Computer Controlled Systems (Introduction to systems and control theory) Lecture 1

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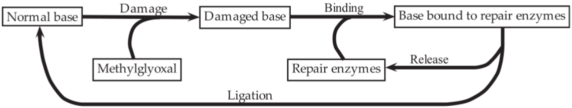
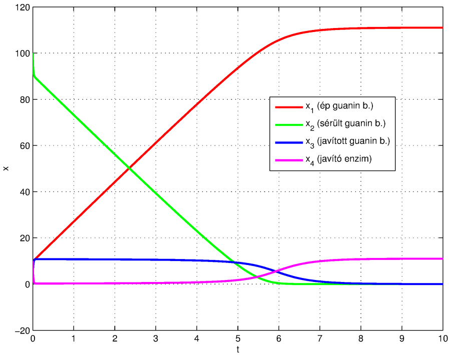
Outline

Quantitative model of a simple DNA-repair mechanism

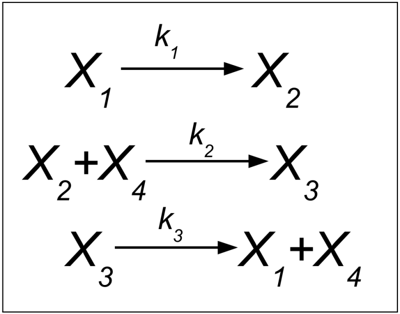
(Karschau et al., Biophysical Journal, 2011)





Introductory example – 2.

Reaction graph:



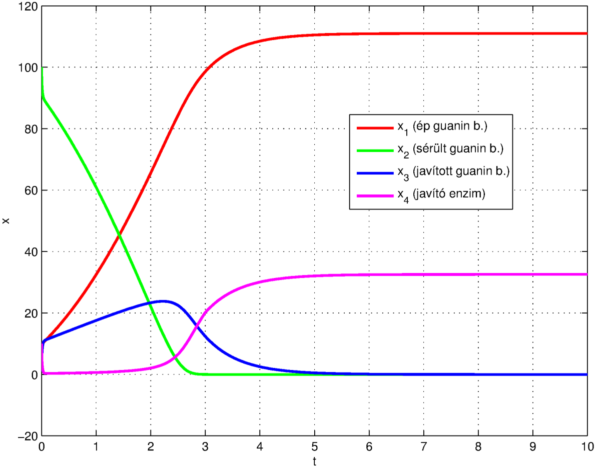
Kinetic equations:

$$\begin{aligned}
\dot{x}_1(t) &= k_3 x_3(t) - k_1 x_1(t) \\
\dot{x}_2(t) &= k_1 x_1(t) - k_2 x_2 x_4(t) \\
\dot{x}_3(t) &= k_2 x_2(t) x_4(t) - k_3 x_3(t) \\
\dot{x}_4(t) &= k_3 x_3(t) - k_2 x_2(t) x_4(t),
\end{aligned}$$

variables:

x_1 - no. of undamaged guanine bases
x_2 - no. of damaged guanine bases
x_3 - no. of guanine bases being repaired
x_4 - no. of free repair enzyme molecules

Intervention (to change the operation of the system):
adding more repair enzymes

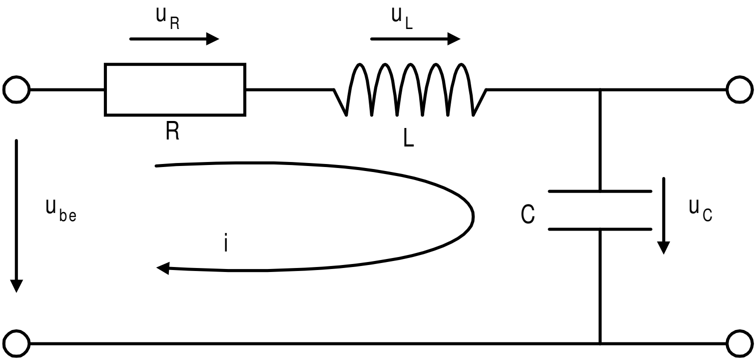


Notion of dynamical models/systems and their application

Dynamical models:

- they are applied to describe [physical] quantities varying in space and/or in time
- they describe the operation of natural or technological processes
- they can be useful to simulate or predict the behaviour of a process
- most often, mathematical models are used to describe dynamics (e.g. ordinary/partial differential equations)
- they can efficiently be solved by computers using various numerical methods
- they are useful to analyse the effect of a given (control) input

Simple RLC circuit



Simple RLC circuit

Kirchhoff's voltage law: $-u_{be} + u_R + u_L + u_C = 0$
Ohm's law: $U_R = R \cdot i$
Operation of the linear capacitor and inductor:

$$u_L = L \cdot \frac{di}{dt}, \quad i = C \cdot \frac{dU_C}{dt}$$

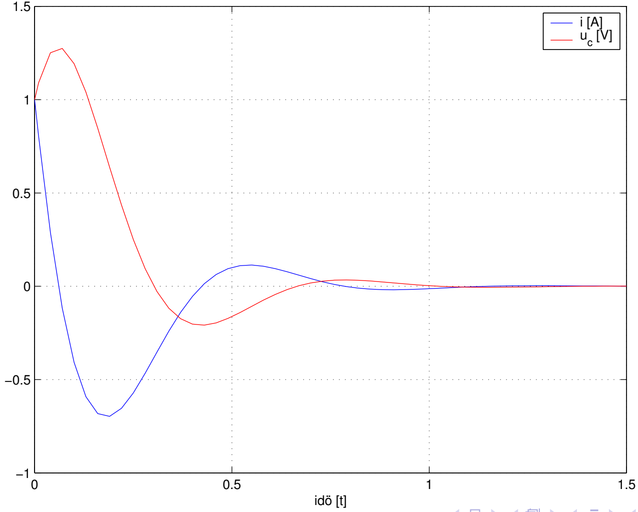
the so-called state equation :

$$\begin{aligned} \frac{di}{dt} &= -\frac{R}{L} \cdot i - \frac{1}{L} u_C + \frac{1}{L} u_{be} \\ \frac{du_C}{dt} &= \frac{1}{C} \cdot i \end{aligned}$$

Simple RLC circuit

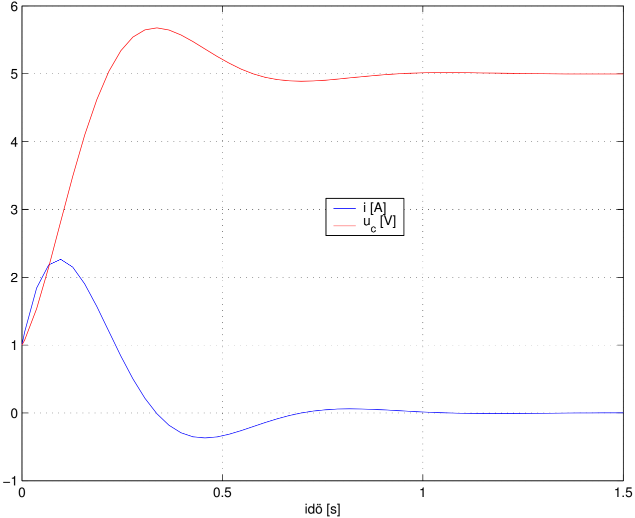
Parameters: $R = 1\ \Omega,\ L = 10^{-1} H,\ C = 10^{-1} F.$

