Lab 05

Basic Image Processing
Fall 2019

Discrete Fourier Transform

- We sample one period of the Fourier transform in evenly spaced frequencies:

$$X(\omega_1, \omega_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1, n_2) e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$

The size of the image in the spatial domain is N_1xN_2

The size of the image in the frequency domain will be the same: N_1xN_2

$$X(k_1, k_2) = X(\omega_1, \omega_2)\Big|_{\omega_1 = \frac{2\pi}{N_1}k_1, \omega_2 = \frac{2\pi}{N_2}k_2}$$

$$k_1 = 0,1\ldots, N_1 - 1$$
$$k_2 = 0,1\ldots, N_2 - 1$$

Only one period is kept

$$X(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1, n_2) e^{-j\frac{2\pi}{N_1}k_1 n_1} e^{-j\frac{2\pi}{N_2}k_2 n_2}$$

Discrete Fourier Transform

- Forward formula: gives the description of the image in the discrete frequency domain

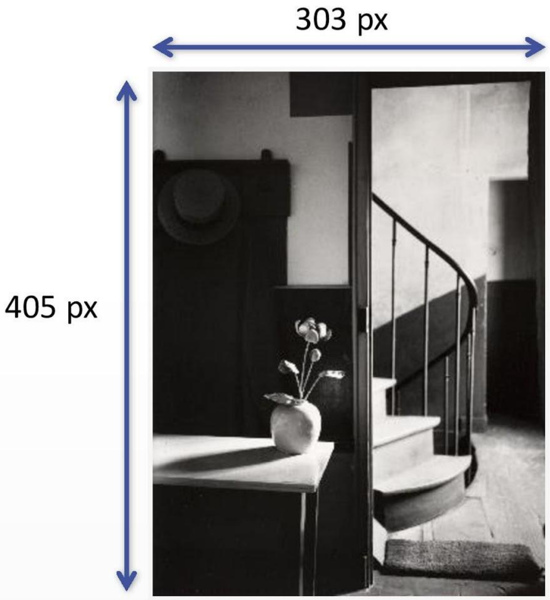
$$X(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1, n_2) e^{-j\frac{2\pi}{N_1}k_1 n_1} e^{-j\frac{2\pi}{N_2}k_2 n_2}$$

- Inverse Fourier transform: maps from the discrete frequency domain back to the discrete spatial domain

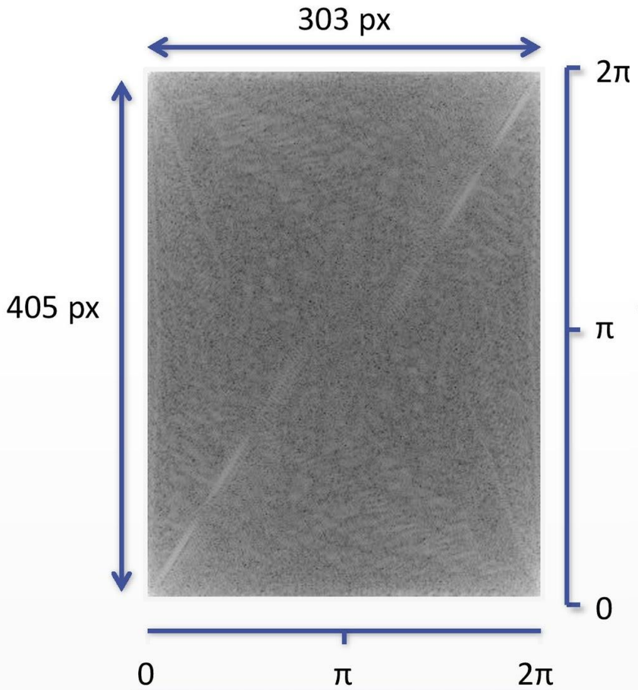
$$x(n_1, n_2) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} X(k_1, k_2) e^{j\frac{2\pi}{N_1}k_1 n_1} e^{j\frac{2\pi}{N_2}k_2 n_2}$$

