Table 1: Rule Forms and Equivalences

rule number			boolean expression	dep	equivalent rules			min
dec	binary	hex			conj	refl	c.r.	
0	00000000	00	0	–––	255	0	255	0
1	00000001	01	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)$	●●●	127	1	127	1
2	00000010	02	$(\bar{a}_{-1}\bar{a}_0 a_1)$	●●●	191	16	247	2
3	00000011	03	$(\bar{a}_{-1}\bar{a}_0)$	●●–	63	17	119	3
4	00000100	04	$(\bar{a}_{-1}a_0\bar{a}_1)$	●●●	223	4	223	4
5	00000101	05	$(\bar{a}_{-1}\bar{a}_1)$	●–●	95	5	95	5
6	00000110	06	$(\bar{a}_{-1}a_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0 a_1)$	●●●	159	20	215	6
7	00000111	07	$(\bar{a}_{-1}\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)$	●●●	31	21	87	7
8	00001000	08	$(\bar{a}_{-1}a_0 a_1)$	●●●	239	64	253	8
9	00001001	09	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0 a_1)$	●●●	111	65	125	9
10	00001010	0a	$(\bar{a}_{-1}a_1)$	●–●	175	80	245	10
11	00001011	0b	$(\bar{a}_{-1}\bar{a}_0)+(\bar{a}_{-1}a_1)$	●●●	47	81	117	11
12	00001100	0c	$(\bar{a}_{-1}a_0)$	●●–	207	68	221	12
13	00001101	0d	$(\bar{a}_{-1}\bar{a}_1)+(\bar{a}_{-1}a_0)$	●●●	79	69	93	13
14	00001110	0e	$(\bar{a}_{-1}a_0)+(\bar{a}_{-1}a_1)$	●●●	143	84	213	14
15	00001111	0f	$(\bar{a}_{-1})$	○––	15	85	85	15
16	00010000	10	$(a_{-1}\bar{a}_0\bar{a}_1)$	●●●	247	2	191	2
17	00010001	11	$(\bar{a}_0\bar{a}_1)$	–●●	119	3	63	3
18	00010010	12	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0 a_1)$	●●●	183	18	183	18
19	00010011	13	$(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)$	●●●	55	19	55	19
20	00010100	14	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0\bar{a}_1)$	●●●	215	6	159	6
21	00010101	15	$(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_1)$	●●●	87	7	31	7
22	00010110	16	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0 a_1)$	●●●	151	22	151	22
23	00010111	17	$(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)$	●●●	23	23	23	23
24	00011000	18	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0 a_1)$	●●●	231	66	189	24
25	00011001	19	$(\bar{a}_{-1}a_0 a_1)+(\bar{a}_0\bar{a}_1)$	●●●	103	67	61	25
26	00011010	1a	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_1)$	●●●	167	82	181	26
27	00011011	1b	$(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_1)$	●●●	39	83	53	27
28	00011100	1c	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0)$	●●●	199	70	157	28
29	00011101	1d	$(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0)$	●●●	71	71	29	29
30	00011110	1e	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0)+(\bar{a}_{-1}a_1)$	○●●	135	86	149	30
31	00011111	1f	$(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1})$	●●●	7	87	21	7
32	00100000	20	$(a_{-1}\bar{a}_0 a_1)$	●●●	251	32	251	32
33	00100001	21	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}\bar{a}_0 a_1)$	●●●	123	33	123	33
34	00100010	22	$(\bar{a}_0 a_1)$	–●●	187	48	243	34
35	00100011	23	$(\bar{a}_{-1}\bar{a}_0)+(\bar{a}_0 a_1)$	●●●	59	49	115	35
36	00100100	24	$(\bar{a}_{-1}a_0\bar{a}_1)+(a_{-1}\bar{a}_0 a_1)$	●●●	219	36	219	36
37	00100101	25	$(a_{-1}\bar{a}_0 a_1)+(\bar{a}_{-1}\bar{a}_1)$	●●●	91	37	91	37
38	00100110	26	$(\bar{a}_{-1}a_0\bar{a}_1)+(\bar{a}_0 a_1)$	●●●	155	52	211	38
39	00100111	27	$(\bar{a}_{-1}\bar{a}_1)+(\bar{a}_0 a_1)$	●●●	27	53	83	27
40	00101000	28	$(a_{-1}\bar{a}_0 a_1)+(\bar{a}_{-1}a_0 a_1)$	●●●	235	96	249	40
41	00101001	29	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}\bar{a}_0 a_1)+(\bar{a}_{-1}a_0 a_1)$	●●●	107	97	121	41
42	00101010	2a	$(\bar{a}_0 a_1)+(\bar{a}_{-1}a_1)$	●●●	171	112	241	42
43	00101011	2b	$(\bar{a}_{-1}\bar{a}_0)+(\bar{a}_0 a_1)+(\bar{a}_{-1}a_1)$	●●●	43	113	113	43
44	00101100	2c	$(a_{-1}\bar{a}_0 a_1)+(\bar{a}_{-1}a_0)$	●●●	203	100	217	44
45	00101101	2d	$(a_{-1}\bar{a}_0 a_1)+(\bar{a}_{-1}\bar{a}_1)+(\bar{a}_{-1}a_0)$	○●●	75	101	89	45
46	00101110	2e	$(\bar{a}_{-1}a_0)+(\bar{a}_0 a_1)$	●●●	139	116	209	46

