1. Csörgő 9. hét

1.1. feladat

$$f(x,y) = \sin\left(x^2+y^2\right)$$

Legyen $x = r\cos\theta, y = r\sin\theta$, ekkor $D(r,\theta) = r$. Az integrál

$$\iint_R \sin\left(x^2+y^2\right) \mathrm{d}(x,y) = \int_0^1 \int_0^{\frac{\pi}{2}} r\sin r^2 \,\mathrm{d}\theta\,\mathrm{d}r = \frac{1}{2}\pi\sin^2\frac{1}{2}$$

1.2. feladat

$$f(x,y) = x^2+y^2$$

Legyen $x = r\cos\theta, y = r\sin\theta$, ekkor $D(r,\theta) = r$. Az integrál

$$\iint_R \left(x^2+y^2\right) \mathrm{d}(x,y) = \int_0^{2\pi} \int_\theta^{2\theta} r^3 \,\mathrm{d}r\,\mathrm{d}\theta = 24\pi^5$$

1.3. feladat

$$f(x,y) = x^2+y^2$$

Legyen $x = r\cos\theta, y = r\sin\theta$, ekkor $D(r,\theta) = r$.

(0,0) középpontú kör

$$\iint_R \left(x^2+y^2\right) \mathrm{d}(x,y) = \int_0^2 \int_0^{2\pi} r^3 \,\mathrm{d}\theta\,\mathrm{d}r = 8\pi$$

(2,0) középpontú kör

$$\int_R \left((x+2)^2+y^2\right) \mathrm{d}(x,y) = \int_0^2 \int_0^{2\pi} \left(r^3+4r^2\cos\theta+4r\right) \mathrm{d}\theta\,\mathrm{d}r = 24\pi$$

(0,2) középpontú kör

$$\int_R \left(x^2+(y+2)^2\right) \mathrm{d}(x,y) = \int_0^2 \int_0^{2\pi} \left(r^3+4r^2\sin\theta+4r\right) \mathrm{d}\theta\,\mathrm{d}r = 24\pi$$

1.4. feladat

A terület kiszámításához bontsuk a tartományt két normáltartományra. Legyen

$$R = \left\{(x,y)\in\mathbb{R}^2 \middle| y \in \left[0, \frac{\sqrt{3}}{2}\right], x \in \left[\frac{1}{\sqrt{3}}y, \sqrt{3}y\right]\right\} \cup$$

$$\cup \left\{(x,y)\in\mathbb{R}^2 \middle| x \in \left[\frac{1}{2}, \frac{3}{2}\right], y \in \left[\frac{\sqrt{3}}{2}, \sqrt{1-(x-1)^2}\right]\right\}.$$

Ekkor a tartomány területe

$$\iint_R \mathrm{d}R = \int_0^{\frac{\sqrt{3}}{2}} \int_{\frac{1}{\sqrt{3}}y}^{\sqrt{3}y} \mathrm{d}x\,\mathrm{d}y + \int_{\frac{1}{2}}^{\frac{3}{2}} \int_{\frac{\sqrt{3}}{2}}^{\sqrt{1-(x-1)^2}} \mathrm{d}y\,\mathrm{d}x = \frac{\pi}{6}.$$

1.5. feladat

A feladat az

$$\iint_{x^2+y^2=1} \left(4-y^2\right) \mathrm{d}(x,y)$$

kettős integrál kiszámítása.

$$\iint_{x^2+y^2=1} \left(4-y^2\right) \mathrm{d}(x,y) = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(4-y^2\right) \mathrm{d}y \, \mathrm{d}x = \frac{15}{4}\pi$$

1.6. feladat

1.6.1. a)

$$f(x,y,z) = 2x - 4y + 6z - 3 \qquad R = [0,2] \times [0,1] \times [0,3]$$

$$\iiint_R f(x,y,z) \, \mathrm{d}(x,y,z) = \int_0^2 \int_0^1 \int_0^3 (2x - 4y + 6z - 3) \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x = 36$$