  - algorithmically it has the same structure as the forward transform,

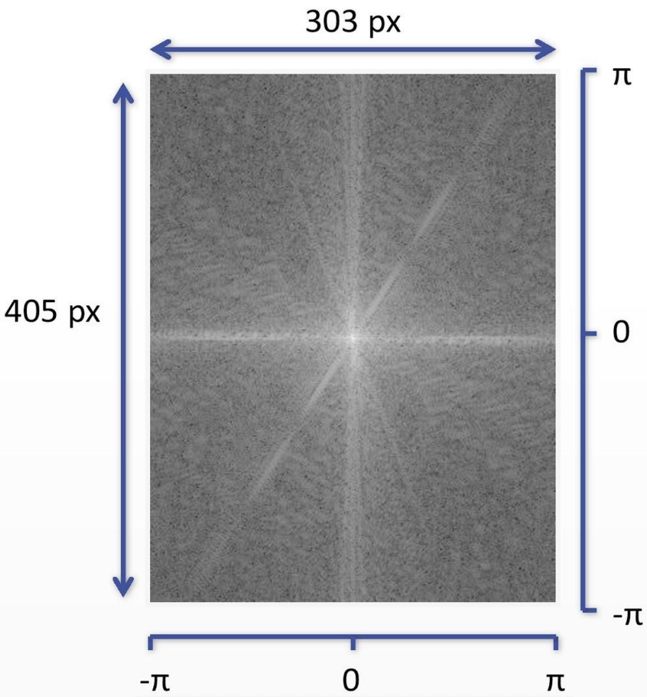
Discrete Fourier Transform



Original Image*



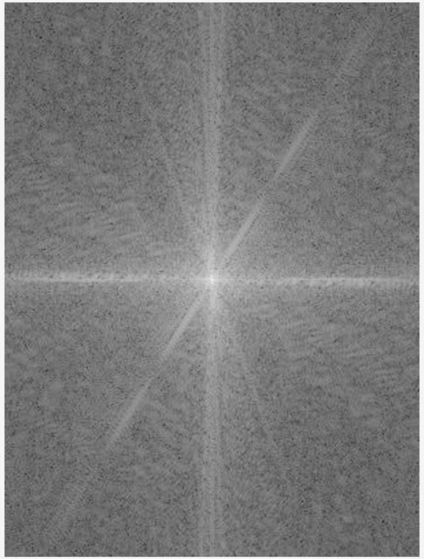
Magnitude of the
Discrete Fourier Transform



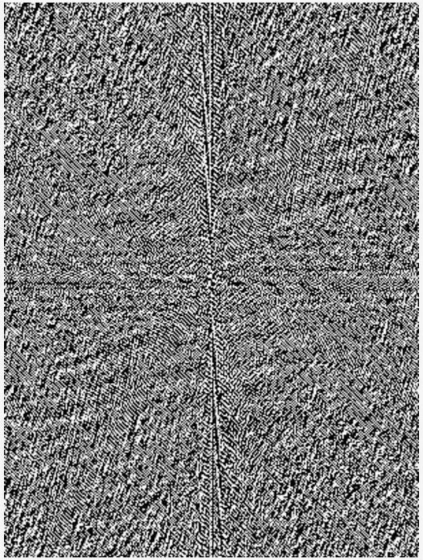
Magnitude of the **centered**
Discrete Fourier Transform