rule number			boolean expression	dep	equivalent rules			min
dec	binary	hex			conj	refl	c.r.	
47	00101111	2f	$(\bar{a}_0 a_1) + (\bar{a}_{-1})$	●●●	11	117	81	11
48	00110000	30	$(a_{-1}\bar{a}_0)$	●●–	243	34	187	34
49	00110001	31	$(\bar{a}_0\bar{a}_1) + (a_{-1}\bar{a}_0)$	●●●	115	35	59	35
50	00110010	32	$(a_{-1}\bar{a}_0) + (\bar{a}_0 a_1)$	●●●	179	50	179	50
51	00110011	33	$(\bar{a}_0)$	–○–	51	51	51	51
52	00110100	34	$(\bar{a}_{-1}a_0\bar{a}_1) + (a_{-1}\bar{a}_0)$	●●●	211	38	155	38
53	00110101	35	$(\bar{a}_{-1}\bar{a}_1) + (a_{-1}\bar{a}_0)$	●●●	83	39	27	27
54	00110110	36	$(\bar{a}_{-1}a_0\bar{a}_1) + (a_{-1}\bar{a}_0) + (\bar{a}_0 a_1)$	●○●	147	54	147	54
55	00110111	37	$(\bar{a}_{-1}\bar{a}_1) + (\bar{a}_0)$	●●●	19	55	19	19
56	00111000	38	$(\bar{a}_{-1}a_0 a_1) + (a_{-1}\bar{a}_0)$	●●●	227	98	185	56
57	00111001	39	$(\bar{a}_{-1}a_0 a_1) + (\bar{a}_0\bar{a}_1) + (a_{-1}\bar{a}_0)$	●○●	99	99	57	57
58	00111010	3a	$(a_{-1}\bar{a}_0) + (\bar{a}_{-1}a_1)$	●●●	163	114	177	58
59	00111011	3b	$(\bar{a}_{-1}a_1) + (\bar{a}_0)$	●●●	35	115	49	35
60	00111100	3c	$(a_{-1}\bar{a}_0) + (\bar{a}_{-1}a_0)$	○○–	195	102	153	60
61	00111101	3d	$(\bar{a}_{-1}\bar{a}_1) + (a_{-1}\bar{a}_0) + (\bar{a}_{-1}a_0)$	●●●	67	103	25	25
62	00111110	3e	$(\bar{a}_{-1}a_1) + (a_{-1}\bar{a}_0) + (\bar{a}_{-1}a_0)$	●●●	131	118	145	62
63	00111111	3f	$(\bar{a}_0) + (\bar{a}_{-1})$	●●–	3	119	17	3
64	01000000	40	$(a_{-1}a_0\bar{a}_1)$	●●●	253	8	239	8
65	01000001	41	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1) + (a_{-1}a_0\bar{a}_1)$	●●●	125	9	111	9
66	01000010	42	$(a_{-1}a_0\bar{a}_1) + (\bar{a}_{-1}\bar{a}_0 a_1)$	●●●	189	24	231	24
67	01000011	43	$(a_{-1}a_0\bar{a}_1) + (\bar{a}_{-1}\bar{a}_0)$	●●●	61	25	103	25
68	01000100	44	$(a_0\bar{a}_1)$	–●●	221	12	207	12
69	01000101	45	$(\bar{a}_{-1}\bar{a}_1) + (a_0\bar{a}_1)$	●●●	93	13	79	13
70	01000110	46	$(\bar{a}_{-1}\bar{a}_0 a_1) + (a_0\bar{a}_1)$	●●●	157	28	199	28
71	01000111	47	$(a_0\bar{a}_1) + (\bar{a}_{-1}\bar{a}_0)$	●●●	29	29	71	29
72	01001000	48	$(a_{-1}a_0\bar{a}_1) + (\bar{a}_{-1}a_0 a_1)$	●●●	237	72	237	72
73	01001001	49	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1) + (a_{-1}a_0\bar{a}_1) + (\bar{a}_{-1}a_0 a_1)$	●●●	109	73	109	73
74	01001010	4a	$(a_{-1}a_0\bar{a}_1) + (\bar{a}_{-1}a_1)$	●●●	173	88	229	74
75	01001011	4b	$(a_{-1}a_0\bar{a}_1) + (\bar{a}_{-1}\bar{a}_0) + (\bar{a}_{-1}a_1)$	○●●	45	89	101	45
76	01001100	4c	$(a_0\bar{a}_1) + (\bar{a}_{-1}a_0)$	●●●	205	76	205	76
77	01001101	4d	$(\bar{a}_{-1}\bar{a}_1) + (a_0\bar{a}_1) + (\bar{a}_{-1}a_0)$	●●●	77	77	77	77
78	01001110	4e	$(a_0\bar{a}_1) + (\bar{a}_{-1}a_1)$	●●●	141	92	197	78
79	01001111	4f	$(a_0\bar{a}_1) + (\bar{a}_{-1})$	●●●	13	93	69	13
80	01010000	50	$(a_{-1}\bar{a}_1)$	●–●	245	10	175	10
81	01010001	51	$(\bar{a}_0\bar{a}_1) + (a_{-1}\bar{a}_1)$	●●●	117	11	47	11
82	01010010	52	$(\bar{a}_{-1}\bar{a}_0 a_1) + (a_{-1}\bar{a}_1)$	●●●	181	26	167	26
83	01010011	53	$(a_{-1}\bar{a}_1) + (\bar{a}_{-1}\bar{a}_0)$	●●●	53	27	39	27
84	01010100	54	$(a_{-1}\bar{a}_1) + (a_0\bar{a}_1)$	●●●	213	14	143	14
85	01010101	55	$(\bar{a}_1)$	––○	85	15	15	15
86	01010110	56	$(\bar{a}_{-1}\bar{a}_0 a_1) + (a_{-1}\bar{a}_1) + (a_0\bar{a}_1)$	●●○	149	30	135	30
87	01010111	57	$(\bar{a}_{-1}\bar{a}_0) + (\bar{a}_1)$	●●●	21	31	7	7
88	01011000	58	$(\bar{a}_{-1}a_0 a_1) + (a_{-1}\bar{a}_1)$	●●●	229	74	173	74
89	01011001	59	$(\bar{a}_{-1}a_0 a_1) + (\bar{a}_0\bar{a}_1) + (a_{-1}\bar{a}_1)$	●●○	101	75	45	45
90	01011010	5a	$(a_{-1}\bar{a}_1) + (\bar{a}_{-1}a_1)$	○–○	165	90	165	90
91	01011011	5b	$(\bar{a}_{-1}\bar{a}_0) + (a_{-1}\bar{a}_1) + (\bar{a}_{-1}a_1)$	●●●	37	91	37	37
92	01011100	5c	$(a_{-1}\bar{a}_1) + (\bar{a}_{-1}a_0)$	●●●	197	78	141	78
93	01011101	5d	$(\bar{a}_{-1}a_0) + (\bar{a}_1)$	●●●	69	79	13	13
94	01011110	5e	$(\bar{a}_{-1}a_0) + (a_{-1}\bar{a}_1) + (\bar{a}_{-1}a_1)$	●●●	133	94	133	94
95	01011111	5f	$(\bar{a}_1) + (\bar{a}_{-1})$	●–●	5	95	5	5