1.6.2. b)

$$f(x,y,z) = x - 2y + 4z \quad R = \left\{(x,y,z) \in \mathbb{R}^3 \middle| x \in [0,1], y \in [0,1-x], z \in [0,1-x-y]\right\}$$

$$\iiint_R f(x,y,z) \, \mathrm{d}(x,y,z) = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} (x - 2y + 4z) \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x = \frac{1}{8}$$

1.6.3. c)

$$f(x,y,z) = x^2 + 2y + z^2 \qquad R = \left\{(x,y,z) \in \mathbb{R}^3 \middle| x, y \in [0,2], z \in [0,2-x]\right\}$$

$$\iiint_R f(x,y,z) \, \mathrm{d}(x,y,z) = \int_0^2 \int_0^2 \int_0^{2-x} \left(x^2 + 2y + z^2\right) \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x = \frac{40}{3}$$

1.6.4. d)

$$f(x,y,z) = e^{x+y+z} \qquad R = \left\{(x,y,z) = \in \mathbb{R}^3 \middle| x \in [0,1], y \in [0,x], z \in [0,x+y]\right\}$$

$$\iiint_R f(x,y,z) \, \mathrm{d}(x,y,z) = \int_0^1 \int_0^x \int_0^{x+y} e^{x+y+z} \, \mathrm{d}z \, \mathrm{d}y \, \mathrm{d}x = \frac{1}{8}(e-1)^3(e+3)$$

1.7. feladat

$$f(x,y,z) = x^2 + y^2 \qquad R = \left\{(x,y,z) \in \mathbb{R}^3 \middle| x^2 + y^2 \in [0,4], z \in [0,8]\right\}$$

Áttérve hengerkoordinátákra

$$R' = \left\{(r,\theta,z) \in \mathbb{R}^3 \middle| r \in [0,2], \theta \in [0,2\pi], z \in [0,8]\right\}.$$

Ekkor az integrál

$$\iiint_{R'} \left(r^2 \cos^2\theta + r^2 \sin^2\theta\right) r \, \mathrm{d}(r,\theta,z) = \int_0^2 \int_0^{2\pi} \int_0^8 r^3 \, \mathrm{d}z \, \mathrm{d}\theta \, \mathrm{d}r = \int_0^2 r^3 \, \mathrm{d}r \int_0^{2\pi} \mathrm{d}\theta \int_0^8 \mathrm{d}z = 64\pi.$$

1.8. feladat

$$f(x,y,z) = x^2 + y^2 \qquad R = \left\{(x,y,z) \in \mathbb{R}^3 \middle| z \in [0,M], x^2+y^2 \in [0, M(1-z)]\right\}$$

Áttérve hengerkoordinátákra

$$R' = \left\{(r,\theta,z) \in \mathbb{R}^3 \middle| \theta \in [0,2\pi], z \in [0,M], r \in \left[0, \sqrt{M(1-z)}\right]\right\}.$$

Ekkor az integrál

$$\iint_{R'} \left(r^2\cos^2\theta + r^2\sin^2\theta\right) r \,\mathrm{d}(r,\theta,z) = \int_0^{2\pi}\int_0^M\int_0^{\sqrt{M(1-z)}} r^3 \,\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\theta =$$

$$= \frac{1}{2}\pi \int_0^M r^4 \Big|_0^{\sqrt{M(1-z)}} \mathrm{d}z = \frac{1}{2}M^2\pi\left(z^2 - 2z + 1\right)\Big|_0^M = \frac{1}{6}M^5\pi - \frac{1}{2}M^4\pi + \frac{1}{2}M^3\pi.$$

1.9. feladat

$$f(x,y,z) = x^2yz \qquad R = \left\{(x,y,z) \in \mathbb{R}^3 \middle| x,y,z \geq 0, x^2+y^2+z^2 \in [0,1]\right\}$$

Áttérve gömbi polárkoordinátákra

$$R' = \left\{(r,\theta,\varphi) \in \mathbb{R}^3 \middle| r \in [0,1], \theta,\varphi \in \left[0, \frac{\pi}{2}\right]\right\}.$$