Discrete Fourier Transform

Original Image*



Magnitude of the **DFT**

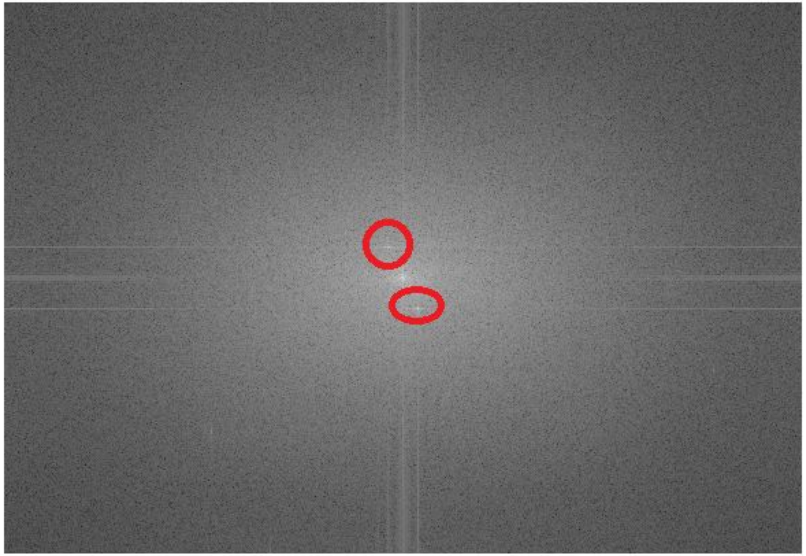


Phase of the **DFT**

Example application I. – Noise filtering

original input with noise

original magnitude of DFT

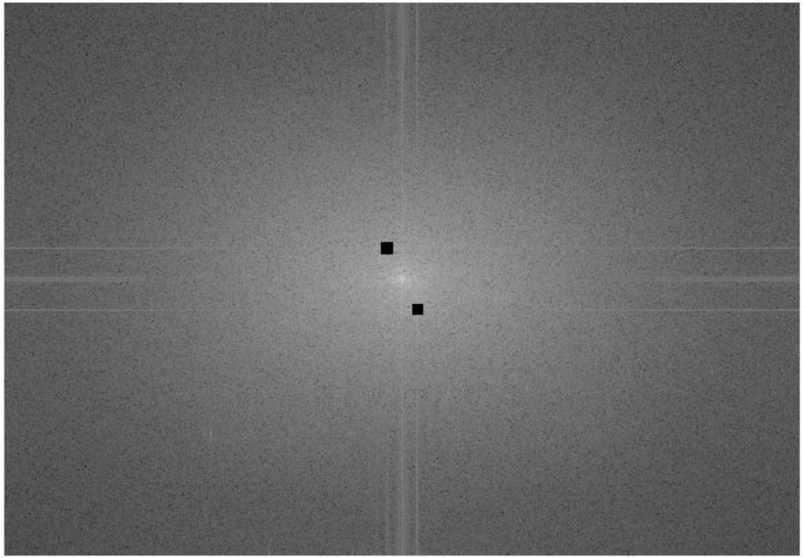


On the left: original image (Robert Capa: Lovers Parting Near Nicosia, Sicily) + some sinusoidal noise
On the right: magnitude part of the frequency domain
Thanks to the regular form of the noise, we can see it as two concentrated frequency-points on the DFT-image (circled with red).

Example application I. – Noise filtering

What if we hide these two, intensive-regions from the frequency-domain?
(Hide := decrease their significance, actually I set their value to complex zero, 0+0i.) The noise disappears!

modified magnitude of DFT



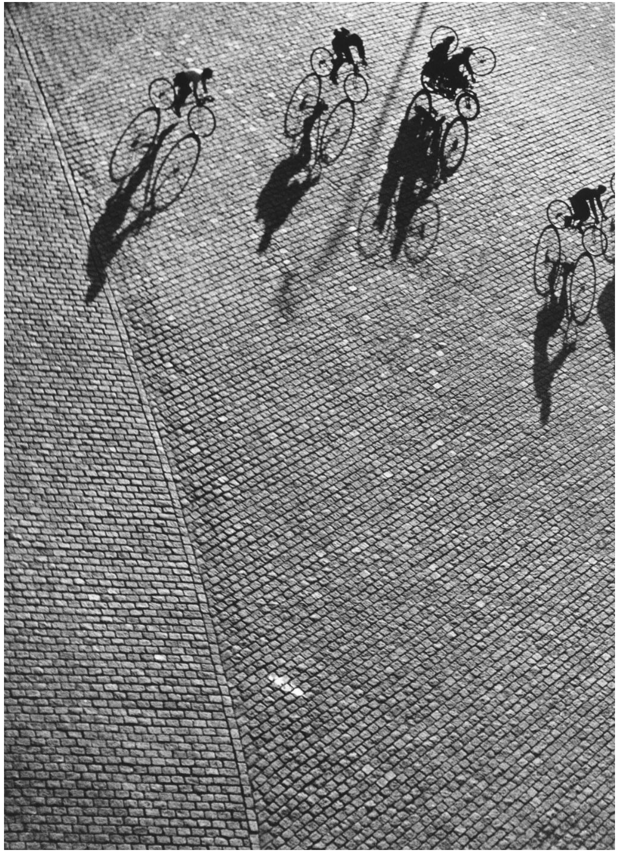
filtered output

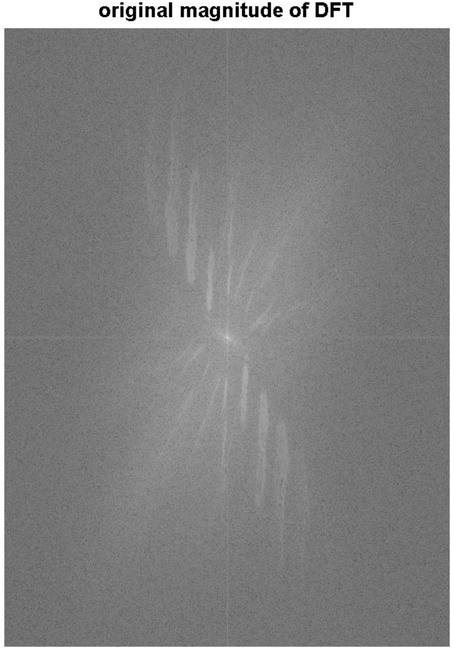
Example application I. – Noise filtering