rule number			boolean expression	dep	equivalent rules			min
dec	binary	hex			conj	refl	c.r.	
96	01100000	60	$(a_{-1}a_0\bar{a}_1)+(a_{-1}\bar{a}_0a_1)$	●●●	249	40	235	40
97	01100001	61	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_0\bar{a}_1)+(a_{-1}\bar{a}_0a_1)$	●●●	121	41	107	41
98	01100010	62	$(a_{-1}a_0\bar{a}_1)+(\bar{a}_0a_1)$	●●●	185	56	227	56
99	01100011	63	$(a_{-1}a_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)+(\bar{a}_0a_1)$	●○●	57	57	99	57
100	01100100	64	$(a_{-1}\bar{a}_0a_1)+(a_0\bar{a}_1)$	●●●	217	44	203	44
101	01100101	65	$(a_{-1}\bar{a}_0a_1)+(\bar{a}_{-1}\bar{a}_1)+(a_0\bar{a}_1)$	●●○	89	45	75	45
102	01100110	66	$(a_0\bar{a}_1)+(\bar{a}_0a_1)$	−○○	153	60	195	60
103	01100111	67	$(\bar{a}_{-1}\bar{a}_0)+(a_0\bar{a}_1)+(\bar{a}_0a_1)$	●●●	25	61	67	25
104	01101000	68	$(a_{-1}a_0\bar{a}_1)+(a_{-1}\bar{a}_0a_1)+(\bar{a}_{-1}a_0a_1)$	●●●	233	104	233	104
105	01101001	69	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_0\bar{a}_1)+(a_{-1}\bar{a}_0a_1)+(\bar{a}_{-1}a_0a_1)$	○○○	105	105	105	105
106	01101010	6a	$(a_{-1}a_0\bar{a}_1)+(\bar{a}_0a_1)+(\bar{a}_{-1}a_1)$	●●○	169	120	225	106
107	01101011	6b	$(a_{-1}a_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)+(\bar{a}_0a_1)+(\bar{a}_{-1}a_1)$	●●●	41	121	97	41
108	01101100	6c	$(a_{-1}\bar{a}_0a_1)+(a_0\bar{a}_1)+(\bar{a}_{-1}a_0)$	●○●	201	108	201	108
109	01101101	6d	$(a_{-1}\bar{a}_0a_1)+(\bar{a}_{-1}\bar{a}_1)+(a_0\bar{a}_1)+(\bar{a}_{-1}a_0)$	●●●	73	109	73	73
110	01101110	6e	$(\bar{a}_{-1}a_0)+(a_0\bar{a}_1)+(\bar{a}_0a_1)$	●●●	137	124	193	110
111	01101111	6f	$(a_0\bar{a}_1)+(\bar{a}_0a_1)+(\bar{a}_{-1})$	●●●	9	125	65	9
112	01110000	70	$(a_{-1}\bar{a}_1)+(a_{-1}\bar{a}_0)$	●●●	241	42	171	42
113	01110001	71	$(\bar{a}_0\bar{a}_1)+(a_{-1}\bar{a}_1)+(a_{-1}\bar{a}_0)$	●●●	113	43	43	43
114	01110010	72	$(a_{-1}\bar{a}_1)+(\bar{a}_0a_1)$	●●●	177	58	163	58
115	01110011	73	$(a_{-1}\bar{a}_1)+(\bar{a}_0)$	●●●	49	59	35	35
116	01110100	74	$(a_0\bar{a}_1)+(a_{-1}\bar{a}_0)$	●●●	209	46	139	46
117	01110101	75	$(a_{-1}\bar{a}_0)+(\bar{a}_1)$	●●●	81	47	11	11
118	01110110	76	$(a_{-1}\bar{a}_0)+(a_0\bar{a}_1)+(\bar{a}_0a_1)$	●●●	145	62	131	62
