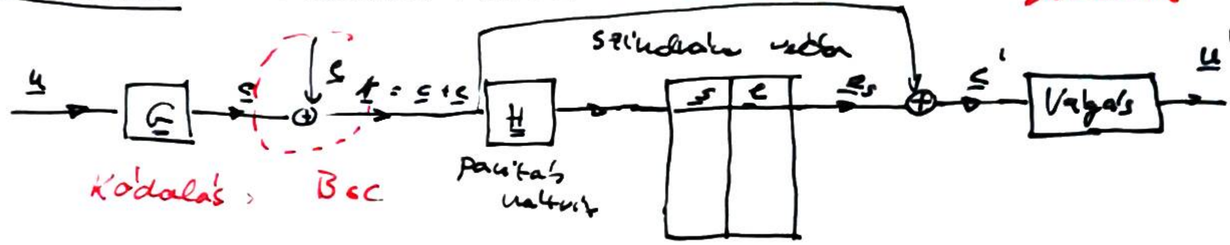
Info Code

ELŐADÁS 4

SZEP 21

Emlékeztető: Lineáris kódok



szisztematikus kódok

$$\underline{c} = (u_1, u_2, \ldots, u_k\ P_{n-k} \ P_n)$$

itt az alsó indexeket nem tudtam elolvasni

$$\underline{\underline{G}} = \left(\underline{\underline{I}}_{n \times k}\ \underline{\underline{B}}_{k-(n-k)}\right) \qquad \underline{\underline{H}} = \left(\underline{\underline{A}}_{(n-k)k}\ \underline{\underline{I}}_{(n-k)(n-n)}\right)$$

$$\underline{\underline{H}}\,\underline{\underline{G}}^T = 0 \Rightarrow \underline{\underline{A}} = -\underline{\underline{B}}^T$$

$\Downarrow$ bináris

$$\underline{\underline{A}} = \underline{\underline{B}}^T$$

"kulcs" egyenlet

$$\boxed{\underline{\underline{H}} \cdot \underline{e}^T = \underline{s}}$$

adott

számolható, adott

$$E_{\underline{s}} = \{ \underline{e} ; \underline{\underline{H}}\,\underline{e}^T = \underline{s} \}$$

$$\underline{e}_s : \min_{e \in E_s} \omega(\underline{e})$$

QoS comm.

$$\gamma, P_b \to t \to C(n,k)$$

Állítás: $C(n,k)$ lineáris $\to d_{min} = \omega_{min}$

ahol $d_{min} = \min_{\substack{c,c' \in C \\ c \neq c'}} d(c,c')$

legkisebb Hamming táv.

$$\omega_{min} = \min_{\underline{c} \in C} \omega(\underline{c})$$

legkisebb súly

Biz. $d_{min} = \min d(c,c') = \min \omega(\underbrace{c+c'}) =$

$c + c' = c''$
(mivel lineáris)

$$= \min_{\substack{c'' \in C \\ c'' \neq 0}} \omega(c'') = \omega_{min}$$

Állítás: $C(n,k)$ Lineáris

$$\boxed{\underline{c} \in E_{\underline{s}} \longrightarrow \underline{e} + \underline{c}^{(i)} \in E_{\underline{s}} \quad \forall i = 0,\dots, 2^{k-1}}$$

Bizonyítás: $\underline{\underline{H}}\, \underline{e}^T = \underline{s} \qquad \underline{\underline{H}}(e + c^{(i)})^T = \underline{s}$

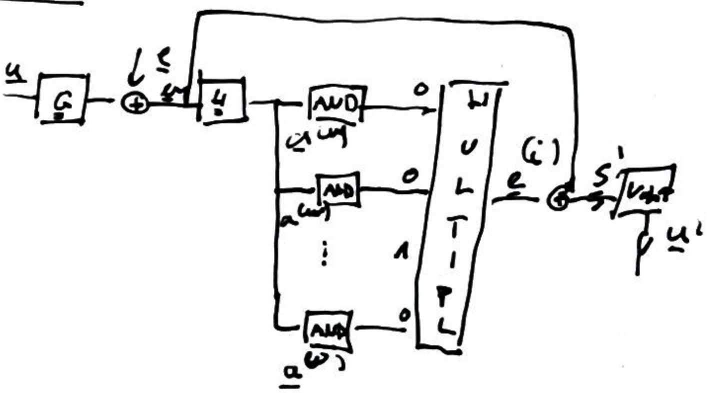
$$\underline{\underline{H}}\, \underline{e}^T + \underbrace{\underline{\underline{H}}\, \underline{c}^{(i)T}}_{=0} = \underline{s}$$

Következmény: $\underline{\underline{H}}\, e^T = \underline{s}$

$$E_s = \{ e; (e + \underline{c}^{(i)})_{j=1\dots}, (e+c)^{2^{(k-1)}} \} \quad 2^k \text{ elem}$$

Minden 1 hibát javító kódok $(t=1)$

Hamming kódok



$$\underline{e}^{(i)} = (0\ 0\ 0 \dots 0\ 1\ 0 \dots 0)$$

„i"

$$\underline{\underline{H}}\, \underline{e}^{(i)T} = \underline{s}^T$$

$$n-k \quad \begin{pmatrix} \vdots & \vdots & \vdots & \vdots \\ a^{(1)T} & a^{(2)T} & a^{(i)T} & a^{(n)T} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow i \qquad \begin{pmatrix} a_1^{(i)} \\ a_2^{(i)} \\ \vdots \\ a_n^{(i)} \end{pmatrix} = \begin{pmatrix} \vdots \\ a^{(i)} \end{pmatrix} = \underline{s}^T$$

Helyes dekódolási feltételek

1.) $\underline{a}^{(i)} \neq \underline{a}^{(j)} \quad \forall i,j = 1 \dots n \quad i \neq j$ [megkülönböztetés]

2.) $a^{(i)} \neq \underline{0}$ [hibajelzés miatt]

$$n = 2^{n-k} - 1$$

↑ Hamming kód

$$n + 1 = 2^{n-k}$$

$$\sum_{i=0}^{1} \binom{n}{i} = 2^{n-k} \to \text{perfekt kód}$$

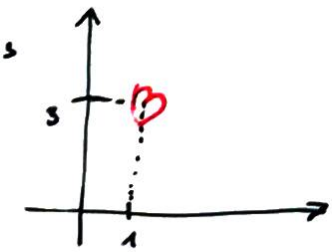
n	k
3	1
7	4
15	11
⋮	⋮

Emlékeztető: $t, n \xrightarrow[\text{kódok}]{\text{Hamming}} k$

Tétel: perfekt kódok

$$\begin{cases} n = 2^k - 1 & k = 2^n - n - 1 \quad t = 1 \quad n \geq 1 \quad \text{H. kód} \\ n = 23 & k = 12 \quad t = 3 \quad \text{Golay kód} \\ n = 2u + 1 & k = 1 \quad t = u \quad \text{ismétléses kód} \end{cases}$$

$C(3,1)$ $d_{min} = 3$ $d_{min} = n - k + 1$ MDS



$3 = 3 - 1 + 1$ ✓ $= 3$ ✓

$$\underset{(n-k)}{\underline{\underline{H}}} = \begin{array}{c} n \\ \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \end{array} \qquad \underset{k \times n}{\underline{\underline{G}}} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

$$\underline{c}^{(0)} = 0 \cdot \underline{\underline{G}} = (0 \ 0 \ 0)$$

$$\underline{c}^{(1)} = 1 \cdot \underline{\underline{G}} = (1 \ 1 \ 1)$$