$u_C(0) = 1$ V, $i(0) = 1$ A, $u_{be}(t) = 0$ V



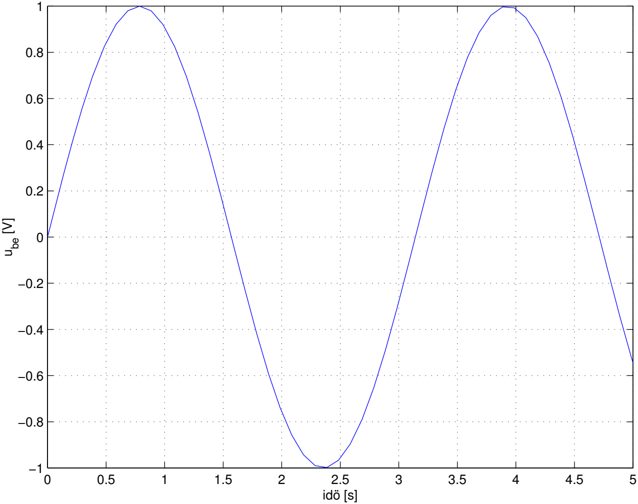
Simple RLC circuit

$u_C(0) = 1$ V, $i(0) = 1$ A, $u_{be}(t) = 5$ V



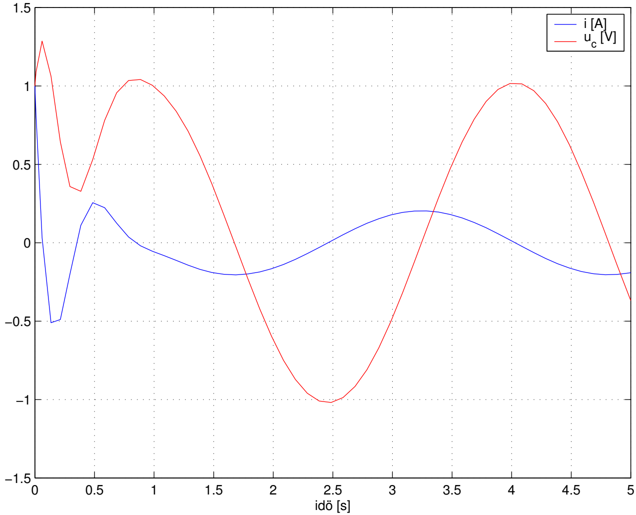
Simple RLC circuit

Periodic input:



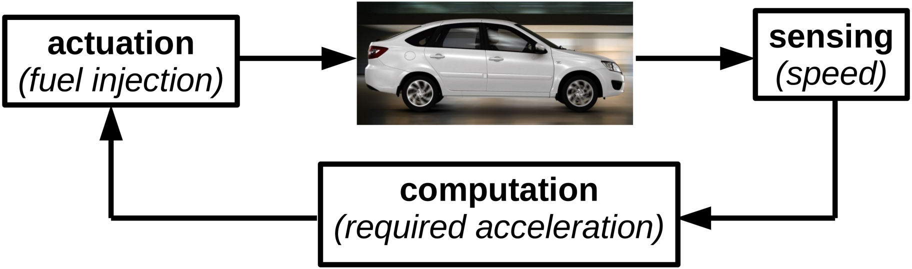
Simple RLC circuit

$u_C(0) = 1$ V, $i(0) = 1$ A



What does control mean? - Example

Control or stabilize the velocity of vehicles (e.g. tempomat)



What does control mean?

To **control** an *object*:

- to **manipulate**
- its **behaviour**
- in order **to reach a goal**.

Manipulation can happen

- through *observing the behaviour* (modeling), then choosing an appropriate control input considering the *desired behaviour*
- through the feedback of the *observed quantities* (measurements) to the input of the system (this can also be model-based)

What does control mean? - Notions

- **System**: What do we want to operate (what are the limits, what are the inputs/outputs)?
- **Control goal**: What kind of behaviour do we want to achieve?
- **System analysis**: Does the problem seem soluble? What can we expect?
- **Sensors**: Detection and monitoring of the the system's behaviour
- **Actuators**: Actual physical intervention (execution)
- **Models**: Mathematical description of the system's operation (over time/space)
- **Control system**: Approach to solve the problem (there can be many solutions based on various principles)
- **Hardware/software**: Controller design and execution of control algorithms

The significance of systems and control theory

- Dynamics : Description of varying quantities in space/time
- Dynamical systems and control systems are present everywhere in our lives: household appliances, vehicles, industrial equipment, communications systems, natural systems (physical, chemical, biological)
- Control becomes mission-critical: if it fails, the whole system may become unusable
- The elements of system theory are (increasingly) utilized by classical sciences
- The principles of control theory has been applied to seemingly distant areas, like economics , biology , drug discovery , etc.

The significance of systems and control theory

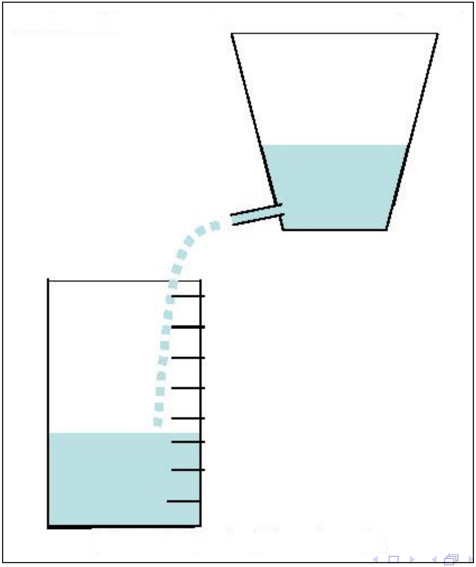
- Systems and control theory is inherently interdisciplinary (construction of mathematical models and analysis; physical components: controlled system, sensors, actuators, communication channels, computers, software)
- Systems theory provides a good environment for the transfer of technology : in general, procedures developed in one area can be useful in other areas, too
- Knowledge and skills obtained in control theory provide a good background for designing and testing complex (technological) systems

Dynamical models (systems) and biology

- dynamics may be essential to understand the operation of important biochemical/biological processes (causes, effects, cross-reactions)
- biology is increasingly available to the traditional engineering approaches (on molecular, cellular and organic levels, too): quantitative modeling, systems theory, computational methods, abstract synthesis methods
- conversely, biological discoveries might serve as a basis for new design methodologies
- a few areas where the dynamics and control have an important role: gene regulation; signal transmission; hormonal, immune and cardiovascular feedbacks; muscle and movement control; active sensing; visual functions; attention; population and disease dynamics

