

# Introduction to artificial intelligence – Midterm examination

- You have 100 minutes to answer the questions.
- Work on your own; any form of communication will result in immediate disqualification.
- To receive full score you have to justify your answers, unless indicated otherwise.
- Midterms with incomplete headers will not get evaluated.

Name: CSUTAK BALAZS

Neptun code: ABDTW 1

Sem. group: Wed 10:15pm ☐

Wed 12:15pm ☐

Fri 12:15pm ☒

Seat: F6

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	+1.	$\Sigma$
16	10	6	6	17	8	6	8	6	9	8	5	105
14	10	2	6	15	8	6	6	6	9	8	0	88

1. (16 pts) Circle the one answer that best answers the question. No justification is needed unless you find the question ambiguous.

a.) A search algorithm is *complete* if it

- ☒ A. always finds the optimal solution  
☒ B. always finds a solution if there is one  
☐ C. finds all possible solutions  
☐ D. never finds a solution

b.) Which of the following search algorithms is not optimal? (assuming that step costs are identical)

- ☒ A. depth-first  
☐ B. uniform cost  
☐ C. breadth-first  
☐ D. iterative deepening

c.) Which of the following search algorithms is *not* heuristic search?

- ☒ A. greedy search  
☐ B. iterative deepening (BFS)  
☐ C. A\*  
☐ D. beam search

d.) In which of the following search algorithms is it possible to confuse a local maximum with a global maximum?

- ☒ A. depth first  
☐ B. A\*  
☒ C. hill climbing  
☐ D. greedy search



e.) What is the difference between the cost function and heuristic function portions of the A\* evaluation function?

- A. A cost function returns the actual cost from current node to goal, while the heuristic function returns the estimated cost from current node to goal
- B. A cost function returns the estimated cost from start node to current node, while the heuristic function returns the estimated cost from current node to goal
- C. A cost function returns the estimated cost from current node to goal, while the heuristic function returns the actual cost from start node to current node
- ☒ D. A cost function returns the actual cost from start node to current node, while the heuristic function returns the estimated cost from current node to goal.

f.) An inference procedure

- A. is a declarative knowledge representation
- ☒ B. provides rules for deriving new facts from existing facts
- C. is a proof
- D. is a type of inheritance

g.) An inference method is *sound* if it

- A. can derive any sentence that is entailed
- ☒ B. only derives entailed sentences
- C. is efficient in both time and space
- D. is not NP-complete

h.) A zero-sum game is one in which

- ☒ A. the feature weights in an evaluation function must sum to zero
- B. the points in any one player's hand must sum to zero
- ☒ C. if one player wins, the other necessarily loses
- D. two players may team up to beat a third

Points of all players must sum to zero.

2. (10 pts) Decide whether the following statements are true or false and justify your answer. Points are only given for the justification.

- a.) Breadth first search is complete if the state space has infinite depth but finite branching factor.

TRUE

If branching factor is finite, the search algorithm reaches any level in a finite number of steps, hence finds the goal (if there is one).



- b.) Assuming that a rook can move one square in any direction on a chessboard (4 directions in all), then Manhattan distance is an admissible heuristic for the problem of moving the rook from square A to square B. (Manhattan distance is the sum of the horizontal and vertical distances between its current position and the desired position in the goal configuration.) TRUE

As the rook can move just horizontally and vertically, manhattan distance is a lower bound for the steps needed. It does not overestimate  $\Rightarrow$  admissible

- c.) The sum of two admissible heuristics is admissible. FALSE

The sum of the two heuristic values can actually overestimate the cost.

- d.)  $\exists x P(x) \vee \exists x \neg P(x)$  is a valid sentence. TRUE

In two valued logic a sentence is either true or false. If  $\exists x : P(x)$ , that means  $P(x)$  is ~~invalid~~ <sup>unsatisfiable</sup>, so the  $\exists x \neg P(x)$  will be true.