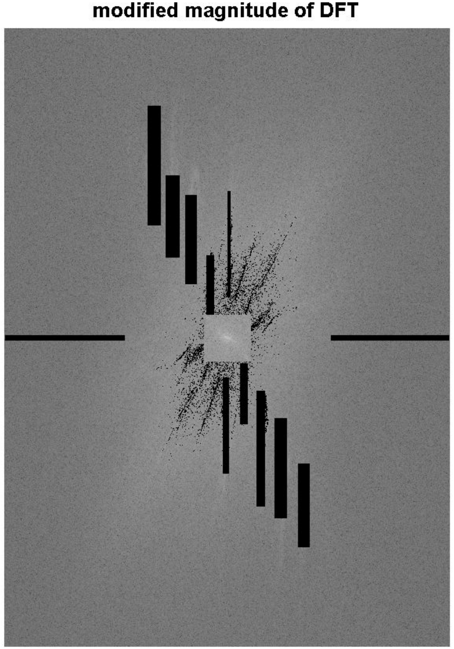
part of orig. input

part of filt. output

Example application II. – Noise filtering



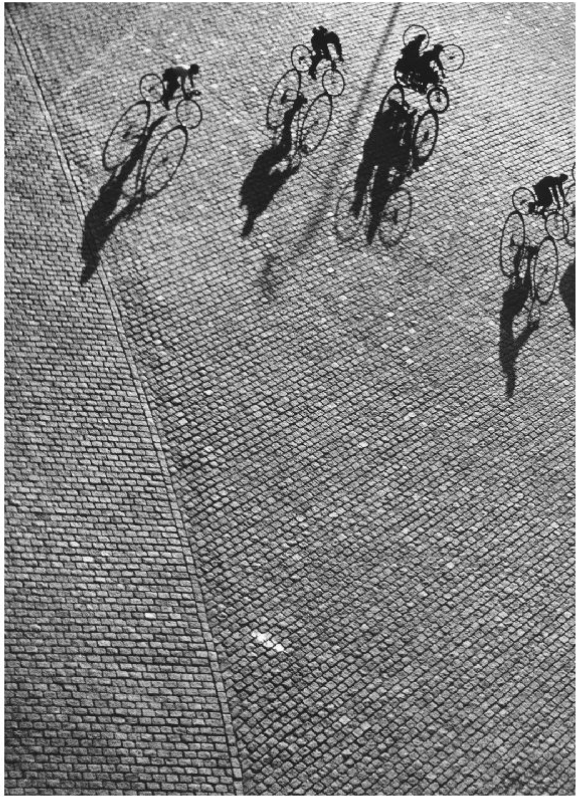




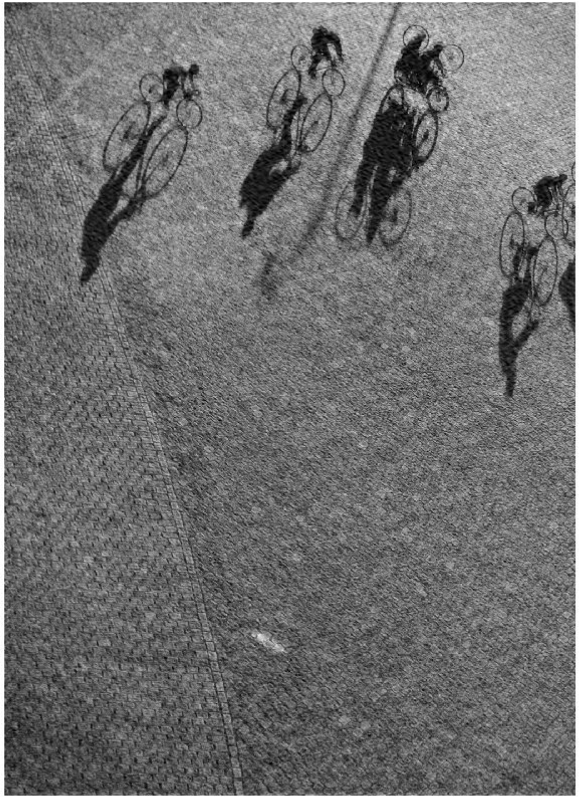
On the left: Lucien Herve: Paris Sans Quitter Ma Fenetre (Les Cyclistes)
On the center: magnitude part of the frequency domain
On the right: modified magnitude (again, the spec. values are replaced with complex zeros)