119	01110111	77	$(\bar{a}_1)+(\bar{a}_0)$	−●●	17	63	3	3
120	01111000	78	$(\bar{a}_{-1}a_0a_1)+(a_{-1}\bar{a}_1)+(a_{-1}\bar{a}_0)$	○●●	225	106	169	106
121	01111001	79	$(\bar{a}_{-1}a_0a_1)+(\bar{a}_0\bar{a}_1)+(a_{-1}\bar{a}_1)+(a_{-1}\bar{a}_0)$	●●●	97	107	41	41
122	01111010	7a	$(a_{-1}\bar{a}_0)+(a_{-1}\bar{a}_1)+(\bar{a}_{-1}a_1)$	●●●	161	122	161	122
123	01111011	7b	$(a_{-1}\bar{a}_1)+(\bar{a}_{-1}a_1)+(\bar{a}_0)$	●●●	33	123	33	33
124	01111100	7c	$(a_{-1}\bar{a}_1)+(a_{-1}\bar{a}_0)+(\bar{a}_{-1}a_0)$	●●●	193	110	137	110
125	01111101	7d	$(a_{-1}\bar{a}_0)+(\bar{a}_{-1}a_0)+(\bar{a}_1)$	●●●	65	111	9	9
126	01111110	7e	$(a_{-1}\bar{a}_1)+(\bar{a}_0a_1)+(\bar{a}_{-1}a_0)$	●●●	129	126	129	126
127	01111111	7f	$(\bar{a}_1)+(\bar{a}_0)+(\bar{a}_{-1})$	●●●	1	127	1	1
128	10000000	80	$(a_{-1}a_0a_1)$	●●●	254	128	254	128
129	10000001	81	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_0a_1)$	●●●	126	129	126	126
130	10000010	82	$(\bar{a}_{-1}\bar{a}_0a_1)+(a_{-1}a_0a_1)$	●●●	190	144	246	130
131	10000011	83	$(a_{-1}a_0a_1)+(\bar{a}_{-1}\bar{a}_0)$	●●●	62	145	118	62
132	10000100	84	$(\bar{a}_{-1}a_0\bar{a}_1)+(a_{-1}a_0a_1)$	●●●	222	132	222	132
133	10000101	85	$(a_{-1}a_0a_1)+(\bar{a}_{-1}\bar{a}_1)$	●●●	94	133	94	94
134	10000110	86	$(\bar{a}_{-1}a_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0a_1)+(a_{-1}a_0a_1)$	●●●	158	148	214	134
135	10000111	87	$(a_{-1}a_0a_1)+(\bar{a}_{-1}\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)$	○●●	30	149	86	30
136	10001000	88	(a_0a_1)	−●●	238	192	252	136
137	10001001	89	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_0a_1)$	●●●	110	193	124	110
138	10001010	8a	$(\bar{a}_{-1}a_1)+(a_0a_1)$	●●●	174	208	244	138
139	10001011	8b	$(\bar{a}_{-1}\bar{a}_0)+(a_0a_1)$	●●●	46	209	116	46
140	10001100	8c	$(\bar{a}_{-1}a_0)+(a_0a_1)$	●●●	206	196	220	140
141	10001101	8d	$(\bar{a}_{-1}\bar{a}_1)+(a_0a_1)$	●●●	78	197	92	78
142	10001110	8e	$(\bar{a}_{-1}a_0)+(\bar{a}_{-1}a_1)+(a_0a_1)$	●●●	142	212	212	142
143	10001111	8f	$(a_0a_1)+(\bar{a}_{-1})$	●●●	14	213	84	14
144	10010000	90	$(a_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_0a_1)$	●●●	246	130	190	130
145	10010001	91	$(a_{-1}a_0a_1)+(\bar{a}_0\bar{a}_1)$	●●●	118	131	62	62