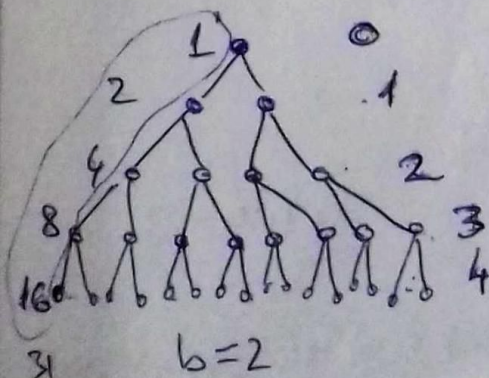
- e.) If  $A \models B$ , then A is true in all interpretations in which B is true. FALSE

$\models$  means every interpretation that is model of A, will be model of B as well. This is the inverse statement: (

3. (6 pts) Given two admissible heuristics,  $h_1$  and  $h_2$ , for a specific problem application with respective costs  $c_1$  and  $c_2$  every time you query the heuristic value of a node. Propose a method to decide which heuristic to use in an A\* search for the entire class of problems.

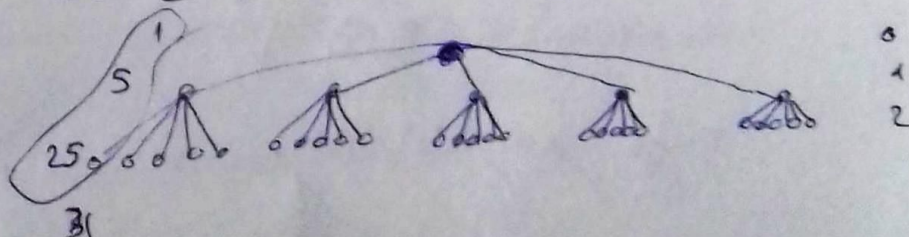
The one that dominates the other (aka. has higher values for the nodes). As both of them are admissible, higher value ( $\max(c_1, c_2)$ ) will be closer to the actual value

4. (6 pts) What is the effective branching rate of the search that gives a solution in  $N=31$  steps at depth  $d=4$ ? What is the effective branching rate if the solution is found at depth  $d=2$ ?



$$d=4 \Rightarrow 1 + b + b^2 + b^3 + b^4 = 31 \Rightarrow b = 2$$

$$d=2 \Rightarrow 1 + b + b^2 = 31 \Rightarrow b = 5$$





For each of the following strategies

- 

Cost of path found:  $1 + 5 + 2 + 4 + 2 = 14$

Cost of path found:  $8+4 = 12$

Cost of path found:  $4 + 1 + 4 = 9$ .

~~Ex 10/11:  $f(x) = p(x)/q(x)$ ,  $f'(x) = \frac{p'(x)q(x) - p(x)q'(x)}{q(x)^2}$ ,  $f''(x) = \frac{p''(x)q(x)^2 - 2p'(x)q'(x)q(x) + p(x)q''(x)}{q(x)^3}$ ,  $f'''(x) = \frac{p'''(x)q(x)^3 - 3p''(x)q'(x)q(x)^2 + 3p'(x)q''(x)q(x) - p(x)q'''(x)}{q(x)^4}$~~

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{40}, x_{41}, x_{42}, x_{43}, x_{44}, x_{45}, x_{46}, x_{47}, x_{48}, x_{49}, x_{50}, x_{51}, x_{52}, x_{53}, x_{54}, x_{55}, x_{56}, x_{57}, x_{58}, x_{59}, x_{60}, x_{61}, x_{62}, x_{63}, x_{64}, x_{65}, x_{66}, x_{67}, x_{68}, x_{69}, x_{70}, x_{71}, x_{72}, x_{73}, x_{74}, x_{75}, x_{76}, x_{77}, x_{78}, x_{79}, x_{80}, x_{81}, x_{82}, x_{83}, x_{84}, x_{85}, x_{86}, x_{87}, x_{88}, x_{89}, x_{90}, x_{91}, x_{92}, x_{93}, x_{94}, x_{95}, x_{96}, x_{97}, x_{98}, x_{99}, x_{100}$$



Fringe: ~~S~~, ~~A~~, ~~B~~, ~~C~~, ~~E~~, ~~F~~, ~~D~~, ~~H~~, ~~G~~, ~~I~~

d.) (2.5 pts) greedy best first search with the given heuristic

Order of node expansion: S C A D H  $\exists$  G<sub>2</sub>

2.5

First path found: S A D H  $\exists$  G<sub>2</sub>

Cost of path found:  $1 + 5 + 2 + 4 + 2 =$

$= 14$

e.) (3.5 pts) A\* search with the given heuristic

Order of node expansion: S, A, C, D, E, H, B, G<sub>1</sub>

First path found: S C B G<sub>1</sub>

Cost of path found:  $4 + 1 + 4 = 9$

Fringe: S, ~~A~~, ~~B~~, ~~C~~, ~~D~~, ~~E~~, ~~F~~, ~~G~~, ~~H~~, ~~I~~, ~~J~~, ~~K~~, ~~L~~, ~~M~~, ~~N~~, ~~O~~, ~~P~~, ~~Q~~, ~~R~~, ~~S~~, ~~T~~, ~~U~~, ~~V~~, ~~W~~, ~~X~~, ~~Y~~, ~~Z~~

