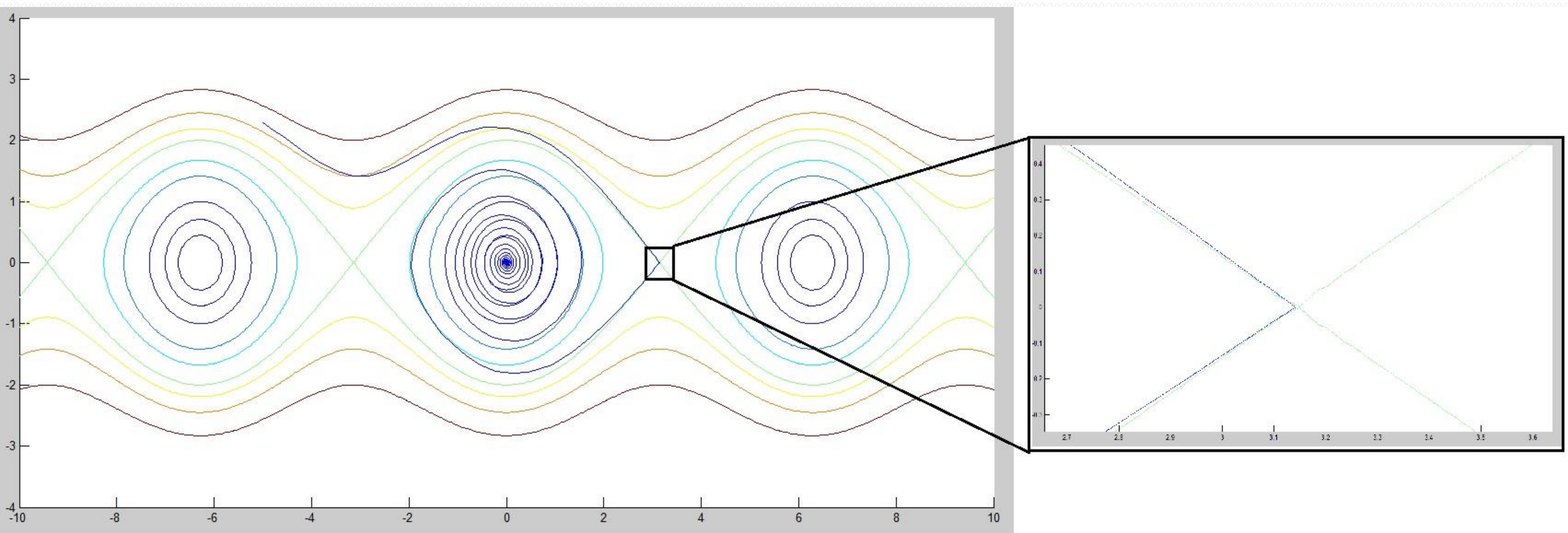


Bolzano shooting

- Approach a saddle point with 1.0e-6 precision! (Give the initial x,y coordinates of the trajectory!)



Thank you for your attention!