rule number			boolean expression	dep	equivalent rules			min
dec	binary	hex			conj	refl	c.r.	
146	10010010	92	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0a_1)+(a_{-1}a_0a_1)$	●●●	182	146	182	146
147	10010011	93	$(a_{-1}a_0a_1)+(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)$	●○●	54	147	54	54
148	10010100	94	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0\bar{a}_1)+(a_{-1}a_0a_1)$	●●●	214	134	158	134
149	10010101	95	$(a_{-1}a_0a_1)+(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_1)$	●●○	86	135	30	30
150	10010110	96	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0a_1)+(a_{-1}a_0a_1)$	○○○	150	150	150	150
151	10010111	97	$(a_{-1}a_0a_1)+(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)$	●●●	22	151	22	22
152	10011000	98	$(a_{-1}\bar{a}_0\bar{a}_1)+(a_0a_1)$	●●●	230	194	188	152
153	10011001	99	$(\bar{a}_0\bar{a}_1)+(a_0a_1)$	−○○	102	195	60	60
154	10011010	9a	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_1)+(a_0a_1)$	●●○	166	210	180	154
155	10011011	9b	$(\bar{a}_{-1}\bar{a}_0)+(\bar{a}_0\bar{a}_1)+(a_0a_1)$	●●●	38	211	52	38
156	10011100	9c	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0)+(a_0a_1)$	●○●	198	198	156	156
157	10011101	9d	$(\bar{a}_{-1}a_0)+(\bar{a}_0\bar{a}_1)+(a_0a_1)$	●●●	70	199	28	28
158	10011110	9e	$(a_{-1}\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_0)+(\bar{a}_{-1}a_1)+(a_0a_1)$	●●●	134	214	148	134
159	10011111	9f	$(\bar{a}_0\bar{a}_1)+(a_0a_1)+(\bar{a}_{-1})$	●●●	6	215	20	6
160	10100000	a0	$(a_{-1}a_1)$	●−●	250	160	250	160
161	10100001	a1	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_1)$	●●●	122	161	122	122
162	10100010	a2	$(\bar{a}_0a_1)+(a_{-1}a_1)$	●●●	186	176	242	162
163	10100011	a3	$(\bar{a}_{-1}\bar{a}_0)+(a_{-1}a_1)$	●●●	58	177	114	58
164	10100100	a4	$(\bar{a}_{-1}a_0\bar{a}_1)+(a_{-1}a_1)$	●●●	218	164	218	164
165	10100101	a5	$(\bar{a}_{-1}\bar{a}_1)+(a_{-1}a_1)$	○−○	90	165	90	90
166	10100110	a6	$(\bar{a}_{-1}a_0\bar{a}_1)+(\bar{a}_0a_1)+(a_{-1}a_1)$	●●○	154	180	210	154
167	10100111	a7	$(\bar{a}_{-1}\bar{a}_0)+(\bar{a}_{-1}\bar{a}_1)+(a_{-1}a_1)$	●●●	26	181	82	26
168	10101000	a8	$(a_{-1}a_1)+(a_0a_1)$	●●●	234	224	248	168
169	10101001	a9	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_1)+(a_0a_1)$	●●○	106	225	120	106
170	10101010	aa	(a_1)	−−○	170	240	240	170
171	10101011	ab	$(\bar{a}_{-1}\bar{a}_0)+(a_1)$	●●●	42	241	112	42
172	10101100	ac	$(\bar{a}_{-1}a_0)+(a_{-1}a_1)$	●●●	202	228	216	172
173	10101101	ad	$(\bar{a}_{-1}a_0)+(\bar{a}_{-1}\bar{a}_1)+(a_{-1}a_1)$	●●●	74	229	88	74
174	10101110	ae	$(\bar{a}_{-1}a_0)+(a_1)$	●●●	138	244	208	138
175	10101111	af	$(\bar{a}_{-1})+(a_1)$	●−●	10	245	80	10
176	10110000	b0	$(a_{-1}\bar{a}_0)+(a_{-1}a_1)$	●●●	242	162	186	162
177	10110001	b1	$(\bar{a}_0\bar{a}_1)+(a_{-1}a_1)$	●●●	114	163	58	58
178	10110010	b2	$(a_{-1}\bar{a}_0)+(\bar{a}_0a_1)+(a_{-1}a_1)$	●●●	178	178	178	178
179	10110011	b3	$(a_{-1}a_1)+(\bar{a}_0)$	●●●	50	179	50	50
180	10110100	b4	$(\bar{a}_{-1}a_0\bar{a}_1)+(a_{-1}\bar{a}_0)+(a_{-1}a_1)$	○●●	210	166	154	154
181	10110101	b5	$(a_{-1}\bar{a}_0)+(\bar{a}_{-1}\bar{a}_1)+(a_{-1}a_1)$	●●●	82	167	26	26
182	10110110	b6	$(\bar{a}_{-1}a_0\bar{a}_1)+(a_{-1}\bar{a}_0)+(\bar{a}_0a_1)+(a_{-1}a_1)$	●●●	146	182	146	146
183	10110111	b7	$(\bar{a}_{-1}\bar{a}_1)+(a_{-1}a_1)+(\bar{a}_0)$	●●●	18	183	18	18
184	10111000	b8	$(a_{-1}\bar{a}_0)+(a_0a_1)$	●●●	226	226	184	184
185	10111001	b9	$(a_{-1}\bar{a}_0)+(\bar{a}_0\bar{a}_1)+(a_0a_1)$	●●●	98	227	56	56
186	10111010	ba	$(a_{-1}\bar{a}_0)+(a_1)$	●●●	162	242	176	162
187	10111011	bb	$(\bar{a}_0)+(a_1)$	−●●	34	243	48	34
188	10111100	bc	$(a_{-1}a_1)+(a_{-1}\bar{a}_0)+(\bar{a}_{-1}a_0)$	●●●	194	230	152	152
189	10111101	bd	$(\bar{a}_0\bar{a}_1)+(a_{-1}a_1)+(\bar{a}_{-1}a_0)$	●●●	66	231	24	24
190	10111110	be	$(a_{-1}\bar{a}_0)+(\bar{a}_{-1}a_0)+(a_1)$	●●●	130	246	144	130
191	10111111	bf	$(\bar{a}_0)+(\bar{a}_{-1})+(a_1)$	●●●	2	247	16	2
192	11000000	c0	$(a_{-1}a_0)$	●●−	252	136	238	136
193	11000001	c1	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_0)$	●●●	124	137	110	110
194	11000010	c2	$(\bar{a}_{-1}\bar{a}_0a_1)+(a_{-1}a_0)$	●●●	188	152	230	152
195	11000011	c3	$(\bar{a}_{-1}\bar{a}_0)+(a_{-1}a_0)$	○○−	60	153	102	60