f.) (3.5 pts) RBFS search with the given heuristic

Order of node expansion: S

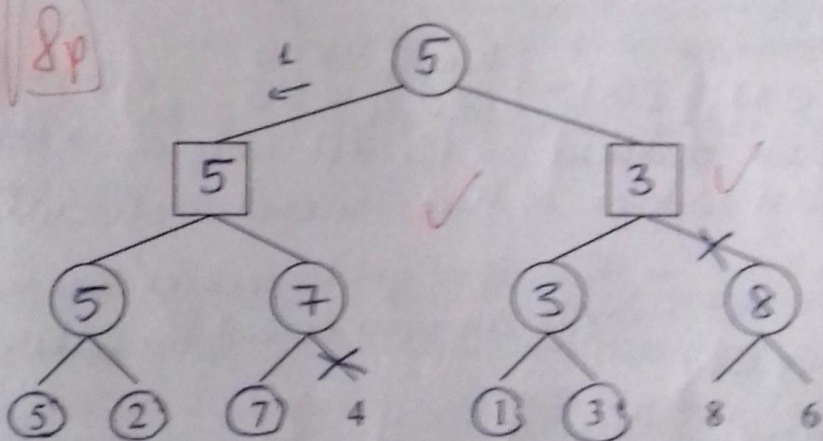
First path found:

Cost of path found:

Fringe: A

6. (8 pts) Let us consider the zero sum game represented by the following tree.

MAX



min

MAX

a.) (3 pts) Supposing that both players play optimally, perform minimax search on the tree by writing game theoretic values into the nodes. What is the next optimal step? What is the game theoretic value of the game? Optimal step is to the left (arrow 1). Game theoretic value is 5.

b.) (5 pts) Repeat the search using alpha-beta pruning from left to right. Circle each leaf node that will be examined. Do not circle pruned nodes, instead mark pruned subtrees by writing an 'X' on the edge above them. What is the game theoretic value of the game in this case? The pruning does not influence the game theoretic value, so it's five.



7. (6 pts) Convert the following sentences to first order predicate logic form using the given predicates and functions.

- a.) (3p) There is a barber in the city who shaves every man in the city who does not shave himself. (BARBER(x), CITIZEN(x), SHAVES(x,y))

$$\exists x \text{ BARBER}(x) \wedge \text{CITIZEN}(x) \wedge (\forall y \text{ CITIZEN}(y) \wedge \neg \text{SHAVES}(y,y) \rightarrow \text{SHAVES}(x,y))$$

- b.) (3pts) A prime number is a natural number greater than 1 that cannot be formed by multiplying two smaller natural numbers. (PRIME(x), NATURAL(x), LESSTHAN(x,y) or  $x < y$ , PRODUCT(x,y,z) or  $z = x * y$ )

$$\forall x \text{ PRIME}(x) \leftrightarrow [\text{NATURAL}(x) \wedge \text{LESSTHAN}(1,x) \wedge \neg \exists y \text{ LESSTHAN}(y,x) \wedge \text{PRODUCT}(y,x,z)]$$

8. (8 pts) Give a domain and an interpretation that makes the first logic sentence true but the second one false, or prove if this is impossible.

a.)  $\forall x (f(x) \rightarrow h(x)) \wedge \forall x (g(x) \rightarrow h(x))$

$\exists x (f(x) \wedge g(x))$

$x \in \mathbb{N}, f(x) = \begin{cases} \text{true, if } x < 0 \\ \text{false, if } x \geq 0 \end{cases}, g(x) = f(x)$

As false statement on the left side of  $\rightarrow$  leads to true result, first sentence is true. Second is obviously false.