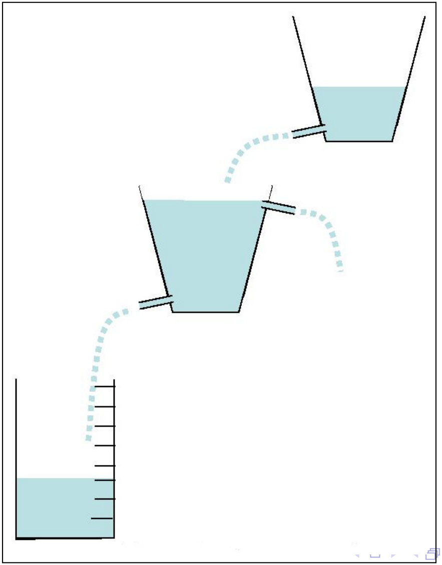
Simple water clock

Before 1000 BC



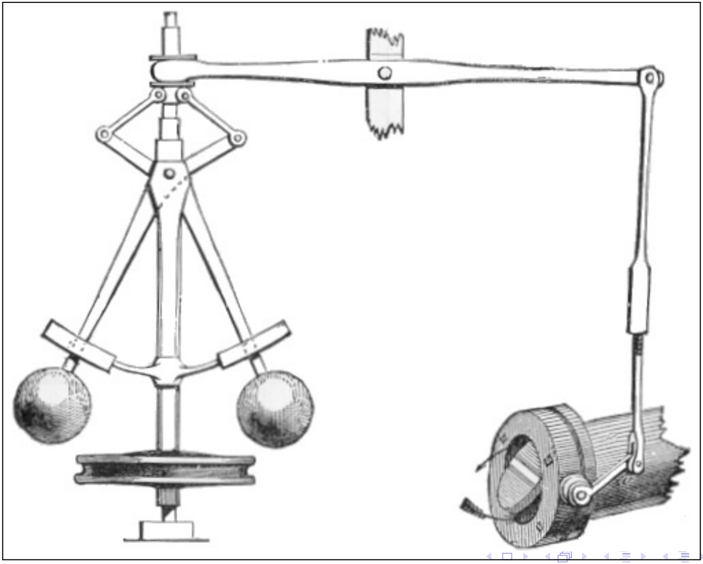
Water clock with water flow rate control

3rd century B.C.



Flyball governor

James Watt, 1788



Birth of systems and control theory as a distinct discipline (approx. 1940-1957)

- 1940-45: Intensive military research (unfortunately); recognizing common principles and representations (radar systems, optimal shooting tables, air defense artillery positioning, autopilot systems, electronic amplifiers, industrial production of uranium etc.).
- Representation of system components using block diagrams
- Analysis and solution of linear differential equations using Laplace transformation, theory of complex functions and frequency domain analysis
- The results of the research in the military were quickly used in other industries as well
- Independent research and teaching of control theory began
- 1957: The International Federation of Automatic Control (IFAC) was founded

The next stage of development (about 1957-1980)

- Motivation: military and industrial application requirements, development of mathematics and computer sciences
- Space Race – space research competition (spacecraft Sputnik, 1957)
- The first computer-controlled oil refinery in 1959
- The use of digital computers for simulation and control systems implementation
- Mathematical precision becomes more important
- The appearance of state-space model based methods

Modern and postmodern control theory (about 1980-)

- Birth of nonlinear systems and control theory based on differential algebra
- The explosive development of numerical optimization methods + computing capacity becomes cheaper
- Handling model uncertainties (robust control)
- Model predictive control (MPC)
- "Soft computing" techniques: fuzzy logics, neural networks etc.
- Energy-based linear and nonlinear control (electrical, mechanical, thermodynamical foundations)
- Control of hybrid systems
- Theory of positive systems
- Control theory and its application to networked systems ("cyber-physical" system)

Controlled technological systems

- thermostat + heating: temperature
- dynamic speed limits on highways: the number of cars passing through during a time unit, exhaust emissions
- power plants' (thermal) power: required electric power
- movement of robotic arms and mobile robots: follow prescribed tracks (guidance)
- aircraft landing/take off: height, speed
- air traffic control: time of landings/take-offs and their order
- re-scheduling of timetables: to minimize all delays
- oxygenation of wastewater treatment plants: speed of bioreactions
- washing machine: weight control, water amount control
- ABS, ESP systems in vehicles: torque, braking force
- CPU clock speed, fan speed: temperature

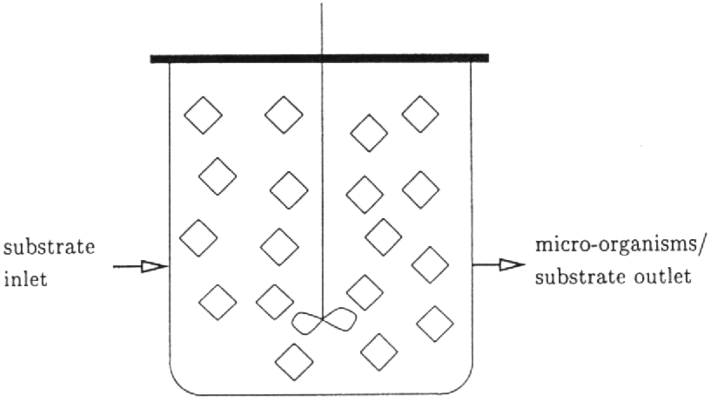
Management of society and economics

- *laws* (including their execution): social life
- *banking systems*: quantity of money in circulation
- *media*: reviews, public taste, agreed standards, overemphasized and concealed informations
- *advertisement*: consumer habits

Control in nature

- control of gene expression (transcription, translation)
- body temperature regulation of warm-blooded animals
- blood glucose control
- hormonal and neural control in organisms/living entities
- swarm of moving animals (birds, insects, fish): speed
- synchronized flashing of light emitting insects
- movement, human walking

Bioreactor–model



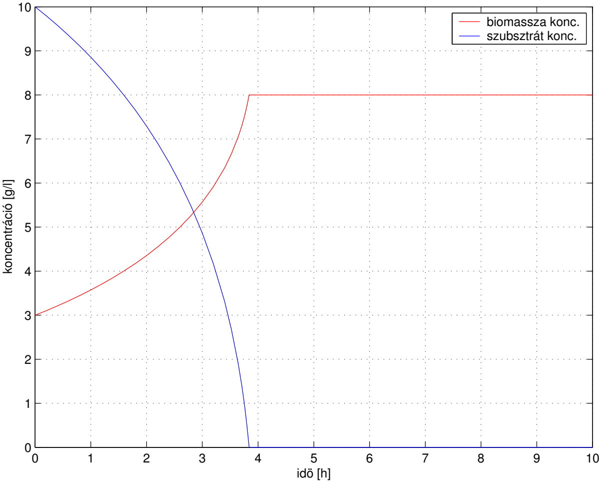
Bioreactor–model

$$\frac{dX}{dt} = \mu(S)X - \frac{XF}{V}$$
$$\frac{dS}{dt} = -\frac{\mu(S)X}{Y} + \frac{(S_F - S)F}{V}$$
$$\text{ahol pl.} \quad \mu(S) = \mu_{max}\frac{S}{K_2S^2 + S + K_1}$$