Example application II. – Noise filtering

original input

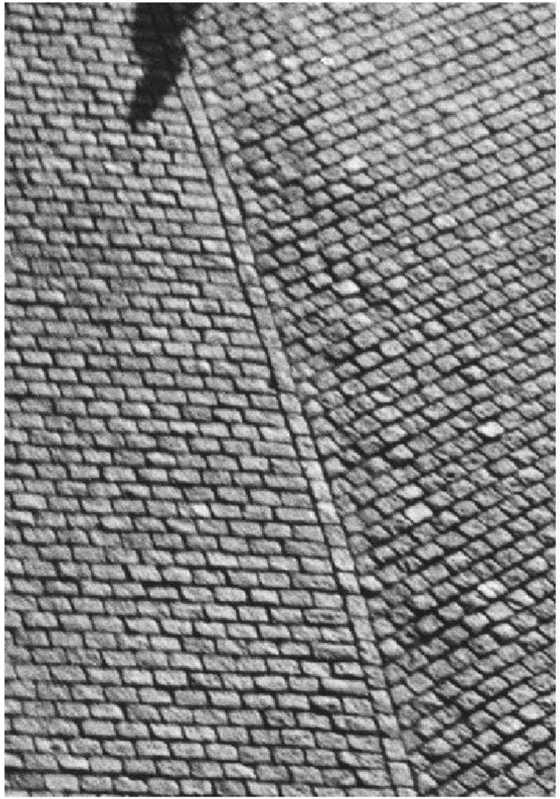


filtered output

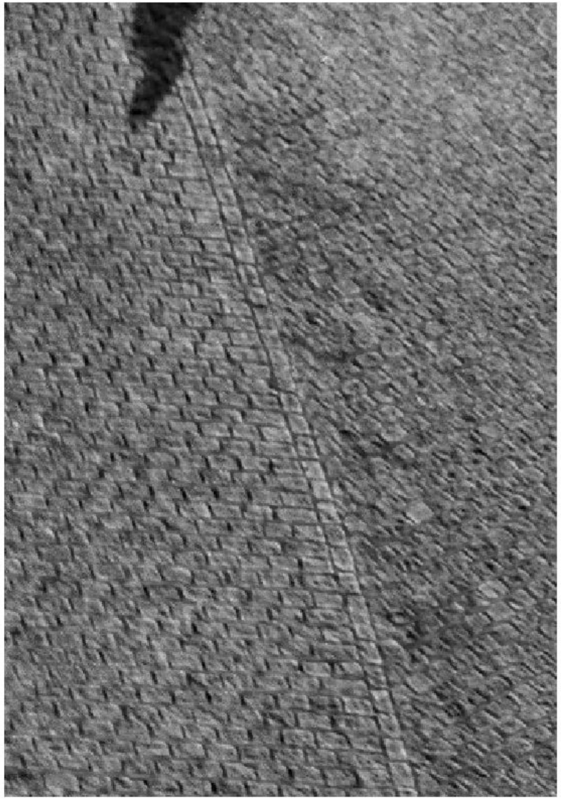


Example application II. – Noise filtering

part of orig. input



part of filtered output



Now please

download the ‘Lab 05’ code package

from the

[submission system]