- b.)  $\forall x \exists y f(x,y)$  - there is a greater number than  $x$  (for all  $x$ )  $\rightarrow$  T  
 $\exists y \forall x f(x,y)$  - exists an upper bound for natural numbers  $\rightarrow$  false  
 $x, y \in \mathbb{N}$

$f(x,y) : \text{true, iff } x < y$

c.)  $\forall x (f(x) \rightarrow g(A))$   $\forall x \neg f(x) \vee g(A)$

$(\forall x (f(x)) \rightarrow g(A))$

$\neg \forall x f(x) \vee g(A) \leftrightarrow \exists x \neg f(x) \vee g(A)$

~~There is no~~

As first sentence is more general, and implies the second one, there is no solution for the problem.

d.)  $\exists x (f(x) \rightarrow g(A))$   $\exists x \neg f(x) \vee g(A)$

$(\exists x (f(x)) \rightarrow g(A))$

$(\neg \exists x f(x)) \vee g(A) \leftrightarrow \forall x \neg f(x) \vee g(A)$

$g(A)$  - false

$f$  : true, iff  $x$  is prime

$x \in \mathbb{N}$ .

(1) there are non-prime numbers  $\rightarrow$  true

(2) for a prime number there exist prime numbers, but right side is



9. (6 pts) For each pair of literals below, specify a most general unifier, or indicate if they are not unifiable.

a)  $k(x, b(x, y), b(y, b(x, y)))$  and  $k(x, w, b(w, z))$  same arity ✓

First arg: OK.

Second argument:  $w \rightarrow b(x, y)$ , result  $k(x, b(x, y), b(y, b(x, y)))$

Third: as  $y \rightarrow b(x, y)$  is not possible, there are not unifiable.

b)  $k(a, x, g(b(y, A)))$  and  $k(z, g(z), g(w))$

$z \rightarrow a$ :  $k(a, x, g(b(y, A)))$   $k(a, g(a), g(w))$

$x \rightarrow g(a)$ :  $k(a, g(a), g(b(y, A)))$   $k(a, g(a), g(w))$

$w \rightarrow b(y, a)$ :  $k(a, g(a), g(b(y, A)))$   $k(a, g(a), g(b(y, A)))$  ✓

10. (9 pts) Consider the following text message:

'Hi Sean, can we meet an hour later? I have just received another folder to finish today. It happens every time I have a dinner appointment.'

a) (3pts) Formalize the premises and the conclusions of the entailment using logical expressions. Which inference scheme was used?

Premise: **I** have a dinner appointment. ✓

Conclusion: I receive another folder to finish that day.

Name of inference scheme: induction ✓

b) (3pts) Explain how this scheme might result in logically unsound sentences.

Something being true multiple times in the past does not mean it is a general truth. If his boss happens to be in good mood, he might not get extra work and doesn't have to cancel dinner.

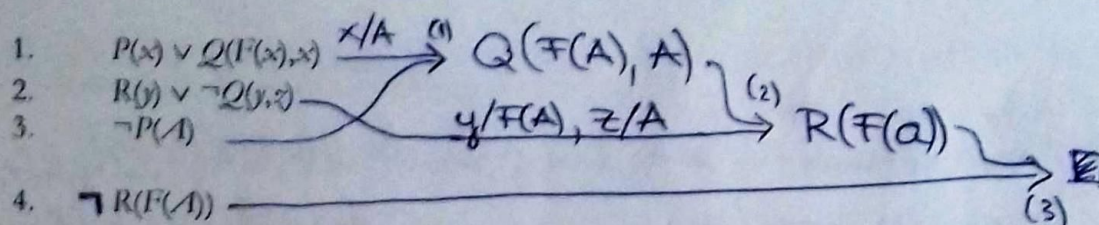
c) (3pts) Why is this inference scheme still useful and hence used by intelligent agents?

Because - taking in consideration probability - a statement being true multiple times can be a base for a conclusion being true in most of the cases.

This is not a real logical consequence, but can solve problems certain times. ✓



11. (8 pts) Use resolution refutation to prove sentence 4 from sentences 1-3. Number your steps and give the number of the lines used for resolution in each step.



Step 1:  $\left. \begin{array}{l} \text{statement 1 } (x/A) \\ \text{statement 3} \end{array} \right\} \rightarrow Q(F(A), A)$

Step 2:  $\left. \begin{array}{l} \text{result of step 1} \\ \text{statement 2} \end{array} \right\} \rightarrow R(F(A))$

Step 3:  $\left. \begin{array}{l} \text{negated statement 4} \\ R(F(A)) \end{array} \right\} \rightarrow \text{empty clause.}$

+1. (5 pts) Give the total number of possible interpretations in a binary algebra that has one unary and a commutative binary operator.