rule number			boolean expression	dep	equivalent rules			min
dec	binary	hex			conj	refl	c.r.	
196	11000100	c4	$(a_0\bar{a}_1)+(a_{-1}a_0)$	●●●	220	140	206	140
197	11000101	c5	$(\bar{a}_{-1}\bar{a}_1)+(a_{-1}a_0)$	●●●	92	141	78	78
198	11000110	c6	$(\bar{a}_{-1}\bar{a}_0a_1)+(a_0\bar{a}_1)+(a_{-1}a_0)$	●○●	156	156	198	156
199	11000111	c7	$(\bar{a}_{-1}\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)+(a_{-1}a_0)$	●●●	28	157	70	28
200	11001000	c8	$(a_{-1}a_0)+(a_0a_1)$	●●●	236	200	236	200
201	11001001	c9	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_0)+(a_0a_1)$	●○●	108	201	108	108
202	11001010	ca	$(a_{-1}a_0)+(\bar{a}_{-1}a_1)$	●●●	172	216	228	172
203	11001011	cb	$(\bar{a}_{-1}a_1)+(\bar{a}_{-1}\bar{a}_0)+(a_{-1}a_0)$	●●●	44	217	100	44
204	11001100	cc	(a_0)	−○−	204	204	204	204
205	11001101	cd	$(\bar{a}_{-1}\bar{a}_1)+(a_0)$	●●●	76	205	76	76
206	11001110	ce	$(\bar{a}_{-1}a_1)+(a_0)$	●●●	140	220	196	140
207	11001111	cf	$(\bar{a}_{-1})+(a_0)$	●●−	12	221	68	12
208	11010000	d0	$(a_{-1}\bar{a}_1)+(a_{-1}a_0)$	●●●	244	138	174	138
209	11010001	d1	$(\bar{a}_0\bar{a}_1)+(a_{-1}a_0)$	●●●	116	139	46	46
210	11010010	d2	$(\bar{a}_{-1}\bar{a}_0a_1)+(a_{-1}\bar{a}_1)+(a_{-1}a_0)$	○●●	180	154	166	154
211	11010011	d3	$(a_{-1}\bar{a}_1)+(\bar{a}_{-1}\bar{a}_0)+(a_{-1}a_0)$	●●●	52	155	38	38
212	11010100	d4	$(a_{-1}\bar{a}_1)+(a_0\bar{a}_1)+(a_{-1}a_0)$	●●●	212	142	142	142
213	11010101	d5	$(a_{-1}a_0)+(\bar{a}_1)$	●●●	84	143	14	14
214	11010110	d6	$(\bar{a}_{-1}\bar{a}_0a_1)+(a_{-1}\bar{a}_1)+(a_0\bar{a}_1)+(a_{-1}a_0)$	●●●	148	158	134	134
215	11010111	d7	$(\bar{a}_{-1}\bar{a}_0)+(a_{-1}a_0)+(\bar{a}_1)$	●●●	20	159	6	6
216	11011000	d8	$(a_{-1}\bar{a}_1)+(a_0a_1)$	●●●	228	202	172	172
217	11011001	d9	$(a_{-1}a_0)+(\bar{a}_0\bar{a}_1)+(a_0a_1)$	●●●	100	203	44	44
218	11011010	da	$(a_{-1}a_0)+(a_{-1}\bar{a}_1)+(\bar{a}_{-1}a_1)$	●●●	164	218	164	164
219	11011011	db	$(\bar{a}_0\bar{a}_1)+(\bar{a}_{-1}a_1)+(a_{-1}a_0)$	●●●	36	219	36	36
220	11011100	dc	$(a_{-1}\bar{a}_1)+(a_0)$	●●●	196	206	140	140
221	11011101	dd	$(\bar{a}_1)+(a_0)$	−●●	68	207	12	12
222	11011110	de	$(a_{-1}\bar{a}_1)+(\bar{a}_{-1}a_1)+(a_0)$	●●●	132	222	132	132
223	11011111	df	$(\bar{a}_1)+(\bar{a}_{-1})+(a_0)$	●●●	4	223	4	4
224	11100000	e0	$(a_{-1}a_0)+(a_{-1}a_1)$	●●●	248	168	234	168
225	11100001	e1	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_0)+(a_{-1}a_1)$	○●●	120	169	106	106
226	11100010	e2	$(a_{-1}a_0)+(\bar{a}_0a_1)$	●●●	184	184	226	184
227	11100011	e3	$(a_{-1}a_1)+(\bar{a}_{-1}\bar{a}_0)+(a_{-1}a_0)$	●●●	56	185	98	56
228	11100100	e4	$(a_0\bar{a}_1)+(a_{-1}a_1)$	●●●	216	172	202	172
229	11100101	e5	$(a_{-1}a_0)+(\bar{a}_{-1}\bar{a}_1)+(a_{-1}a_1)$	●●●	88	173	74	74
230	11100110	e6	$(a_{-1}a_0)+(a_0\bar{a}_1)+(\bar{a}_0a_1)$	●●●	152	188	194	152
231	11100111	e7	$(\bar{a}_{-1}\bar{a}_1)+(\bar{a}_0a_1)+(a_{-1}a_0)$	●●●	24	189	66	24
232	11101000	e8	$(a_{-1}a_0)+(a_{-1}a_1)+(a_0a_1)$	●●●	232	232	232	232
233	11101001	e9	$(\bar{a}_{-1}\bar{a}_0\bar{a}_1)+(a_{-1}a_0)+(a_{-1}a_1)+(a_0a_1)$	●●●	104	233	104	104
234	11101010	ea	$(a_{-1}a_0)+(a_1)$	●●●	168	248	224	168
235	11101011	eb	$(\bar{a}_{-1}\bar{a}_0)+(a_{-1}a_0)+(a_1)$	●●●	40	249	96	40
236	11101100	ec	$(a_{-1}a_1)+(a_0)$	●●●	200	236	200	200
237	11101101	ed	$(\bar{a}_{-1}\bar{a}_1)+(a_{-1}a_1)+(a_0)$	●●●	72	237	72	72
238	11101110	ee	$(a_0)+(a_1)$	−●●	136	252	192	136
239	11101111	ef	$(\bar{a}_{-1})+(a_0)+(a_1)$	●●●	8	253	64	8
240	11110000	f0	(a_{-1})	○−−	240	170	170	170
241	11110001	f1	$(\bar{a}_0\bar{a}_1)+(a_{-1})$	●●●	112	171	42	42
242	11110010	f2	$(\bar{a}_0a_1)+(a_{-1})$	●●●	176	186	162	162
243	11110011	f3	$(\bar{a}_0)+(a_{-1})$	●●−	48	187	34	34
244	11110100	f4	$(a_0\bar{a}_1)+(a_{-1})$	●●●	208	174	138	138