Ekkor az integrál

$$\iiint_{R'} r^2\cos^2\theta\sin^2\varphi r\sin\theta\sin\varphi r\cos\varphi r^2\sin\varphi \,\mathrm{d}(r,\theta,\varphi) =$$

$$= \int_0^1\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} r^6\cos^2\theta\sin\theta\sin^4\varphi\cos\varphi \,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r = \int_0^1 r^6\,\mathrm{d}r \int_0^{\frac{\pi}{2}}\cos^2\theta\sin\theta\,\mathrm{d}\theta \int_0^{\frac{\pi}{2}}\sin^4\varphi\cos\varphi\,\mathrm{d}\varphi =$$

$$= \frac{1}{105}.$$

1.10. feladat

Az R sugarú 2α nyíásszögű gömbcikk térfogata

$$\int_0^R\int_{-\alpha}^{\alpha}\int_0^{\pi} r^2\sin\varphi\,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r = \frac{4}{3}R^3\alpha.$$

1.11. feladat

$$\iint_{x^2+y^2=27-2x^2-2y^2}\left(27-2x^2-2y^2-x^2-y^2\right)\mathrm{d}(x,y) = \iint_{x^2+y^2=9}\left(27-3x^2-3y^2\right)\mathrm{d}(x,y) =$$

$$= \int_0^3\int_0^{2\pi}\left(27r-3r^3\right)\mathrm{d}\theta\,\mathrm{d}r = \frac{243}{2}\pi$$

1.12. feladat

$$\varrho(x,y) = R - y$$

A tömegközéppont koordinátái

$$M\left(\frac{\iint_T x\varrho(x,y)\,\mathrm{d}(x,y)}{\iint_T \varrho(x,y)\,\mathrm{d}(x,y)}, \frac{\iint_T y\varrho(x,y)\,\mathrm{d}(x,y)}{\iint_T \varrho(x,y)\,\mathrm{d}(x,y)}\right).$$

1.12.1. a)

$$T = \Big\{(x,y) \in \mathbb{R}^2 \Big| x^2 + y^2 \in \big[0, R^2\big]\Big\}$$

$$\iint_T \varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^{2\pi} (R - r\sin\theta) r\,\mathrm{d}\theta\,\mathrm{d}r = \int_0^R Rr\theta + r^2\cos\theta \Big|_0^{2\pi} \mathrm{d}r =$$

$$= \int_0^R 2Rr\pi\,\mathrm{d}r = Rr^2\pi \Big|_0^R = R^3\pi$$

$$\iint_T x\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^{2\pi} r\cos\theta(R - r\sin\theta) r\,\mathrm{d}\theta\,\mathrm{d}r =$$

$$= \int_0^R \int_0^{2\pi} \big(Rr^2\cos\theta - r^3\sin\theta\cos\theta\big)\,\mathrm{d}\theta\,\mathrm{d}r = 0$$

$$\iint_T y\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^{2\pi} r\sin\theta(R - r\sin\theta) r\,\mathrm{d}\theta\,\mathrm{d}r = \int_0^R \int_0^{2\pi} \big(Rr^2\sin\theta - r^3\sin^2\theta\big)\,\mathrm{d}\theta\,\mathrm{d}r =$$

$$= -\frac{1}{4}R^4\pi$$

Ebből a tömegközéppont

$$M\bigg(0, -\frac{1}{4}R\bigg).$$

1.12.2. b)

$$T = \Big\{(x,y) \in \mathbb{R}^2 \Big| y \in [0,R], x^2 + y^2 \in \big[0, R^2\big]\Big\}$$

$$\iint_T \varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^{\pi} (R - r\sin\theta) r\,\mathrm{d}\theta\,\mathrm{d}r = \int_0^R Rr\theta + r^2\cos\theta \Big|_0^{\pi} \mathrm{d}r =$$

$$= \int_0^R Rr\pi\,\mathrm{d}r = \frac{1}{2}Rr^2\pi \Big|_0^R = \frac{1}{2}R^3\pi$$

$$\iint_T x\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^{\pi} r\cos\theta(R - r\sin\theta) r\,\mathrm{d}\theta\,\mathrm{d}r =$$

$$= \int_0^R \int_0^{\pi} \big(Rr^2\cos\theta - r^3\sin\theta\cos\theta\big)\,\mathrm{d}\theta\,\mathrm{d}r = 0$$

