Theory of nonlinear dynamic systems
Practice 1

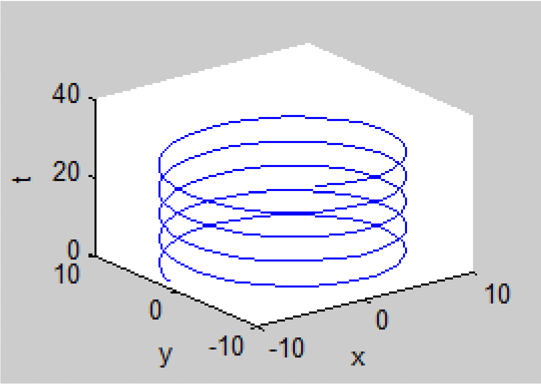
Juhász János
juhasz.janos@.itk.ppke.hu

I. Equation

- Examine the following equation:

$$\ddot{x} + x = 0$$

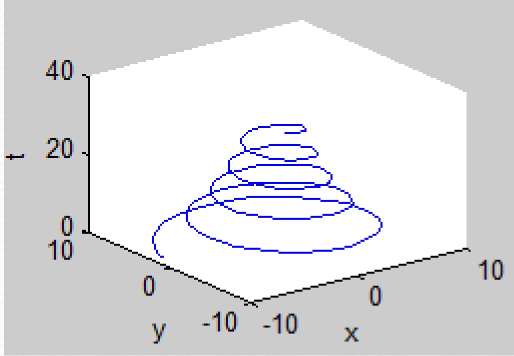
- Simple harmonic oscillator, LC circuit, "perpetual motion"
- Constant energy
- Only the amplitude is changing at changing Xinit, Yinit parameters, no other effect on the characteristics of the trajectory.



II. Equation

- Examine the following equation:

$$\ddot{x} + b\dot{x} + x = 0$$

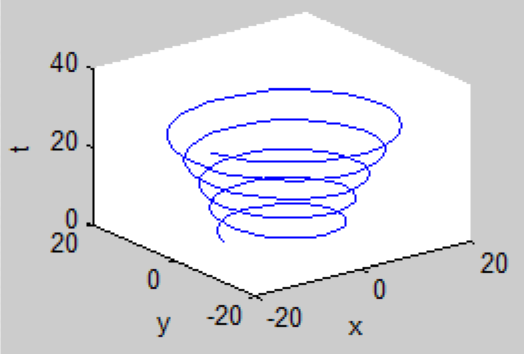


- Damped harmonic oscillator, RLC circuit, spring.
- Changing energy – relaxation.
- Xinit, Yinit has effect to the attenuation
- b: most important parameter, attenuation rate.
  - b < 2: soft damping, spiral fixed point
  - b > 2: hard damping, sink fixed point

III. Equation

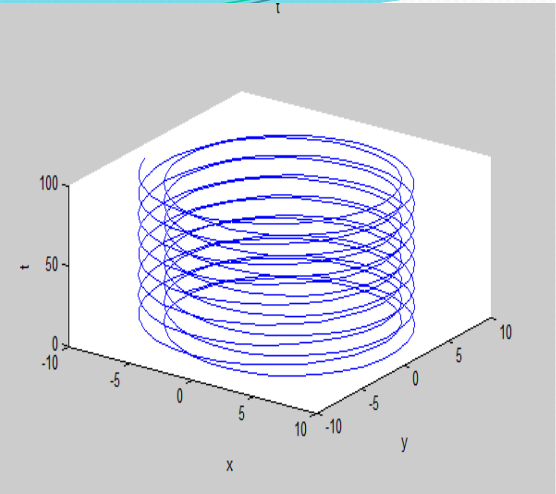
- Examine the following equation:

$$\ddot{x} + x = \cos t$$



- Excitation of the classical harmonic oscillation, RC circuit, excitation by alternate current (AC), resonant case
- Energy increses, the system „explodes”
- Settings of Xinit, Yinit parameters
- cos(t): excitation factor