X	biomass concentration		$[\frac{g}{l}]$	Y	kin.par.	0.5	-
S	substrate concentration		$[\frac{g}{l}]$	μ_{max}	kin.par.	1	$[\frac{1}{h}]$
F	input flow rate		$[\frac{l}{h}]$	K_1	kin.par.	0.03	$[\frac{g}{l}]$
V	volume	4	$[l]$	K_2	kin.par	0.5	$[\frac{l}{g}]$
S_F	substrate feed concentration	10	$[\frac{g}{l}]$				

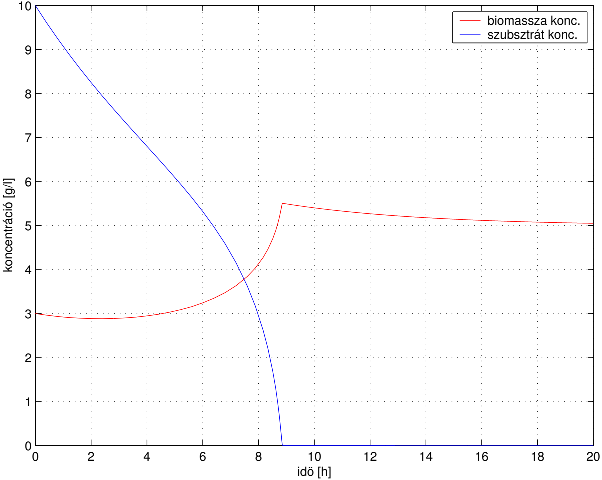
Bioreactor–model

$F = 0 \frac{l}{h}$



Bioreactor–model

$F = 0.8 \frac{l}{h}$



Simple ecological system

$$\frac{dx}{dt} = k \cdot x - a \cdot x \cdot y$$
$$\frac{dy}{dt} = -l \cdot y + b \cdot x \cdot y$$

x – number of preys in a closed area

y – the number of predators in a closed area

k – the natural growth rate of preys in the absence of predators

a – "meeting" rate of predators and preys

l – natural mortality rate of predators in the absence of preys

b – reproduction rate of predators for each consumed pray animal

Parameters:

$k = 2 \ \frac{1}{\text{month}}$

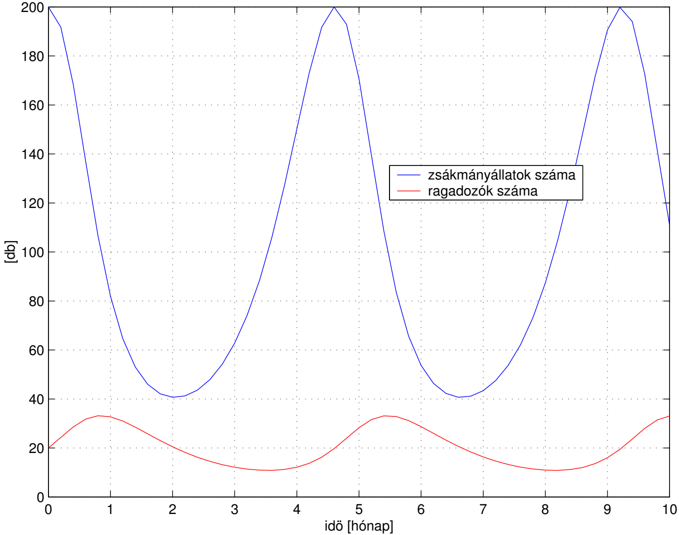
$a = 0.1 \ \frac{1}{\text{pieces} \cdot \text{month}}$

$l = 1 \ \frac{1}{\text{month}}$

$b = 0.01 \ \frac{1}{\text{pieces} \cdot \text{month}}$

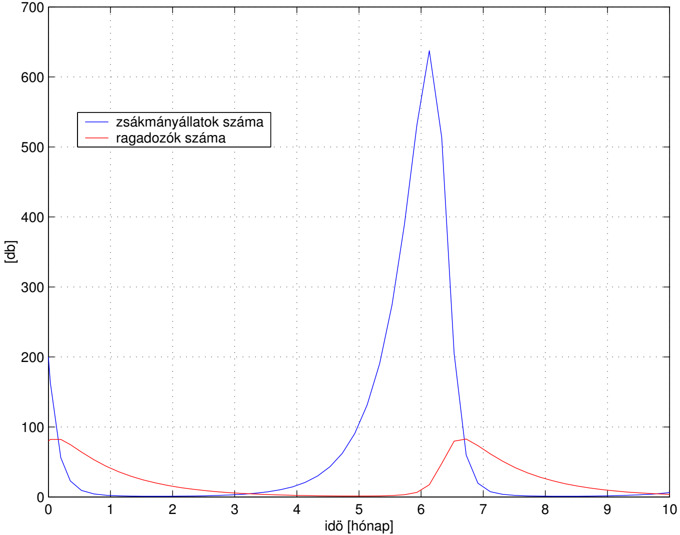
Simple ecological system

$x(0) = 200, \ y(0) = 20$



Simple ecological system

$x(0) = 200, \; y(0) = 80$



SIR disease spreading model

Healing/spreading mechanism:

$$S + I \xrightarrow{b} 2I$$
$$I \xrightarrow{k} R$$

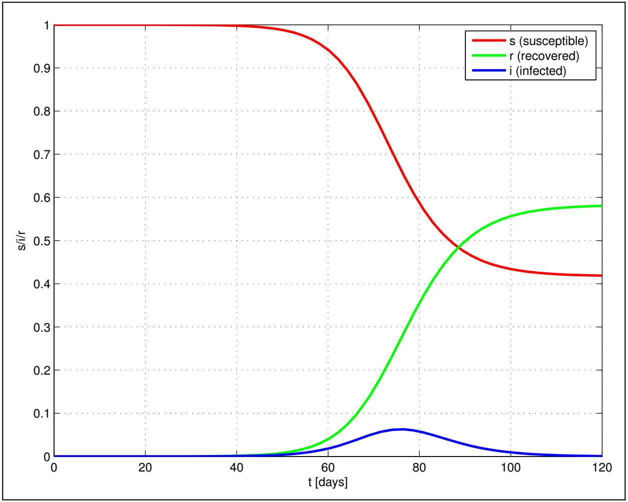
S: susceptible human individuals
I: infected human individuals
R: recovered human individuals
N: number of population
$s = S/N$, $i = I/N$, $r = R/N$
mathematical model:

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -b \cdot s(t) \cdot i(t)$$
$$\frac{\mathrm{d}r}{\mathrm{d}t} = k \cdot i(t)$$
$$\frac{\mathrm{d}i}{\mathrm{d}t} = b \cdot s(t) \cdot i(t) - k \cdot i(t)$$

b, k: constant parameters

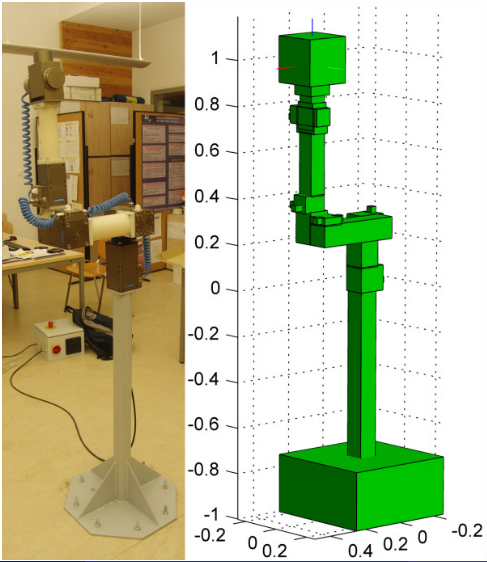
SIR disease spreading model

$N = 10^7$, $S(0) = 9999990$, $I(0) = 10$, $R(0) = 0$, $k = 1/3$, $b = 1/2$



6 degree of freedom robotic arm

(doctoral work of Ferenc Lombai)



6 degree of freedom robotic arm

Planning and execution of a throwing movement

(videos/6dof_dob_1.avi)
(videos/6dof_dob_2.avi)
(videos/6dof_dob_3.avi)

Flexible robotic joint

Controlled flexor-extensor mechanism with 2 stepper motor
(doctoral work of József Veres)

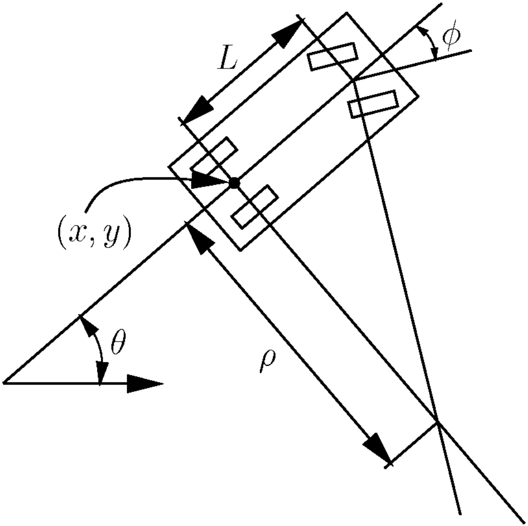
http://www.youtube.com/watch?v=qBMs_36gZMg

Simultaneous Localization & Mapping (SLAM)

Task: Active localization of a mobile robot (parallel movement and mapping)
Students' Scientific Conference assignment of
János Rudan and Zoltán Tuza

(videos/SLAM_TDK.mpeg)

Steered car model – 1



Steered car model – 2

Configuration space: $\mathbb{R}^2 \times \mathbb{S}^1$

Configuration: $q = (x, y, \theta)$

Parameters:

S: signed longitudinal direction, speed

ϕ: steering angle

L: distance between front and rear axles

ρ: turning radius for a fixed steering angle ϕ

The dynamical model describes how x, y and θ change in time:

$$\begin{aligned} \dot{x} &= f_1(x, y, \theta, s, \phi) \\ \dot{y} &= f_2(x, y, \theta, s, \phi) \\ \dot{\theta} &= f_3(x, y, \theta, s, \phi) \end{aligned}$$

Steered car model – 3

The most simple control model:

Manipulate input (simplistic assumptions): velocity (u_s), steering angle (u_ϕ), namely $u = (u_s, u_\phi)$

The equations:

$$\begin{aligned} \dot{x} &= u_s \cos\theta \\ \dot{y} &= u_s \sin\theta \\ \dot{\theta} &= \frac{u_s}{L} \tan u_\phi \end{aligned}$$

More accurate (realistic) model using acceleration dynamics:

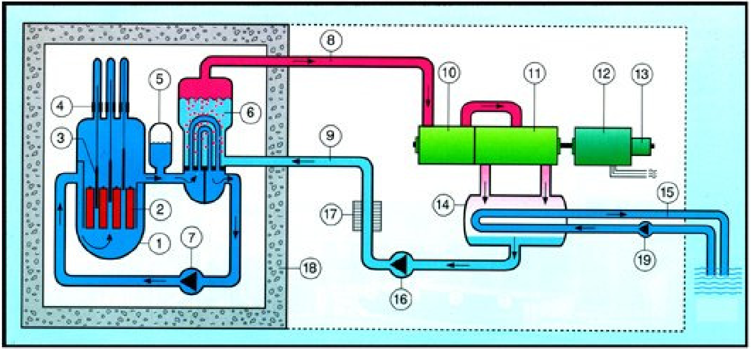
$$\begin{aligned} \dot{x} &= s \cos\theta \\ \dot{y} &= s \sin\theta \\ \dot{\theta} &= \frac{u_s}{L} \tan u_\phi \\ \dot{s} &= u_t \end{aligned}$$

Autonomous and cooperative vehicles

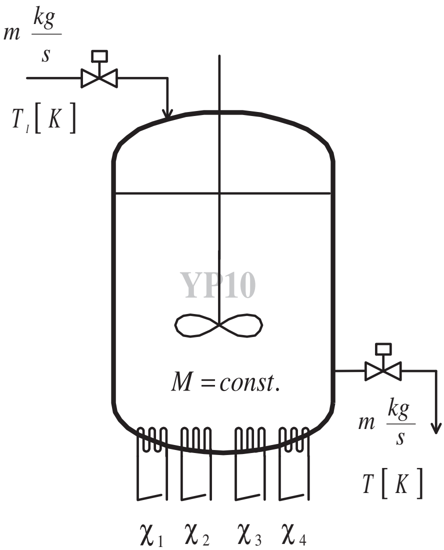
- Following prescribed trajectories (guidance) (`videos/car_track.avi`)
- Chasing of moving objects, simulations: Gábor Faludi (`videos/ref_car.avi`)
- (flight) movement in formations (`videos/formation.avi`)
- Formation change (`videos/chg_form.avi`)
- Obstacle avoidance (`videos/obstacle.avi`)

Power system application: primary circuit pressure control

Structure of pressurized water reactor unit



Primary circuit pressure control



pressurizer tank

Primary circuit pressure control

Modeling assumptions:

- two perfectly stirred balance volumes: water and the wall of the tank
- constant mass in the two balance volumes
- constant physico-chemical properties
- vapor-liquid equilibrium in the tank

Equations:

water

$$\frac{\mathrm{d}U}{\mathrm{d}t} = c_p m T_I - c_p m T + K_W(T_W - T) + W_{HE} \cdot \chi$$

wall of the tank

$$\frac{\mathrm{d}U_W}{\mathrm{d}t} = K_W(T - T_W) - W_{loss}$$

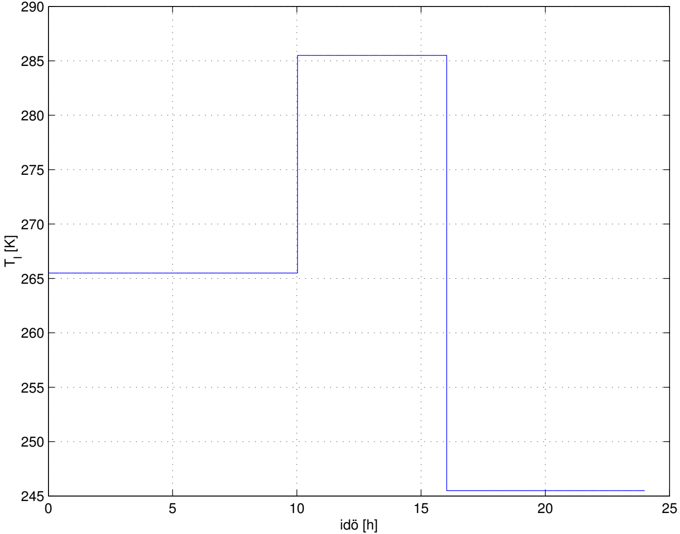
Primary circuit pressure control

Variables and parameters:

T	water temperature	$^\circ$C
T_W	wall temperature	$^\circ$C
c_p	specific heat of water	$\frac{J}{kg^\circ C}$
U	internal energy of water	J
U_W	internal energy of the wall	J
m	water inflow rate	$\frac{kg}{s}$
T_I	temperature of incoming water	$^\circ$C
M	mass of water	kg
C_{pW}	heat capacity of the wall	$\frac{J}{^\circ C}$
W_{HE}	max. power of heaters	W
χ	portion of heaters turned on	-
K_W	heat transfer coefficient of the wall	$\frac{W}{^\circ C}$
W_{loss}	the system's heat loss	W

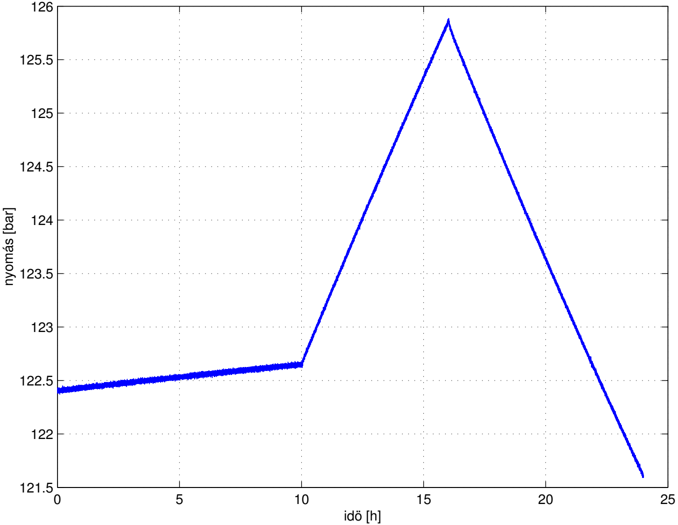
Primary circuit pressure control

Temperature of water inflow

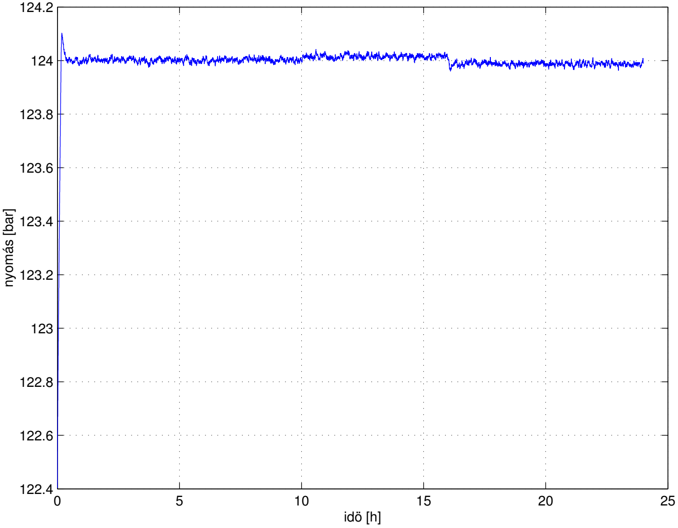


Primary circuit pressure control

Pressure without control

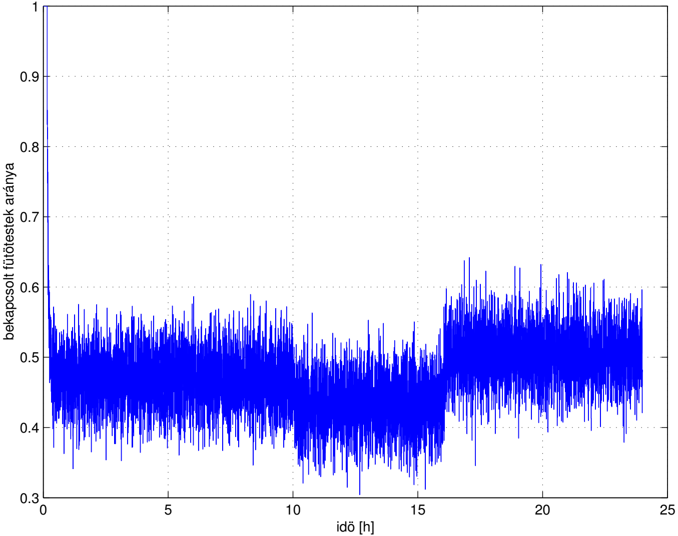


Pressure using a control system



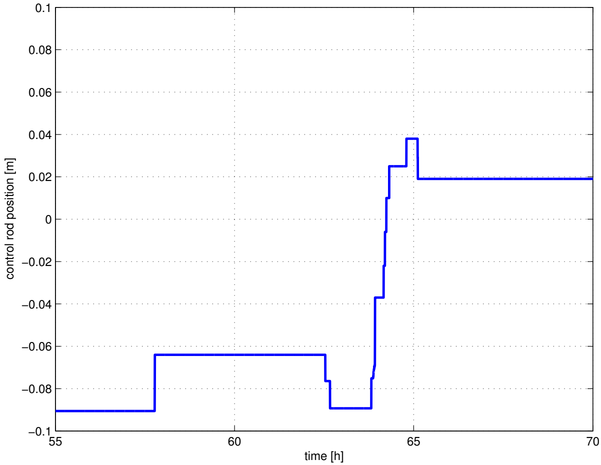
Primary circuit pressure control

Heating power applied by the the controller



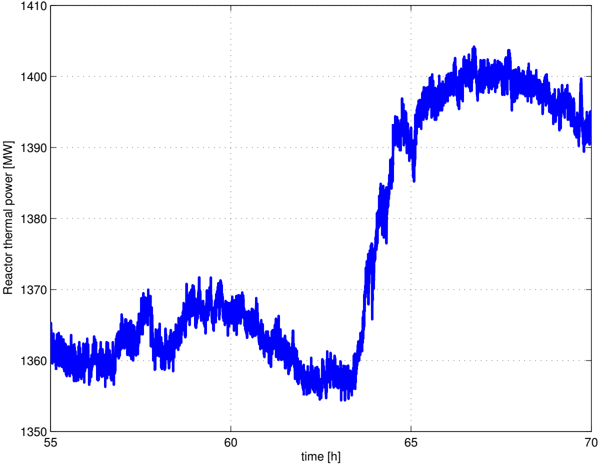
Primary circuit pressure control

Smaller transient: position of control rods



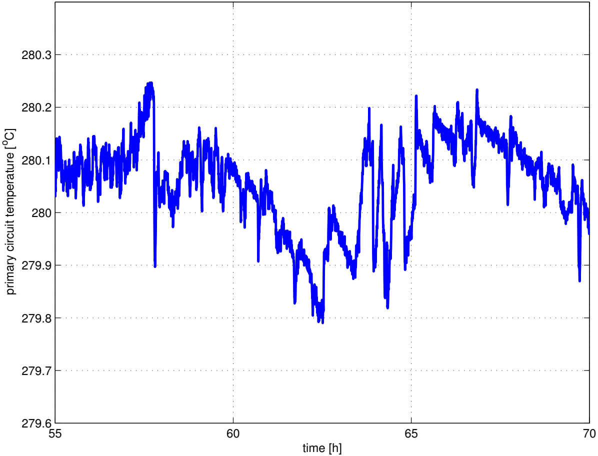
Primary circuit pressure control

Thermal power of the reactor



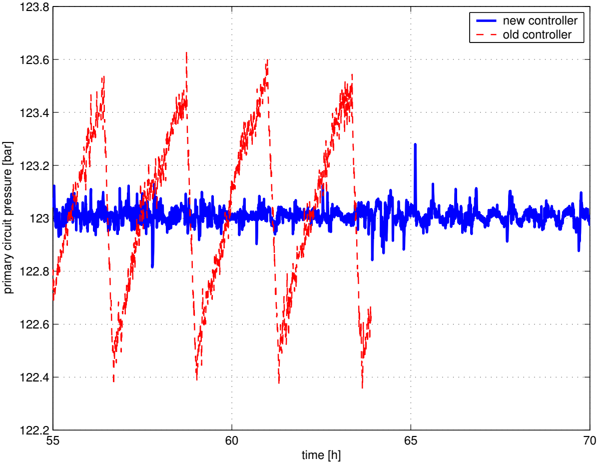
Primary circuit pressure control

Temperature in the primary circuit



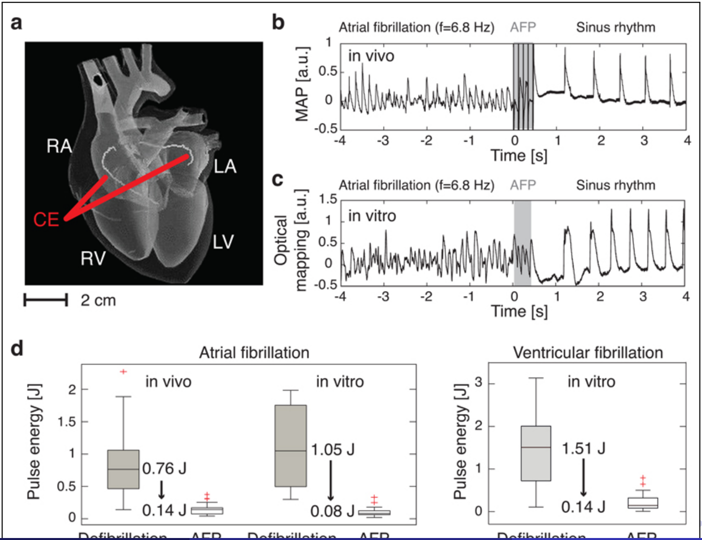
Primary circuit pressure control

primary circuit pressure with the old and the new controller

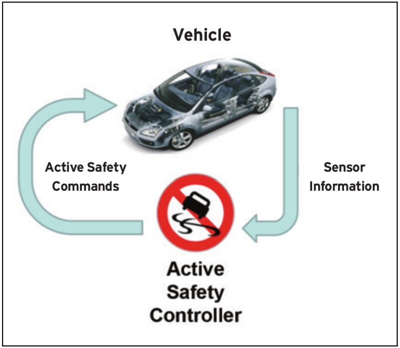


Non-conventional defibrillation

(S. Luther et al. Nature. 2011 Jul 13;475(7355):235-9)
Foundation: a detailed 3D mathematical model of the heart



Vehicle safety



- anti blocking system (ABS)
- traction control (TC)
- electronic stability control (ESC)

There is a 4-times payback of the development costs with the avoidance of accidents

Typically, model-based controllers are used

Traffic control on highways

- Australia (Monash Freeway), 2008
- model-based ramp metering control
- problem-free implementation

- traffic jams disappeared
- throughput increased by 4.7 and 8.4% in the morning and afternoon peak period, resp.
- average speed increased by 24.5 and 58.6% in the morning and afternoon, resp.

Why do we study this course?

- primary goal: basic knowledge in **systems theory**
  - ability to observe, analyse and separate systems of the surrounding world