$$\iint_T y\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^{\pi} r\sin\theta(R - r\sin\theta) r\,\mathrm{d}\theta\,\mathrm{d}r = \int_0^R \int_0^{\pi} \big(Rr^2\sin\theta - r^3\sin^2\theta\big)\,\mathrm{d}\theta\,\mathrm{d}r =$$

$$= -\frac{1}{8}R^4\pi$$

Ebből a tömegközéppont

$$M\bigg(0, -\frac{1}{8}R\bigg).$$

1.12.3. c)

$$\Gamma = \Big\{ \gamma(t) = \big(\cos t, \sin t \big) \Big| t \in [0, \pi] \Big\}$$

$$\int_\Gamma \varrho(x,y)\,\mathrm{d}(x,y) = \int_0^\pi \varrho(R\cos t, R\sin t)\sqrt{R^2\sin^2 t + R^2\cos^2 t}\,\mathrm{d}t = \int_0^\pi (R - R\sin t)R\,\mathrm{d}t = (\pi - 2)R^2$$

$$\int_\Gamma x\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^\pi R\cos t(R - R\sin t)R\,\mathrm{d}t = 0$$

$$\int_\Gamma y\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^\pi R\sin t(R - R\sin t)R\,\mathrm{d}t = \int_0^\pi R^3\sin t - R^3\sin^2 t = \left(2 - \frac{\pi}{2}\right)R^3$$

Ebből a tömegközéppont

$$M\left(0, \frac{4-\pi}{2\pi - 4}R\right).$$

1.13. feladat

$$T = \big\{ (x,y) \in \mathbb{R}^2 \big| x^2 + y^2 \in \big[0, R^2\big], y \in [0, R] \big\}$$

A tömegközéppont koordinátái

$$M\left(\frac{\iint_T x\varrho(x,y)\,\mathrm{d}(x,y)}{\iint_T \varrho(x,y)\,\mathrm{d}(x,y)}, \frac{\iint_T y\varrho(x,y)\,\mathrm{d}(x,y)}{\iint_T \varrho(x,y)\,\mathrm{d}(x,y)} \right).$$

1.13.1. a)

$$\varrho(x,y) = k$$

$$\iint_T \varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^\pi kr\,\mathrm{d}\theta\,\mathrm{d}r = \frac{1}{2}kR^2\pi$$

$$\iint_T x\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^\pi kr^2\cos\theta\,\mathrm{d}\theta\,\mathrm{d}r = 0$$

$$\iint_T y\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^\pi kr^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}r = \frac{2}{3}kR^3$$

Ebből a tömegközéppont

$$M\left(0, \frac{4}{3\pi}R\right).$$

1.13.2. b)

$$\varrho(x,y) = k\sqrt{x^2 + y^2}$$

$$\iint_T \varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^\pi kr^2\,\mathrm{d}\theta\,\mathrm{d}r = \frac{1}{3}kR^3\pi$$

$$\iint_T x\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^\pi kr^3\cos\theta\,\mathrm{d}\theta\,\mathrm{d}r = 0$$

$$\iint_T y\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^\pi kr^3\sin\theta\,\mathrm{d}\theta\,\mathrm{d}r = \frac{1}{2}kR^4$$

Ebből a tömegközéppont

$$M\left(0, \frac{3}{2\pi}R\right).$$

1.13.3. c)

$$\varrho(x,y) = k|y|$$

$$\iint_T \varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^\pi kr^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}r = \frac{2}{3}kR^3$$

$$\iint_T x\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^\pi kr^3 \sin\theta\cos\theta\,\mathrm{d}\theta\,\mathrm{d}r = 0$$

$$\iint_T y\varrho(x,y)\,\mathrm{d}(x,y) = \int_0^R \int_0^\pi kr^3 \sin^2\theta\,\mathrm{d}\theta\,\mathrm{d}r = \frac{1}{8}kR^4\pi$$

