Fuzzy logic

Artificial intelligence

Kristóf Karacs
PPKE-ITK

Fuzzy logic

- Lotfi Zadeh
- Concept: truth values may apply partially
- Not able to express uncertainty (rather truthfulness)
- Logical statements are derived from natural language statements

Fuzzy sets

- Sets are identified by linguistic identifiers
  - "tall", "young", "bigger"
- Grade of membership

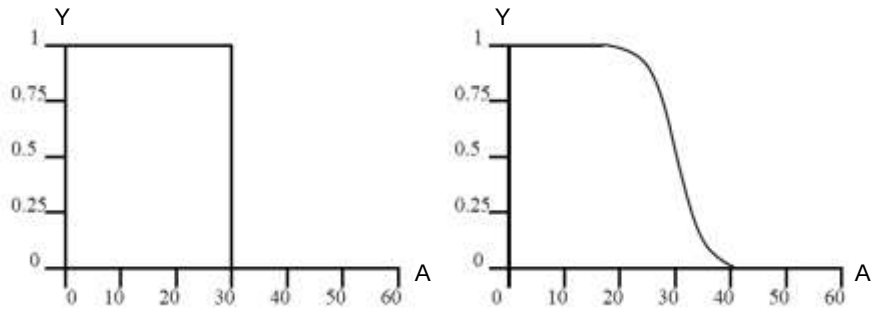
$$\mu(x) \quad , \quad x \in U \quad , \quad 0 \leq \mu(x) \leq 1$$

- Fuzzy set A

$$A = \{\langle x, \mu(x)\rangle\} = \{\langle x, \mu(x_1)\rangle, \langle x, \mu(x_2)\rangle, \ldots, \langle x, \mu(x_n)\rangle\}$$

Membership function

- Age – "young"



Crisp Fuzzy

Types of membership functions

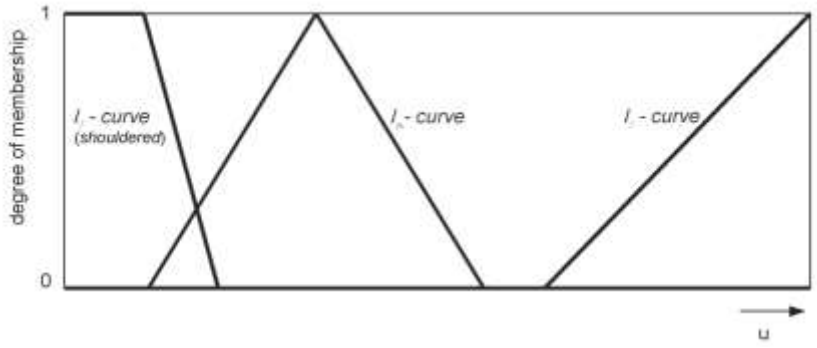
- Piecewise linear
  - Straight lines (increasing, decreasing)

$$\ell_{/}(x_\ell, x_r, x) = \begin{cases} 0, & x < x_\ell \\ \frac{x - x_\ell}{x_r - x_\ell}, & x_\ell \le x \le x_r \\ 1, & x > x_r \end{cases}$$

  - Triangular

$$\ell_{\wedge}(x_\ell, x_c, x_r, x) = \begin{cases} 0, & x < x_\ell \\ \frac{x - x_\ell}{x_c - x_\ell}, & x_\ell \le x \le x_c \\ 1 - \frac{x - x_c}{x_r - x_c}, & x_c \le x \le x_r \\ 0, & x > x_r \end{cases}$$

Linear curves



Types of membership functions

- Smooth curves
  - s-curve

$$s(x_0,b,x) = \begin{cases} 0, & x < x_0 - b \\ \frac{1}{2} + \frac{1}{2}cos(\frac{x-x_0}{b}\pi), & x_0 - b \le x \le x_0 \\ 1, & x > x_0 \end{cases}$$

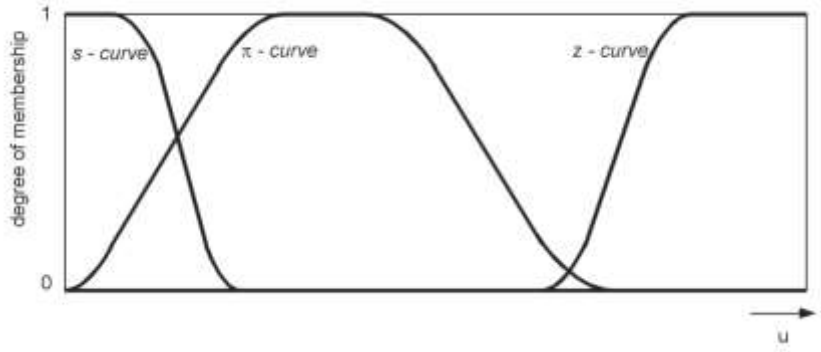
  - z-curve

$$z(x_0,b,x) = \begin{cases} 0, & x < x_0 \\ \frac{1}{2} + \frac{1}{2}cos(\frac{x-x_0}{b}\pi), & x_0 \le x \le x_0 + b \\ 1, & x > x_0 + b \end{cases}$$

  - π-curve

$$\pi(x_1,x_2,b,x) = min(s(x_1,b,x), z(x_2,b,x))$$

Smooth curves



Operations

Given $A = \{\langle x, \mu_A(x)\rangle\}$ and $B = \{\langle y, \mu_B(y)\rangle\}$ on a joint universe U.