rule number			boolean expression	dep	equivalent rules			min
dec	binary	hex			conj	refl	c.r.	
245	11110101	f5	$(\bar{a}_1)+(a_{-1})$	•–•	80	175	10	10
246	11110110	f6	$(a_0\bar{a}_1)+(\bar{a}_0a_1)+(a_{-1})$	•••	144	190	130	130
247	11110111	f7	$(\bar{a}_1)+(\bar{a}_0)+(a_{-1})$	•••	16	191	2	2
248	11111000	f8	$(a_0a_1)+(a_{-1})$	•••	224	234	168	168
249	11111001	f9	$(\bar{a}_0\bar{a}_1)+(a_0a_1)+(a_{-1})$	•••	96	235	40	40
250	11111010	fa	$(a_{-1})+(a_1)$	•–•	160	250	160	160
251	11111011	fb	$(\bar{a}_0)+(a_{-1})+(a_1)$	•••	32	251	32	32
252	11111100	fc	$(a_{-1})+(a_0)$	••–	192	238	136	136
253	11111101	fd	$(\bar{a}_1)+(a_{-1})+(a_0)$	•••	64	239	8	8
254	11111110	fe	$(a_{-1})+(a_0)+(a_1)$	•••	128	254	128	128
255	11111111	ff	1	–––	0	255	0	0