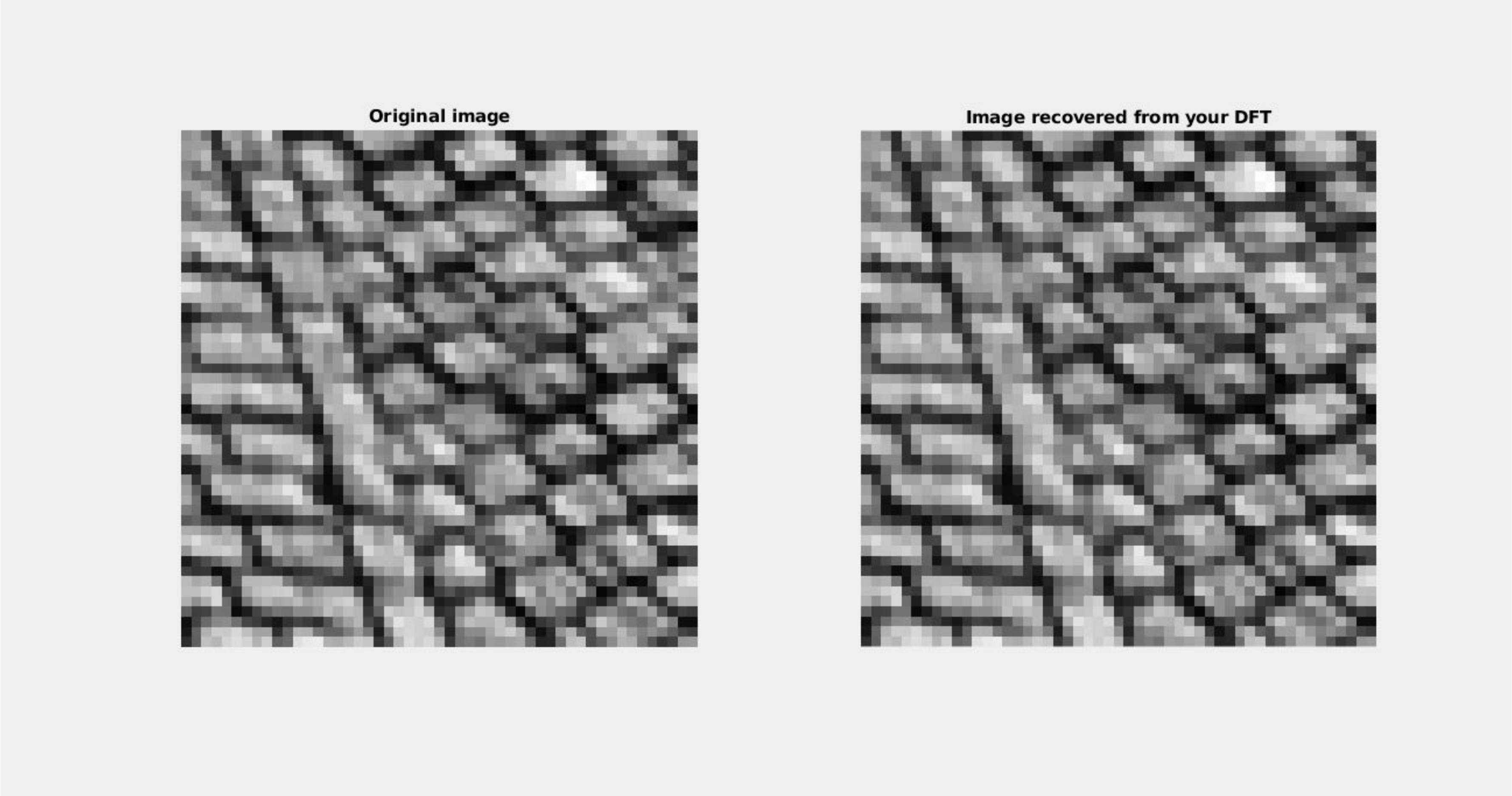
Exercise 1

Implement the function `my_fourier` in which:

- Create the empty `F` as the complex discrete Fourier space. This should be the same size as the input image (`I`).
- With `k1` and `k2` iterate through the Fourier-space (two (nested) `for` loops).
- Compute `F(k1, k2)` which is denoted by $X(k_1, k_2)$ on Slide 3. For this you'll need to use `n1` and `n2` to iterate through the input image (another two (nested) `for` loops inside the previous ones → 4 nested loops)

You can assume that the input of this function is a 2D double matrix. You should return a 2D complex double matrix. **Remember: Matlab has 1-based indexing.**

Run `script1.m` which will test your implementation. **Diff < 10^{-8} is OK.**



Check the console as well:

```
Runtime: 3.654 s
Sum of absoulte difference (in the frequency domain): 3.156e-10
Sum of absoulte difference (in the spatial domain):   7.165e-12
>>
```

Exercise 2

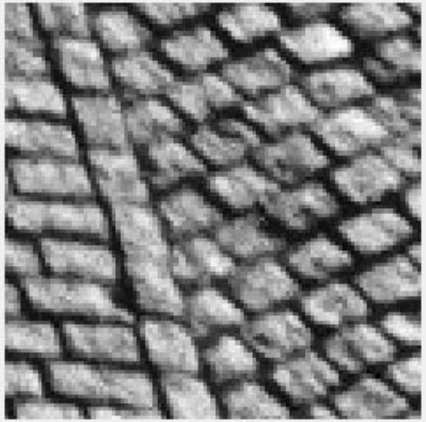
Implement the function `fourier_parts` in which:

- Shift the input F-space matrix to the center (`fftshift`).
- Compute the phase (`P`) of the F matrix (`angle`)
- Compute the magnitude (`M`) of the F matrix (`abs`)

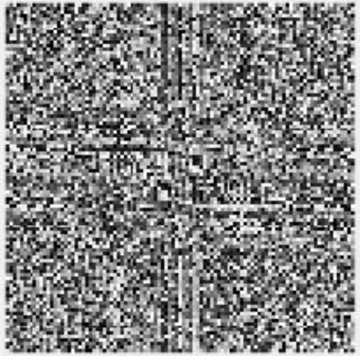
You should return two double matrices (they are NOT *complex* double)!
You can assume that the input is a 2D complex double type matrix.

Run `script2.m` to check your implementation and plot the DFT of an image.

