2019/2020 NDS EXAM — LIST OF TOPICS:

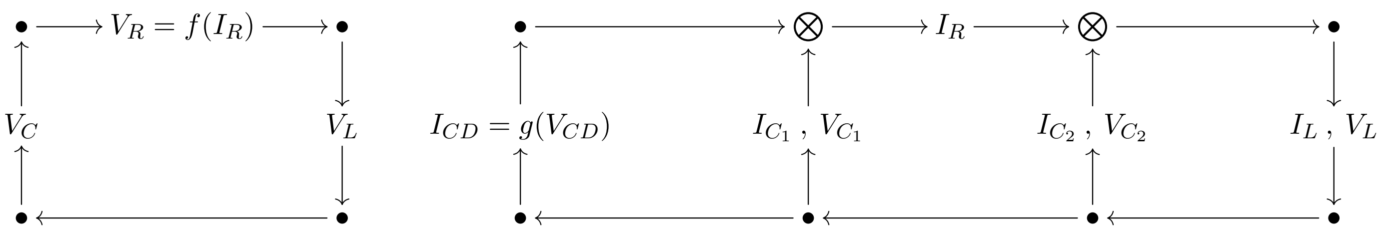
Basic Examples in Their Usual Contexts:

1.) $\dot{x} = x(c_1 + a_1 x + b_1 y)$, $\dot{y} = y(c_2 + a_2 x + b_2 y)$: Lotka–Volterra systems in 2D (appearance of the basic concepts and methods, linear scaling (linear transformation of the variables))

2.) $\dot{x} = f(x)$, $\mathrm{div}\, f \equiv 0$; $X = F(x)$, $\det(F'(x)) \equiv 1$: general and volume/area preserving continuous and discrete time dynamical systems

3.) $\ddot{x} + \frac{1}{10}\,\dot{x} + \sin(x) = \cos(t)$: Linearly damped gravitational pendulum with (co)sinusoidal forcing (combinatorial chaos, phase portraits of related special cases including linear LRC circuits)

4.) $\ddot{x} = -\,\underline{grad}\, V(x)$; $(d = 1)$ $\dot{x} = y$, $\dot{y} = -\,V'(x)$: Newton's second law in potential force field (three different types of Euler method, $X = x + hf(x)$, $X = x + hf(X)$; $X = x + hy$, $Y = y - hV'(X)$)



5.) $\ddot{x} - \mu(1 - x^2)\dot{x} + x = 0$ ($x \sim I_L$, $y = \dot{x} \sim V_L$, $f(x) = \mu(x - x^3/3)$ (for $L = 1$, $C = 1$)) : Van der Pol circuit (relaxation oscillation for μ large), $\dot{x} = \alpha(y - x - g(x))$, $\dot{y} = x - y + z$, $\dot{z} = -\beta y$ ($x \sim V_{C_1}$, $y =\sim V_{C_2}$, $z \sim I_L$, $g(x) = -\,g_1 x + g_3 x^3$) : Chua circuit[1] (symmetric synchronization (on the 3D diagonal) via large diffusive coupling in the first coordinate, chaotic circuit)

6.) $\dot{x} = \sigma(y - x)$, $\dot{y} = rx - y - xz$, $\dot{z} = xy - bz$: Lorenz system (Lorenz peak map, master-slave synchronization ($x \leftrightarrow X$ in the second and third coordinates), secret communication via open channel)

7.) $x_{k+1} = \mu x_k(1 - x_k)$, $k \in \mathbb{N}$; $0 \le \mu \le 4$: Logistic family of maps (Liapunov exponent and period doubling cascade on the bifurcation diagram, case $\mu = 4$: time versus space averages)

8.) $\mathcal{S} = \cap_{k \ge 0} S^k(RT)$ where $S(x) = F^T(x) \cup F^L(x) \cup F^R(x)$, F^V ($V = T, L, R$) is the radial contraction of factor 1/2 centered at vertex V of a regular triangle RT : Sierpinski triangle (as an attractor of an iterated function system, Sierpinski chaos game, Borel's normal number theorem, $\dim(\mathcal{S}) = \frac{\ln(3)}{\ln(2)}$)

[1] Notations $\bullet$ (actually, groups of bullets) and $\otimes$ refer to Kirchhoff's voltage law $V_L + V_R + V_C = 0$, $V_{CD} -\ V_{C_1} = 0$, $V_L + V_{C_2} = 0$ and current law $I_{C_1} + I_{CD} = I_R$, $I_{C_2} + I_R = I_L$ and , respectively.

Basic Types of Dynamical Behaviour:

A.) Stability and attraction of equilibria (and of fixed points), examples showing that stability and attractivity are independent notions, stability analysis via eigenvalues in $\mathbb{R}^d$ esp. for $d = 2$

B.) Saddle structure and stable/unstable manifolds; Grobman–Hartman Lemma in the vicinity of noncritical equilibria, also for discretization

C.) Omega limit sets for bounded trajectories in $\mathbb{R}^d$ esp. for $d = 2$, attractors

D.) Inequalities for $\dot{V}_{(E)}(x)$, Liapunov's Russion doll and trapping region arguments

E.) Chaos from the view–point of topology (Devaney's definition of chaos), measure theory and combinatorics; Liapunov exponent, topological entropy, and box dimension as chaos/fractal indicators

F.) One–step p–th order stepsize h discretization operator for equation $\dot{x} = f(x)$; various aspects of discretization: error estimate on $[0, T]$, the iteration leading to implicit Euler method, implicit Euler method with large stepsize, round–off error and stepsize, adaptive stepsize control

G.) Equivalence and some basic types of objects (vector spaces, graphs/trees, discrete–time dynamical systems) in mathematics. Conjugacy

H.) Topological equivalence and structural stability for autonomous ordinary differential equations. The 1D and the 2D case. Elementary examples for and the abstract notion of bifurcations

ON THE EXAM: A topic from those between 1 and 8 and a topic from those between A and H. A topic for free presentation and a topic for discussion (questions and answers).