Ebből a tömegközéppont

$$M\left(0, \frac{3}{16\pi}R\right).$$

1.14. feladat

$$\varrho(r,\theta) = k \qquad T = \left\{(r,\theta) \in \mathbb{R}^2 \middle| \theta \in \left[\frac{1}{6}\pi, \frac{1}{3}\pi\right], r \in [0, 2\cos\theta]\right\}$$

$$\iint_T \varrho(r,\theta) r d(r,\theta) = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \int_0^{2\cos\theta} kr\,\mathrm{d}r\,\mathrm{d}\theta = \frac{1}{6}k\pi$$

$$\iint_T r\cos\theta\varrho(r,\theta) r d(r,\theta) = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \int_0^{2\cos\theta} kr^2\cos\theta\,\mathrm{d}r\,\mathrm{d}\theta = \frac{2\pi - \sqrt{3}}{12}k$$

$$\iint_T r\sin\theta\varrho(r,\theta) r d(r,\theta) = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \int_0^{2\cos\theta} kr^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta = \frac{1}{3}k$$

Ebből a tömegközéppont

$$M\left(\frac{2\pi - \sqrt{3}}{2\pi}, \frac{2}{\pi}\right).$$

1.15. feladat

$$T = \left\{(x,y) \in \mathbb{R}^2 \middle| x \in \left[0, \sqrt{1-y^2-z^2}\right], y \in \left[0, \sqrt{1-z^2}\right], z \in [0,1]\right\}$$

$$\varrho(x,y,z) = xyz$$

A nyolcadgömb tömegét az

$$\iiint_T \varrho(x,y,z)\,\mathrm{d}(x,y,z)$$

integrállal kaphatjuk meg.

$$\iiint_T \varrho(x,y,z)\,\mathrm{d}(x,y,z) = \iiint_T xyz\,\mathrm{d}(x,y,z) = \int_0^1 \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} r^5 \sin^3\varphi\cos\varphi\sin\theta\cos\theta\,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r =$$

$$= \frac{1}{48}$$

1.16. feladat

$$\varrho(r,\theta,\varphi) = kr|\cos\varphi| \qquad T = \left\{(r,\theta,\varphi)\in\mathbb{R}^3 \middle| r\in[0,3], \theta\in[0,2\pi], \varphi\in\left[0,\frac{\pi}{2}\right] Bigg\}$$

$$\iiint_T \varrho(r,\theta,\varphi) r^2\sin\varphi \,\mathrm{d}(r,\theta,\varphi) = \int_0^3\int_0^{2\pi}\int_0^{\frac{\pi}{2}} kr^3\sin\varphi\cos\varphi \,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r = \frac{81}{4}k\pi$$

$$\iiint_T r\sin\varphi\cos\theta\varrho(r,\theta,\varphi) r^2\sin\varphi \,\mathrm{d}(r,\theta,\varphi) = \int_0^3\int_0^{2\pi}\int_0^{\frac{\pi}{2}} kr^4\sin^2\varphi\cos\varphi\cos\theta \,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r = 0$$

$$\iiint_T r\sin\varphi\sin\theta\varrho(r,\theta,\varphi) r^2\sin\varphi \,\mathrm{d}(r,\theta,\varphi) = \int_0^3\int_0^{2\pi}\int_0^{\frac{\pi}{2}} kr^4\sin^2\varphi\cos\varphi\sin\theta \,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r = 0$$

$$\iiint_T r\cos\varphi\varrho(r,\theta,\varphi) r^2\sin\varphi \,\mathrm{d}(r,\theta,\varphi) = \int_0^3\int_0^{2\pi}\int_0^{\frac{\pi}{2}} kr^4\sin\varphi\cos^2\varphi \,\mathrm{d}\varphi\,\mathrm{d}\theta\,\mathrm{d}r = \frac{162}{5}k\pi$$

Ebből a tömegközéppont

$$M\left(0,0,\frac{8}{5}\right).$$

1.17. feladat

$$F(x,y) = (y,x) \qquad \Gamma = \left\{(-1,1)\to(2,3)\right\}$$

Vegyük észre, hogy a vektormező potenciálos, potenciálja $f(x,y) = xy$ így

$$\int_\Gamma F(\mathbf{r})\,\mathrm{d}\mathbf{r} = 7.$$

1.18. feladat

$$F(x,y) = \left(x^2-2xy, y^2-2xy\right) \qquad \Gamma = \left\{\gamma(t) = \left(t,t^2\right) \middle| t\in[0,1]\right\}$$

$$\int_\Gamma F(\mathbf{r})\,\mathrm{d}\mathbf{r} = \int_0^1 \left\langle \left(t^2-2t^3, t^4-2t^3\right), \left(1,2t\right)\right\rangle \mathrm{d}t = -\frac{7}{30}$$