  - ability to determine a system's inputs, outputs, states
- knowledge of basic **system properties** and their analysis (what can we expect?)
- what options of manipulation do we have in order to reach a certain control goal, and how expensive is it (time, energy)?
- establishing an interdisciplinary perspective (electrical, mechanical, chemical, biological, thermodynamic, ecological, economic systems)

Topics

- System classes, basic system properties
- Input/output and state space models of continuous time, linear time invariant (CT-LTI) systems
- BIBO stability and other stability criteria for CT-LTI systems
- Asymptotic stability of CT-LTI systems, Lyapunov's method
- Controllability and observability of CT-LTI systems
- Joint controllability and observability, minimal realization, system decomposition

Topics

- Control design: PI, PID and pole placement controllers
- Optimal (linear quadratic) regulator
- State observer synthesis
- Sampling, discrete time linear time invariant (DT-LTI) models
- Controllability, observability, stability of DT-LTI systems
- Control design for DT-LTI systems

Relations to other subjects

Preliminary studies

- mathematics (linear algebra, calculus, probability theory, stochastic processes)
- physics (determining physical models)
- signal processing (transfer functions, filters, stability)
- electrical networks/circuits theory (linear circuit models)

Further subjects

- robotics (dynamical modeling, regulations and guidance)
- nonlinear dynamical systems (simulation and stability)
- optimization methods, functional analysis (optimal control design, linear system operators)
- computational systems biology (differential equation models, molecular control loops)
- parameter estimation of dynamical systems (construction of dynamical models based on measurements)

Software-tools (possible choices)

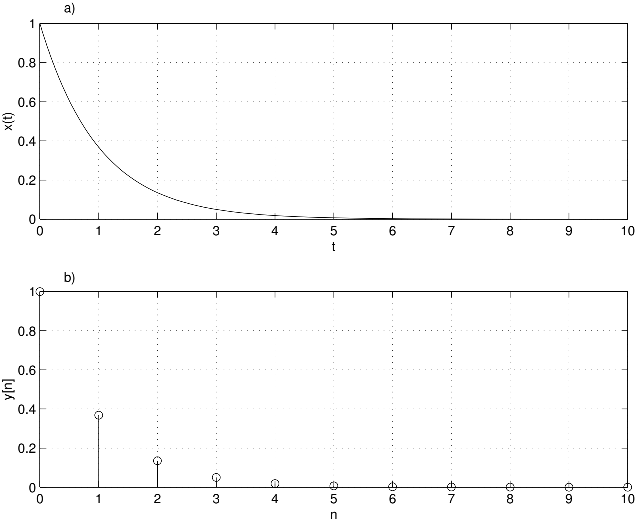
- **Commercial**
  - *Matlab/Simulink*: numerical computations, simulations
    http://www.mathworks.com
  - *Mathematica*: symbolic and numerical computations
    http://www.wolfram.com/
  - *Maple*: symbolic, numerical computations, simulations
    http://www.maplesoft.com/
- **Free**
  - *Scilab/Xcos*: numerical computations, simulations
    http://www.scilab.org/
  - *Sage*: symbolic, numerical computations
    http://sagemath.org/