IV. Equation

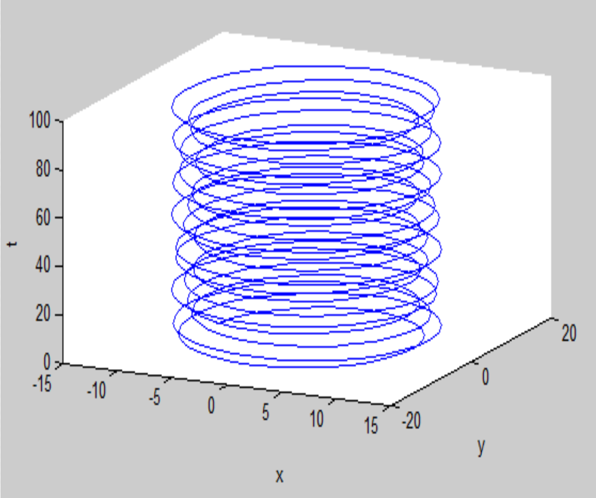


- Examine the following equation:

$$\ddot{x} + x = \cos \omega t$$

- Same to the previous one, but it has a plus frequency component (omega) in the excitation.
- Complex periods
- The energy is changing sinusoidal because of the Inhomogeneous DE
- w_b : oscillation of angular frequency
- $w_k = 1$
- In the case of III. equation : $w_b = w_k = 1$

V. Equation



- Examine the following equation:

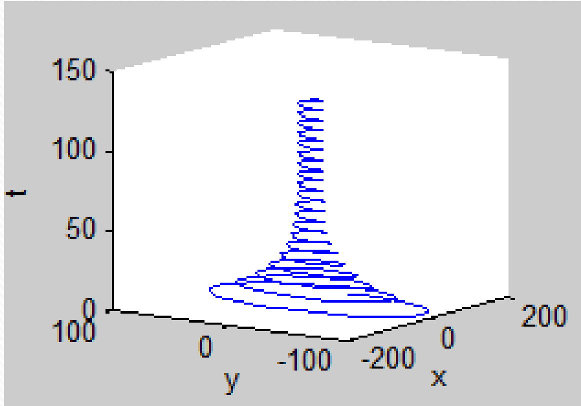
$$\ddot{x} + 2x = \cos t$$

- Equation of harmonic oscillator.
- With different string constant (2)
- General solution formula:

$$c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t$$

- Complex periods
- Energy is oscillating

VI. Equation



- Examine the following equation:

$$\ddot{x} + b\dot{x} + x = \cos t$$

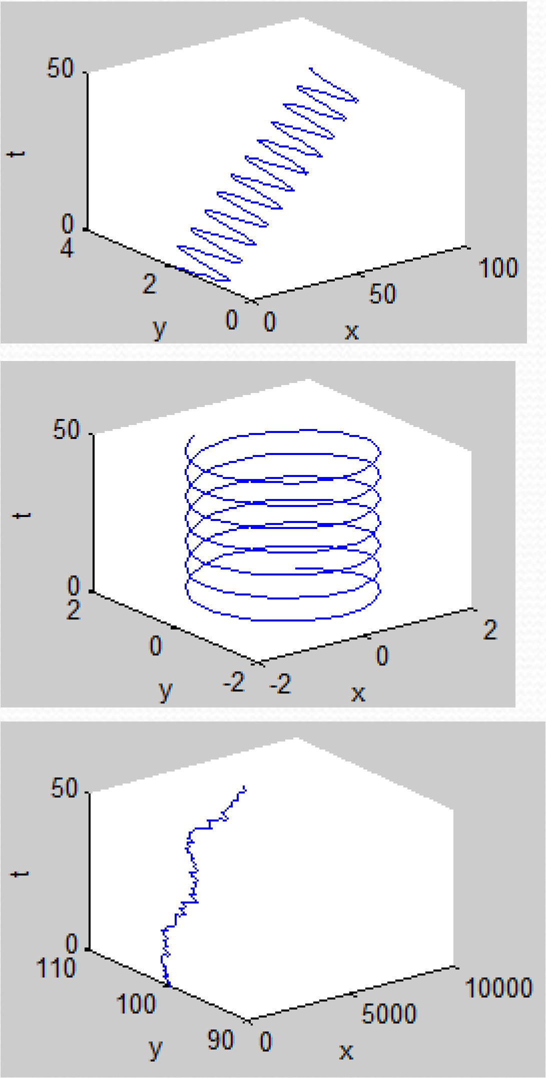
- Damped system under harmonic force, stable periodic orbit after transient state with sustained energy profile
- Initial conditions have no effect the stable orbit (in most cases...)
- b: greater b parameter results in reaching periodic orbit earlier (system lost the initial energy and only the excitation sustains it)