- **fuzzy union**: $A \cup B = A \text{ or } B \triangleq A \max B$
  $\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x))$ for all $x \in U$
- **fuzzy intersection**: $A \cap B = A \text{ and } B \triangleq A \min B$
  $\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$ for all $x \in U$
- **fuzzy complement**: $\neg A = \text{not} A \triangleq 1 - A$
  $\mu_{\neg A}(x) = 1 - \mu_A(x)$ for all $x \in U$

Fuzzy intersection

- Universe (cylinder capacity): $U = \{1.0, 1.2, 1.4, 1.6, 1.8, 2.0\}$
  - **low consumption (LC)**

U	1.0	1.2	1.4	1.6	1.8	2.0
μ_{LC}	1.0	0.9	0.7	0.5	0.2	0.0

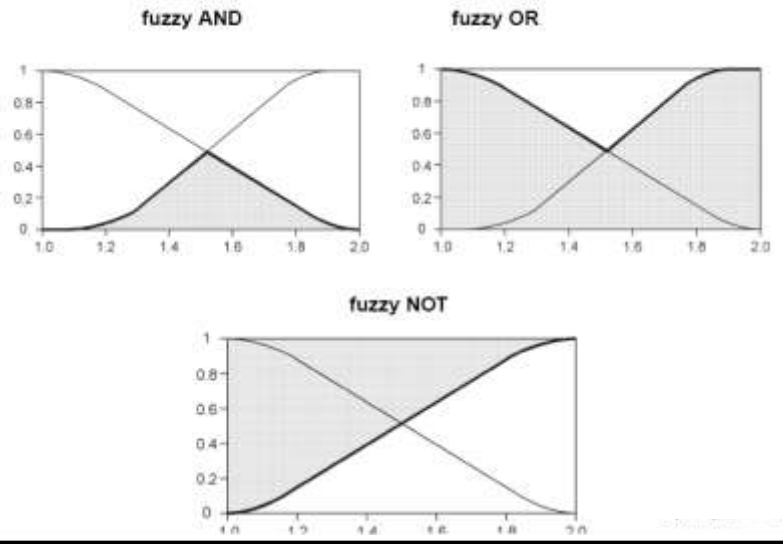
  - **high acceleration (HA)**

U	1.0	1.2	1.4	1.6	1.8	2.0
μ_{HA}	0.0	0.1	0.4	0.5	0.8	1.0

low consumption and *high acceleration*

U	1.0	1.2	1.4	1.6	1.8	2.0
μ_{LC}	1.0	0.9	0.7	0.5	0.2	0.0
μ_{HA}	0.0	0.1	0.4	0.5	0.8	1.0
$\min(\mu_{LC}, \mu_{HA})$	0.0	0.1	0.4	0.5	0.2	0.0

Operations on fuzzy sets



Algebraic properties

- Commutativity
  - a or b = b or a
  - a and b = b and a
- Associativity
  - (a or b) or c = a or (b or c)
  - (a and b) and c = a and (b and c)
- Distributivity
  - a or (b and c) = (a or b) and (b or c)
  - a and (b or c) = (a and b) or (b and c)
- DeMorgan rules
  - not (a and b) = (not a) or (not b)
  - not (a or b) = (not a) and (not b)

Algebraic properties

- Absorption
  - (a and b) or a = a
  - (a or b) and a = a
- Idempotency
  - a or a = a
  - a and a = a
- Exclusion (not satisfied)
  - a or ¬a ≠ 1
  - a and ¬a ≠ ∅

Linguistic modifiers

- **Approximation of Fuzzy Sets**: scalar → fuzzy set, modifying the "base" of a fuzzy set
  - about, around, near and close to
- **Restriction of Fuzzy Sets**: modifying the shape
  - below and above
- **Intensification and Dilution of Fuzzy Sets**
  - *intensification*: very (n = 2) and extremely (n = 3)
  - *dilution*: somewhat (n = 1/2) and greatly (n = 5/7)

$$\mathbf{op}\ \mu(x) = \mu^n(x)$$

Linguistic modifiers

- Graphical representation