Input image

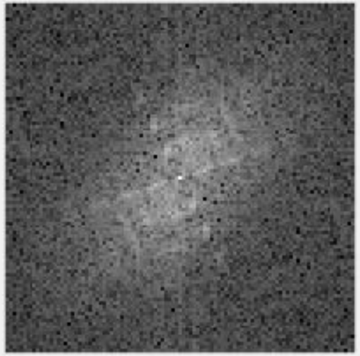


Phase



Magnitude

Log magnitude

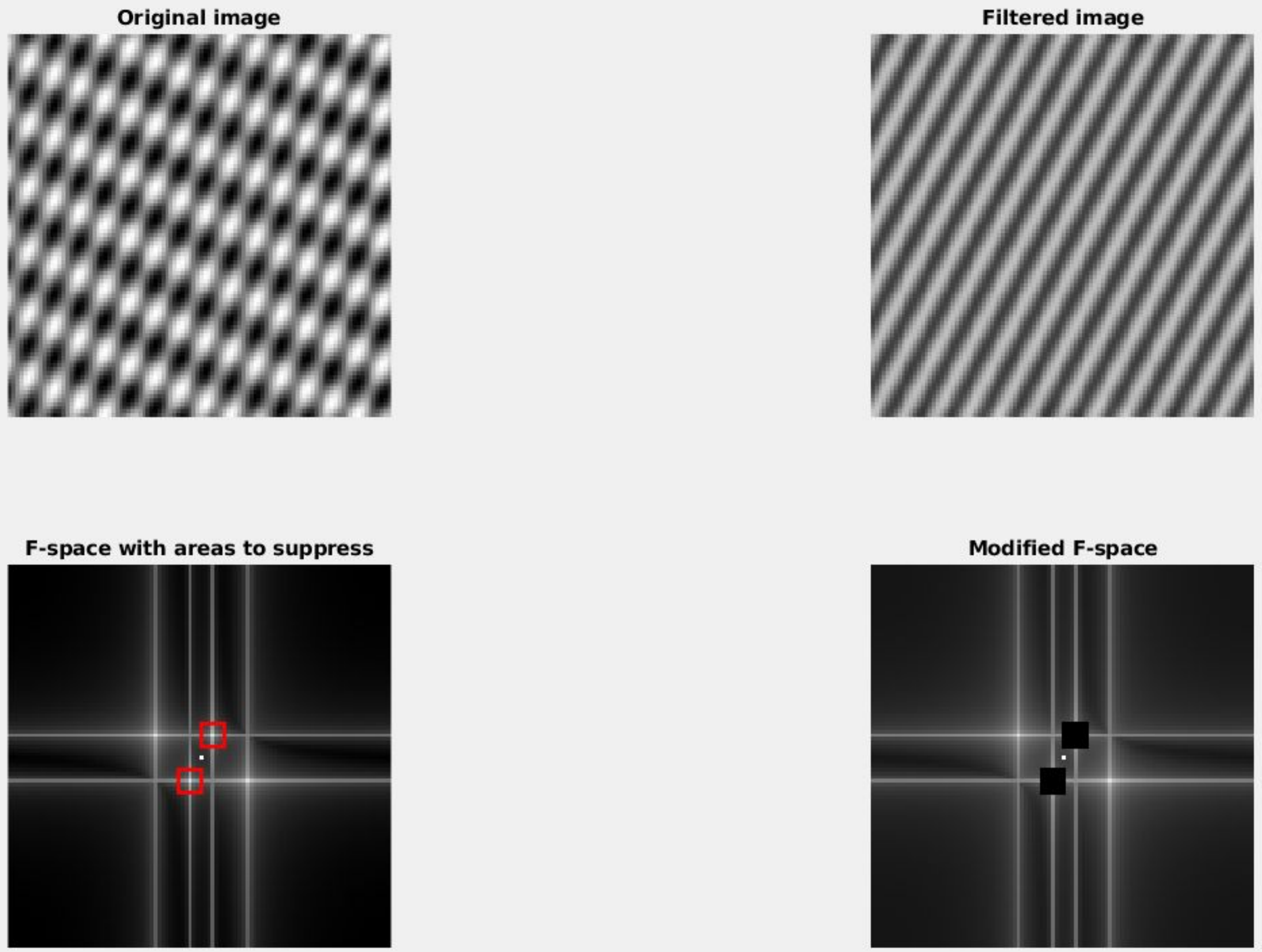


Exercise 3

Implement the function `mask_fourier` in which we replace the neighborhoods around the (x,y) points with complex zeros (frequency filtering).

- Shift the F space to the center (`fftshift`).
- Round the input coordinate vectors (`x` and `y`).
- For each point, set the `r`-radius neighborhood of the point to `0+0i`. This step is exactly the same as in the non maximum suppression function of the previous Lab.
- Undo the shift of the F space (`ifftshift`).