Signal: A (physical) quantity, which depends on time, space or other independent variables

E.g. (in addition to the introductory examples)

- $x : \mathbb{R}_0^+ \mapsto \mathbb{R}, \quad x(t) = e^{-t}$
- $y : \mathbb{N}_0^+ \mapsto \mathbb{R}, \quad y[n] = e^{-n}$
- $X : \mathbb{C} \mapsto \mathbb{C}, \quad X(s) = \frac{1}{s+1}$

Signals – 2



a)
x(t)
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10
t
b)
y[n]
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10
n

Signals – 3

- room temperature: $T(x, y, z, t)$
  (x, y, z: spatial coordinates, t: time)
- image of a color TV: $I : \mathbb{R}^3 \mapsto \mathbb{R}^3$

$$I(x, y, t) = \begin{bmatrix} I_r(x, y, t) \\ I_g(x, y, t) \\ I_b(x, y, t), \end{bmatrix}$$

Classification of signals

- dimension of the independent variable
- dimension of the dependent variable (signal)
- real or complex valued
- continuous vs discrete time
- bounded vs not bounded
- periodic vs aperiodic
- even vs odd

Dirac-δ or the *unit impulse* function

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$$

where $f : \mathbb{R}_0^+ \mapsto \mathbb{R}$ is an arbitrary smooth (infinitely many times continuously differentiable) function.
consequence

$$\int_{-\infty}^{\infty} 1 \cdot \delta(t)dt = 1$$

Signals with particular significance – 1

The physical meaning of the unit impulse:

- current impulse ⇒ charge
- temperature impulse ⇒ energy
- force impulse ⇒ momentum
- pressure impulse ⇒ mass
- density impulse: point mass
- charge impulse: point charge

Signals with particular significance – 1

Heaviside (*unit step*) function

$$\eta(t) = \int_{-\infty}^{t} \delta(\tau) d\tau,$$

in other words:

$$\eta(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } t \geq 0 \end{cases}$$

Basic operations on signals – 1

$$x(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} y_1(t) \\ \vdots \\ y_n(t) \end{bmatrix}$$

- addition:
  $(x+y)(t) = x(t) + y(t), \quad \forall t \in \mathbb{R}_0^+$
- multiplication by a scalar:
  $(\alpha x)(t) = \alpha x(t) \quad \forall t \in \mathbb{R}_0^+, \ \alpha \in \mathbb{R}$
- scalar product:
  $\langle x, y \rangle_\nu(t) = \langle x(t), y(t) \rangle_\nu \quad \forall t \in \mathbb{R}_0^+$

Basic operation on signals – 2

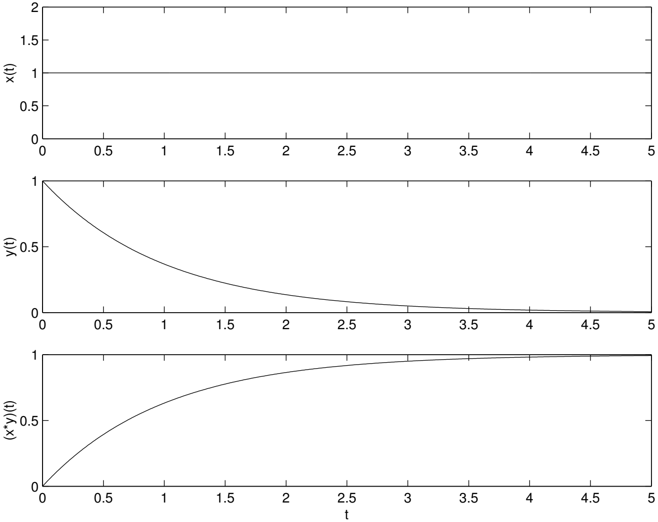
- time shifting:
  $\mathbf{T}_a x(t) = x(t-a) \quad \forall t \in \mathbb{R}_0^+, a \in \mathbb{R}$
- causal time shifting:
  $\mathbf{T}_a^c x(t) = \eta(t-a)x(t-a) \quad \forall t \in \mathbb{R}_0^+, a \in \mathbb{R}$

Convolution – 1

$x, y : \mathbb{R}_0^+ \mapsto \mathbb{R}$

$$(x * y)(t) = \int_0^t x(\tau)y(t-\tau)d\tau, \quad \forall t \geq 0$$

Convolution – 2



Laplace transform

Domain (of interpretation):

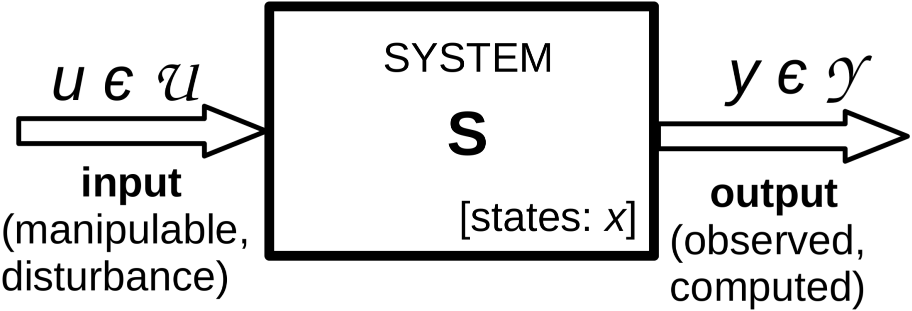
$$\Lambda = \{\ f \mid f : \mathbb{R}_0^+ \mapsto \mathbb{C}, f \text{ is integrable on } [0,a]\ \forall a > 0 \text{ and}$$
$$\exists A_f \geq 0, a_f \in \mathbb{R},\ \text{such that } |f(x)| \leq A_f e^{a_f x}\ \forall x \geq 0\ \}$$

Definition:

$$\mathcal{L}\{f\}(s) = \int_0^{\infty} f(t) e^{-st} dt, \quad f \in \Lambda,\ s \in \mathbb{C}$$

The notion of a system

System: A physical or logical device that performs operations on signals. (Processes input signals, and generates output signals.)



Summary

- changing (physical) quantities: dynamical models
- mathematical representation: differential equations
- system: operator , input-output mapping
- systems theory is interdisciplinary : describes and treats physical, biological, chemical, technological processes in a common framework
- control is present everywhere and is often mission-critical
- control design and implementation requires knowledge from mathematics, physics, hardware, software and computer science
- control principles can be found in purely natural systems as well
- why to study: to be able to describe, understand and influence (control) dynamical processes