1 Homework

1.1 problem

1.1.1 (a)

$$A = \begin{bmatrix} -2 & 0 & \frac{1}{2} \\ 0 & 0 & 1 \\ -1 & -1 & -\frac{5}{4} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad D = 0$$

1.1.2 (b)

The eigenvalues of the matrix A are -1.432 and $-0.909 \pm 0.7552i$. All of the eigenvalues have negative real parts, hence the system is stable.

1.1.3 (c)

The oscillatory behavior is present because the complex conjugate roots.

1.1.4 (d)

From the Bass-Gura formula we get

$$K = (\alpha - a) \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & \frac{1}{2} & -\frac{13}{8} \\ 0 & 1 & -\frac{5}{4} \\ 1 & -\frac{5}{4} & \frac{1}{16} \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 2 & \frac{11}{4} \end{bmatrix}.$$

1.1.5 (e)

With $L = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}$ we get that the eigenvalues of $A - LC$ are $-2, -\frac{5}{4}$ and -1, hence this is stable. (If this were not true, we could use the Bass-Gura formula with a little modification to calculate L.)

1.2 problem

Using the feedback rule we get

$$H(s) = \frac{\frac{s+5}{s-1}}{1 + \frac{\frac{1}{s}}{1+\frac{k}{s}} \frac{s+5}{s-1}} = \frac{s^2 + (5+k)s + 5k}{s^2 + ks + 5 - k}.$$

We can see that $k = 3$ satisfies the prescribed conditions, i.e. the poles are -1 and -2, since $s^2 + ks + 5 - k\Big|_{k=3} = s^2 + 3s + 2 = (s+1)(s+2)$.