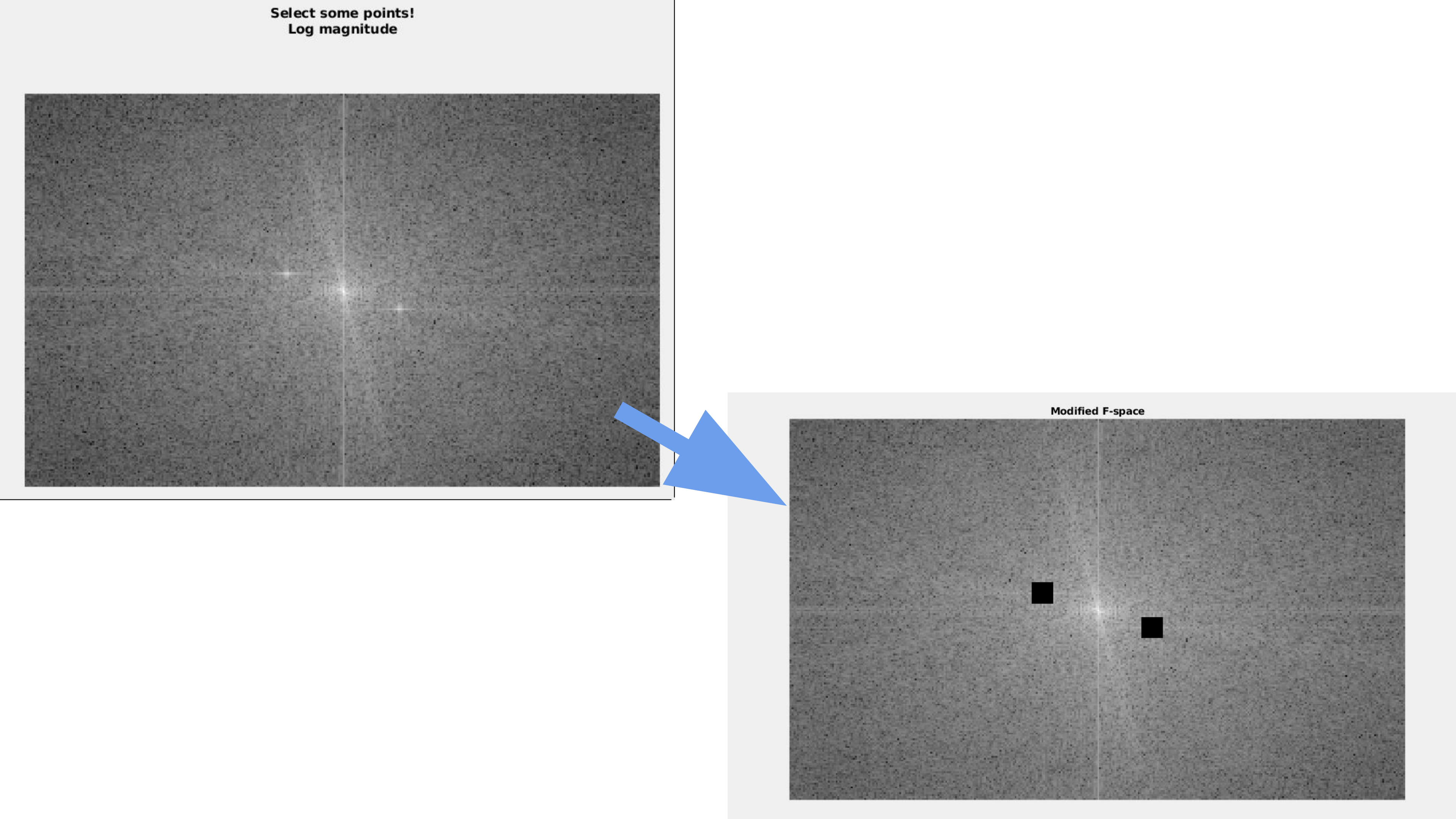
Run `script3.m` and examine the results.


Original image
Filtered image
F-space with areas to suppress
Modified F-space

Exercise 4

Read and understand the script `script4.m`. This script:

- Reads in an image and computes its DFT.
- Plots the log magnitude of the F-space.
- Using `ginput()` it asks the user to select some points on the magnitude plot. If the user is done with selecting points, *Return* (ENTER) key is pressed.
- The selected coordinates (float type) and the F-space is passed to the `mask_fourier` function together with a pre-defined radius value.
- The `mask_fourier` function should set the values in the neighborhood of the selected pixels to complex zero (`0 + 0i`).
- The new F-space (returned by the masking function) is transformed back to the spatial domain and the image is shown to allow visual comparison.


Select some points!
Log magnitude
Modified F-space

Original image

IFFT of the modified F-space

THE END