Forms of rules and equivalences between rules.

The table lists all 256 possible rules for $k = 2$, $r = 1$ one-dimensional cellular automata. Such cellular automata consist of a line of sites, each with value 0 or 1. At each time step, the value a_i of a site at position i is updated according to the rule

$$a_i' = \phi(a_{i-1}, a_i, a_{i+1}).$$

This table lists the $2^{2^3} = 256$ possible choices of ϕ.

Each digit in the binary representation of the rule number gives the value of ϕ for a particular set of (a_{i-1}, a_i, a_{i+1}). The digit corresponding to the coefficient of 2^n in the rule number gives the value of $\phi(n_2, n_1, n_0)$, where $n = 4n_2 + 2n_1 + n_0$. Thus the leftmost digit in the binary representation of the rule number gives $\phi(1, 1, 1)$, the next gives $\phi(1, 1, 0)$, and so on, down to $\phi(0, 0, 0)$.

The table also gives the decimal and hexadecimal representations of the rule numbers.

Each ϕ can be considered a Boolean function of three variables, say a_{-1}, a_0 and a_1. The table gives the minimal disjunctive normal form representations for these Boolean functions. Boolean multiplication and addition are used (corresponding to AND and OR operations). Bar denotes complementation. In each case, the expression with the minimal number of components, using only these operations, is given.

The column labelled "dep" gives the dependence of $\phi(a_{-1}, a_0, a_1)$ on each of the a_{-1}, a_0 and a_1. The symbol – indicates no change in ϕ when the corresponding a_j is changed. The symbol ∘ denotes linear dependence of ϕ on the corresponding a_j: whenever a_j changes, ϕ also changes. The symbol • denotes arbitrary dependence of ϕ. Rules such as 90 in which only ∘ and – dependence occurs, are called additive, and can be represented as linear functions modulo two.

For each rule, the table gives rules equivalent under simple transformations. "conj" denotes conjugation: interchange of the roles of 0 and 1. "refl" denotes reflec-