VII. Equation

- Examine the following equation:

$$\ddot{x} + \sin x = 0$$

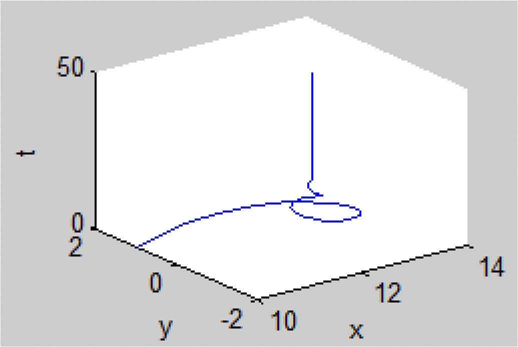
- Simple (rigid) pendulum equation: swinging ball, swinging boat
- Conservation of energy
- Similar to Equation 1.



VIII. Equation

- Examine the following equation:

$$\ddot{x} + b\dot{x} + \sin x = 0$$



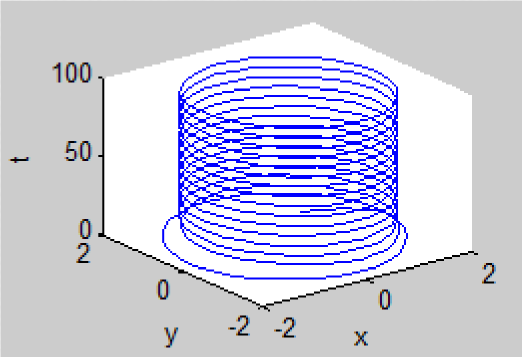
- Damped pendulum
- Energy is decreasing, the system relaxes
- b: defines the speed of relaxation
- Similar to Equation 2.

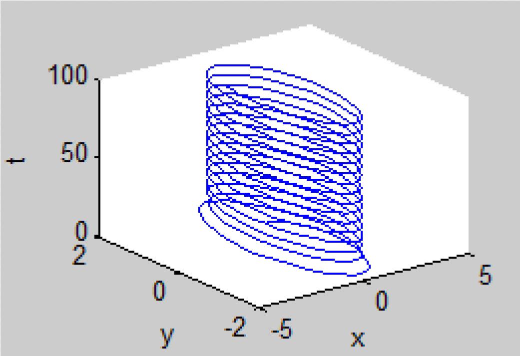
IX. Equation

- Examine the following equation:

$$\ddot{x} + b\dot{x} + \sin x = \cos t$$

- Pendulum, oscillation with excitation and damping
- Similar to Equation 6.
- Chaos ☺
- Figures with Xinit = 1 and Xinit = 1.1 initial conditions – significant difference to small initial changes

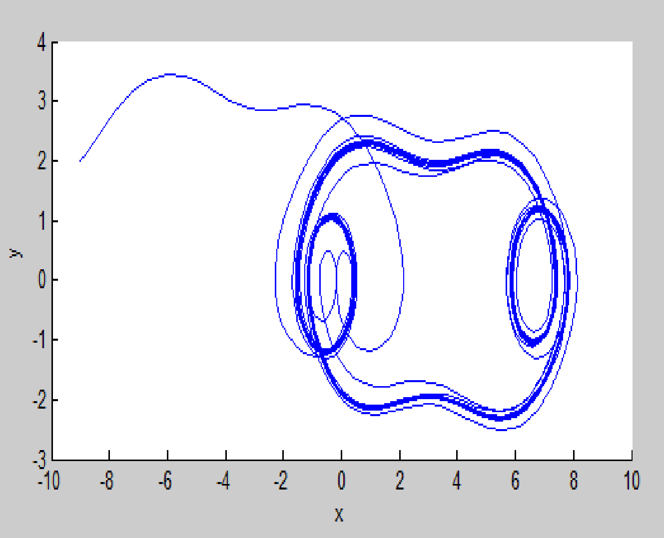




X. Equation

- Examine the following equation:

$$\ddot{x} + b\dot{x} + \sin x = \cos \omega t$$



- Equation 9. + excitation with resonant frequency
- b=0.05
- ω=0.9
- Double period