1.19. feladat

$$F(x,y) = \left(x^2+y^2, x^2-y^2\right) \qquad \Gamma = \left\{\gamma(t) = (t, 1-|1-t|) \middle| t\in[0,2]\right\}$$

$$\int_\Gamma F(\mathbf{r})\underline{r} = \int_0^1 \left\langle \left(2t^2,0\right),\left(1,1\right)\right\rangle \mathrm{d}t + \int_1^2 \left\langle \left(t^2+(2-t)^2, t^2-(2-t)^2\right), \left(1,-1\right)\right\rangle \mathrm{d}t =$$
$$= \frac{4}{3}$$

1.20. feladat

$$F(x,y) = \left(\frac{x+y}{x^2+y^2}, \frac{x-y}{x^2+y^2}\right) \qquad \Gamma = \left\{\gamma(t) = (R\cos t, R\sin t) \middle| t\in[0,2\pi]\right\}$$

$$\int_\Gamma F(\mathbf{r})\,\mathrm{d}\mathbf{r} = \int_0^{2\pi} \left\langle \left(\frac{1}{R}\cos t + \frac{1}{R}\sin t, \frac{1}{R}\cos t - \frac{1}{R}\sin t\right), \left(R\sin t, R\cos t\right)\right\rangle \mathrm{d}t =$$
$$= \int_0^{2\pi} \left(\cos^2 t - \sin^2 t - 2\sin t\cos t\right)\mathrm{d}t = 0$$

1.21. feladat

$$F(x,y) = \big(x^2y - xy\big) \qquad \Gamma = \Big\{\gamma(t) = \big(t^3, t^4\big)\Big| t \in [0,1]\Big\}$$

$$\int_\Gamma F(\mathbf{r})\,\mathrm{d}\mathbf{r} = \int_0^1 \big\langle \big(t^{10}, -t^7\big), \big(3t^2, 4t^3\big)\big\rangle\,\mathrm{d}t = -\frac{19}{143}$$

1.22. feladat

$$F(x,y,z) = \big(y+z, -x^2, -4y^2\big) \qquad \Gamma = \Big\{\gamma(t) = \big(t, t^2, t^4\big)\Big| t \in [0,1]\Big\}$$

$$\int_\Gamma F(\mathbf{r})\,\mathrm{d}\mathbf{r} = \int_0^1 \big\langle \big(t^2 + t^4, -t^2, -4t^8\big), \big(1, 2t, 4t^3\big)\big\rangle\,\mathrm{d}t = -\frac{13}{10}$$

1.23. feladat

$$F(x,y,z) = \big(x^2, xy, z^2\big) \qquad \Gamma = \left\{\gamma(t) = \big(\sin t, \cos t, t^2\big)\middle| t \in \left[0, \frac{\pi}{2}\right]\right\}$$

$$\int_\Gamma F(\mathbf{r})\,\mathrm{d}\mathbf{r} = \int_0^{\frac{\pi}{2}} \big\langle \big(\sin^2 t, \sin t\cos t, t^4\big), \big(\cos t, -\sin t, 2t\big)\big\rangle\,\mathrm{d}t = \frac{\pi^6}{192}$$

1.24. feladat

$$F(x,y,z) = (x, y, -z) \qquad \Gamma = \left\{\gamma(t) = \left(t^2, t\frac{1}{t}\right)\middle| t \in [1,2]\right\}$$

Vegyük észre, hogy F potenciálos, potenciálja $f(x,y,z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 - \frac{1}{2}z^2$. Ekkor

$$\int_\Gamma F(\mathbf{r})\,\mathrm{d}\mathbf{r} = f\left(4, 2, \frac{1}{2}\right) - f(1,1,1) = \frac{75}{8}$$