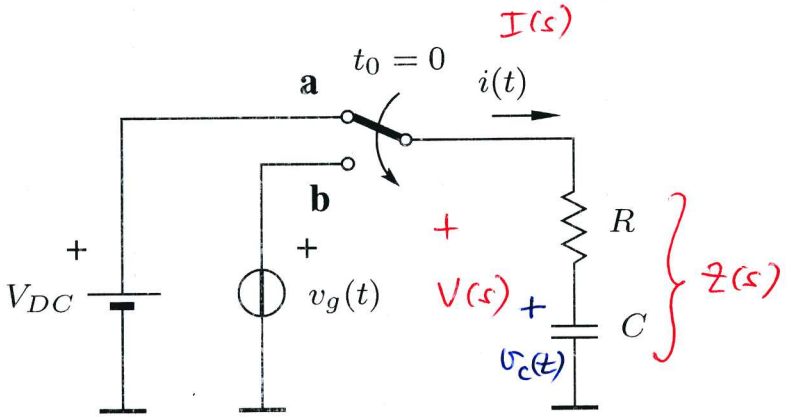
A 2010. december 22-i vizsgaZH/2. feladat megoldása



$R = 500\ \Omega$

$C = 1\ \mu\text{F}$

$V_{DC} = 10\ \text{V}$

$v_g(t) = 10\cos(\omega t)\ \text{V}$

$f = 1\ \text{kHz}$

MEGOLDÁS AZ IMPEDANCIA KONCEPCIÓVAL (AZ IDŐTARTOMÁNYBELI MEGOLDÁS IS OK)

$$V(s) = Z(s)\, I(s) = \left(R + \frac{1}{sC}\right) I(s) = \frac{1+sRC}{sC}\, I(s)$$

$$(1+sRC)\, I(s) = sC\, V(s)$$

(2.1) TRANZIENS: $V(s) = V_g(s) = 0$

$$(1+sRC)\, I(s) = 0 \Rightarrow RC\frac{di}{dt} + i = 0$$

ELSŐRENDŰ RENDSZER TRANZIENS MEGOLDÁSÁT KERESSÜK

$$i = A\, e^{-\frac{t}{\tau}} \Rightarrow \frac{di}{dt} = -\frac{1}{\tau} i \quad \text{ALAKBAN, AHOL } i \neq 0$$

$$RC\left(-\frac{1}{\tau}\right) i + i = 0 \Rightarrow \tau = RC = \underline{0{,}5\ \text{ms}}$$

$$\underline{i_{TRANZIENS}(t) = A\, e^{-\frac{t}{RC}}\ [\text{A}]}$$

(2.2) ÁLLANDÓSULT A KOMPLEX AMPLITÚDÓKKAL:

$$v_g = 10\cos(\omega t) \Rightarrow V_g = \frac{10}{\sqrt{2}}\angle 0° \quad \text{ÉS} \quad f = 1\ \text{kHz}$$

$$Z(s)\Big|_{\substack{s=j\omega \\ f=1\text{kHz}}} = R + \frac{1}{j\omega C}\Big|_{f=1\text{kHz}} = 500 - j\,159 = \underline{525\angle -17{,}6°}\ \Omega$$

$$I = \frac{V_g}{Z(1\text{kHz})} = \frac{10\angle 0°}{\sqrt{2}\cdot 525\angle -17{,}6°} = \frac{19{,}05}{\sqrt{2}}\angle 17{,}6°\ \text{mA}$$

$$\underline{i_{ÁLLANDÓSULT} = 19{,}05\cos(\omega t + 17{,}6°)\ \text{mA} \qquad f = 1\ \text{kHz}}$$

2.3 $t \geq 0$-RA A TELJES VÁLASZ:

$$i(t) = i_{TRANZIENS}(t) + i_{ÁLLANDÓSULT}(t) = A\, e^{-\frac{t}{RC}} + 19{,}05 \cos(\omega t + 17{,}6°) \text{ mA}$$

KEZDETI FELTÉTEL:

$$v_C(0-) = V_{0C} = v_C(0) = v_C(0+)$$

$$i(0) = \frac{v_g(0) - v_C(0)}{R} = \frac{v_g(0) - v_C(0-)}{R} = \frac{10-10}{500} = 0 \text{ A}$$

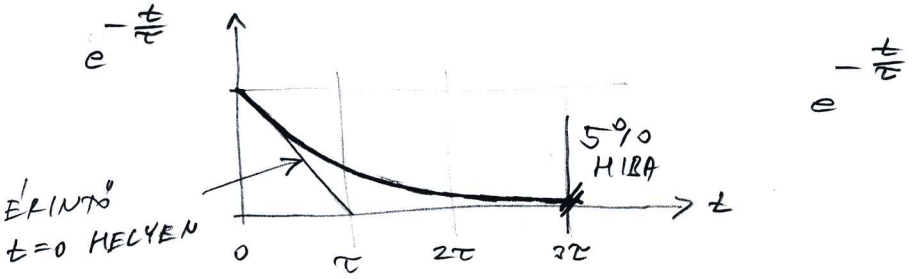
$$i(0) = A + 19{,}05 \cos(17{,}6°) = 0 \Rightarrow A = -18{,}2$$

$$i(t) = -18{,}2\, e^{-\frac{t}{RC}} + 19{,}05 \cos(\omega t + 17{,}6°) \text{ mA}$$

$\tau = RC = 0{,}5$ ms

$f = 1$ kHz

2.4 EMLÉKEZTETŐ AZ EXP. FGV. FELRAJZOLÁSÁHOZ:



$i(t)$ ALAKJA: