

$$1) \int (2e^x - 2\frac{x}{x^2} + \frac{1}{2x}) dx = \int 2e^x - \frac{2}{x} + \frac{1}{2x} dx = 2e^x - 2\ln|x| + \frac{1}{2}\ln|x| + C$$

$$2) \int (\frac{\cosh x}{x} - \frac{3}{\cosh x} + \frac{1}{x^2}) dx = \frac{\sinh x}{x} - 3\th x + 2\operatorname{arsh} x + C$$

$$3) \int (1+\sqrt{x})^4 dx = \text{not a polynomial}$$

$$4) \int (\sqrt{x}+1)(x-\sqrt{x}+1) dx = \int (\sqrt{x}+1)(x-(x-1)) dx = \int (\sqrt{x}+1)(x-x+1) dx = \int (\sqrt{x}+1) dx = \frac{2}{3}\sqrt{x^3} + x + C$$

$$5) \int (1+\sqrt{x})^4 dx = \int (1+\sqrt{x})^2 (1+\sqrt{x})^2 dx = \int (1+2\sqrt{x}+x)(1+2\sqrt{x}+x) dx = \int (1+4\sqrt{x}+5x+2x\sqrt{x}+x^2) dx = \int (1+4\sqrt{x}+5x+2x^{3/2}+x^2) dx = \frac{x^3}{3} + 3x^2 + \frac{8}{5}\sqrt{x^5} + \frac{4}{3}\sqrt{x^3} + x + C$$

$$6) \int \frac{x^2-5x}{x^3} dx = \int (\frac{x^2}{x^3} - \frac{5x}{x^3}) dx = \int (\frac{1}{x} - \frac{5}{x^2}) dx = \ln|x| + \frac{5}{x} + C$$

$$7) \int \frac{x^2+1}{x^2+1} dx = \int 1 dx = x + C$$

$$8) \int \frac{1+2x^2}{x^2(1+x^2)} dx = \int (\frac{1}{x^2} + \frac{2x^2}{x^2(1+x^2)}) dx = \int (\frac{1}{x^2} + \frac{2}{1+x^2}) dx = -\frac{1}{x} + 2\arctan x + C$$

$$9) \int \frac{\sqrt{x^2-9}-1}{x^2-1} dx = \int (\frac{\sqrt{x^2-9}}{x^2-1} - \frac{1}{x^2-1}) dx = 3x - \operatorname{arsh}(x) + C : x > 1$$

$$10) \int \frac{1+\cos(2x)}{1+\cos(x)-\sin^2(x)} dx = \int \frac{1+\cos(2x)-\sin(x)+\sin(x)}{1+\cos(x)-\sin^2(x)} dx = \int \frac{\sin x}{1+\cos(x)-\sin^2(x)} dx = \int \frac{\sin x}{2\cos(x)} dx = \frac{1}{2} \int \frac{\sin x}{\cos(x)} dx = -\frac{1}{2} \ln|\cos(x)| + C$$

$$= \int 1 + \frac{\sin(x)}{2\cos(x)} dx = \int 1 dx + \frac{1}{2} \int \frac{\sin(x)}{\cos(x)} dx = x + \frac{1}{2} \int \frac{-\cos(x)}{\cos(x)} dx = x - \frac{1}{2} \ln|\cos(x)| + C$$

$$g(x) = \sin(x) \quad g'(x) = \cos(x)$$

$$4'(x) = \frac{1}{\cos^2(x)} \quad 4(x) = \tan(x)$$

$$= \frac{1}{2} \int \cos(x) \cdot \frac{\sin(x)}{\cos^2(x)} dx + x + C = (-\frac{1}{2}) \cdot \cos(x) + x + C = x - \frac{1}{2} \cos(x) + C$$

$$11) \int \frac{x+\cos^2 x}{x+\cos^2 x} dx = \int 1 dx = x + C$$

$$12) \int x^3 \sin x dx = -\frac{1}{4} \cos(x) + \int 3x^2 (\cos(x)) dx = -\frac{1}{4} \cos(x) + 3x^2 \sin x + \int 6x \sin x dx =$$

$$g(x) = x^3 \quad g'(x) = 3x^2 \quad g(x) = 3x^2 \quad g'(x) = 6x \quad g(x) = 6x \quad g'(x) = 6$$

$$4'(x) = \sin x \quad 4(x) = -\cos(x) \quad 4'(x) = \cos(x) \quad 4(x) = \sin(x) \quad 4'(x) = \cos(x) \quad 4(x) = \sin(x)$$

$$= (-\frac{1}{4} \cos(x) + 3x^2 \sin x - 6x \sin x) + \int 6 \sin(x) dx = -\frac{1}{4} \cos(x) + 3x^2 \sin x - 6x \sin x + 6 \cos(x) + C$$

$$13) \int x \cdot 5^x dx = x \frac{5^x}{\ln 5} - \int \frac{5^x}{\ln 5} dx = x \frac{5^x}{\ln 5} - \frac{1}{\ln 5} \cdot \frac{5^x}{\ln 5} + C = \frac{5^x}{\ln 5} (x - \frac{1}{\ln 5})$$

$$g(x) = x \quad g'(x) = 1$$

$$4'(x) = 5^x \quad 4(x) = \frac{5^x}{\ln 5}$$

$$13) \int (x^3 - 3x - 1)e^x dx = (x^3 - 3x - 1)e^x - \int (3x^2 - 3)e^x dx = (x^3 - 3x - 1)e^x - \int 6x e^x dx$$

$$g(x) = x^3 - 3x - 1 \quad \begin{cases} g'(x) = 3x^2 - 3 \\ f(x) = e^x \end{cases} \quad g(x) = 3x^2 - 3 \quad \begin{cases} g'(x) = 6x \\ f(x) = e^x \end{cases}$$

$$= (x^3 - 3x - 1)e^x - (3x^2 - 3)e^x - \int 6x e^x dx = e^x(x^3 - 3x - 1 - 3x^2 + 3 + 6x) - \int 6e^x dx$$

$$g(x) = 6x \quad \begin{cases} g'(x) = 6 \\ f(x) = e^x \end{cases}$$

$$= e^x(x^3 - 3x^2 + 3x - 4) - 6e^x = e^x(x^3 - 3x^2 + 3x - 10) + C$$

$$14) \int (x-1)^3 \log_2 x dx = \log_2 x \cdot \frac{(x-1)^4}{4} - \int \frac{1}{x \ln 2} \cdot \frac{(x-1)^4}{4} dx = \frac{1}{4} \log_2 x (x-1)^4 - \frac{1}{4 \ln 2} \int \frac{(x-1)^4}{x} dx$$

$$g(x) = \log_2 x \quad \begin{cases} g'(x) = \frac{1}{x \ln 2} \\ f(x) = (x-1)^4 \end{cases}$$

$$= \frac{1}{4} \log_2 x (x-1)^4 - \frac{1}{4 \ln 2} \int \frac{(x^4 - 4x^3 + 6x^2 - 4x + 1)}{x} dx$$

$$= \frac{1}{4} \log_2 x (x-1)^4 - \frac{1}{4 \ln 2} \int (x^3 - 4x^2 + 6x - 4 + \frac{1}{x}) dx$$

$$= \frac{1}{4} \log_2 x (x-1)^4 - \frac{1}{4 \ln 2} \left(\frac{x^4}{4} - \frac{4x^3}{3} + \frac{6x^2}{2} - 4x + \ln|x| \right) + C$$

$$15) \int \ln(x^2 + 1) dx = \int \ln(x^2 + 1) \cdot 2x \cdot \frac{1}{2x} dx =$$

$$e^{\ln(x^2+1)} = x^2 + 1$$

$$16) \int x \arctan x dx = \frac{1}{2} \int \frac{1}{1+x^2} \cdot x^2 dx = \frac{1}{2} \int \frac{x^2}{1+x^2} dx = \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx$$

$$g(x) = \arctan x \quad \begin{cases} g'(x) = \frac{1}{1+x^2} \\ f(x) = x \end{cases}$$

$$17) \int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx = e^x \sin x - (e^x \cos x + \int e^x \sin x dx)$$

$$g(x) = \sin x \quad \begin{cases} g'(x) = \cos x \\ f(x) = e^x \end{cases} \quad g(x) = \cos x \quad \begin{cases} g'(x) = -\sin x \\ f(x) = e^x \end{cases}$$

$$2 \int e^x \sin x dx = e^x (\sin x - \cos x) + C \Rightarrow \int e^x \sin x dx = \frac{e^x (\sin x - \cos x)}{2} + C$$

$$15) \int \ln(x^2 + 1) dx = \int 1 \cdot \ln(x^2 + 1) dx = x \ln(x^2 + 1) - \int \frac{2x^2}{x^2 + 1} dx =$$

$$g(x) = \ln(x^2 + 1) \quad \begin{cases} g'(x) = \frac{1}{x^2 + 1} \cdot 2x \\ f(x) = 1 \end{cases}$$

$$= x \ln(x^2 + 1) - 2 \int \frac{x^2 + 1 - 1}{x^2 + 1} dx = x \ln(x^2 + 1) - 2 \int \left(1 - \frac{1}{x^2 + 1} \right) dx = x \ln(x^2 + 1) - 2(x - \arctan x) + C$$

$$= x \ln(x^2 + 1) - 2x + 2 \arctan x + C$$

$$16) \int x \arctan(x) dx = \frac{x^2}{2} \arctan(x) - \int \frac{1}{x^2+1} \cdot \frac{x^2}{2} dx = \frac{x^2}{2} \arctan(x) - \frac{1}{2} \int \frac{x^2+1-1}{x^2+1} dx$$

$$g(x) = \arctan(x) \quad g'(x) = \frac{1}{x^2+1}$$

$$f'(x) = x \quad f(x) = \frac{x^2}{2}$$

$$= \frac{x^2}{2} \arctan(x) - \frac{1}{2} \int 1 - \frac{1}{x^2+1} dx = \frac{x^2}{2} \arctan(x) - \frac{1}{2} (x - \arctan(x))$$

$$= \frac{1}{2} (\arctan(x)(x^2+1) - x) + c$$

$$18) \int \sin(x) \cdot \cos(x) dx = \sin(x) \cdot \sin(x) - \int \cos(x) \sin(x) dx = \sin(x) \sin(x) - \cos(x) \cos(x) + \int \sin(x) \cos(x) dx$$

$$g(x) = \sin(x) \quad g'(x) = \cos(x)$$

$$f(x) = \cos(x) \quad f'(x) = -\sin(x)$$

$$\Rightarrow 2 \int \sin(x) \cos(x) dx = \sin(x) \sin(x) - \cos(x) \cos(x)$$

$$\int \sin(x) \cos(x) dx =$$

$$18) \int (5x-4)^6 dx = \frac{1}{5} \frac{(5x-4)^7}{7} + c = \frac{1}{35} (5x-4)^7 + c$$

$$20) \int \cos(7x+2) dx = \frac{1}{7} \int \cos(7x+2) \cdot 7 dx = \frac{1}{7} \sin(7x+2) + c$$

$$21) \int \frac{1}{\ln^2(3x+1)} dx = \frac{1}{3} \int \frac{1}{\ln^2(3x+1)} \cdot 3 dx = \frac{1}{3} \ln(3x+1) + c$$

$$22) \int \frac{\ln^5 x}{x^2} dx = \int \left(\frac{1}{x^2} \cdot \ln^5 x \right) dx = \ln^5 x - \int 5 \ln^4 x \cdot \frac{1}{x} dx = \ln^5 x - 5 \frac{\ln^3 x}{3} + c$$

$$g(x) = \ln^5 x \quad g'(x) = 5 \ln^4 x$$

$$f(x) = \frac{1}{x} \quad f'(x) = -\frac{1}{x^2}$$

$$= \ln^5 x \left(1 - \frac{5}{3} \right) + c$$

$$23) \int \frac{x}{x^2-1} dx = \frac{1}{2} \int 2x (x^2-1)^{-1} dx = \frac{1}{2} \frac{(x^2-1)^{-1/2}}{-1/2} = -\frac{1}{(x^2-1)^{1/2}} + c$$

$$24) \int (x+1) \sqrt{x^2+2x-3} dx = \frac{1}{2} \int \sqrt{x^2+2x-3} (2x+2) dx = \frac{2}{3} (x^2+2x-3)^{3/2} + c$$

$$25) \int \frac{\ln^2 x}{x} dx = \frac{\ln^3 x}{3} + c$$

$$26) \int \frac{2x-1}{x^2-x+9} dx = \ln|x^2-x+9| + c$$

$$27) \int \frac{x^2}{x^2+1} dx = \frac{1}{3} \ln|x^2+1| + c$$

$$28) \int \frac{e^{x+1}}{e^x+x} dx = \ln|e^x+x| + c$$

$$29) \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = -\ln |\cos x| + c$$

$$30) \int \frac{1}{x \ln x} \, dx = \int \frac{1}{x} \cdot \frac{1}{\ln x} \, dx = \int \frac{\frac{1}{x}}{\ln x} \, dx = \ln |\ln x| + c$$

$$31) \int x \cdot e^{-x^2} \, dx = -\frac{1}{2} \int (-2x) \cdot e^{-x^2} = -\frac{1}{2} e^{-x^2}$$

$$32) \int \frac{\frac{d}{dx} x^3}{\sin^2 x} \, dx = \int \frac{d}{dx} x^3 \cdot \frac{1}{\sin^2 x} \, dx = \frac{\frac{d}{dx} x^3}{\sin^2 x} + c$$

$$33) \int 2^{\ln x} \cdot \ln x \, dx = \frac{2^{\ln x}}{\ln 2} + c$$

$$34) \int \frac{\arcsin^2 x}{\sqrt{1-x^2}} \, dx = \int \arcsin^2 x \cdot \frac{1}{\sqrt{1-x^2}} \, dx = \frac{\arcsin^3 x}{3} + c$$

$$35) \int (4x^2 + 4x + 5) \, dx = \frac{4x^3}{3} + 2x^2 + 5x + c$$

$$9) \int (\sin x)^4 \cos x \, dx = \frac{4}{5} \frac{1}{5} \sin^5 x + C$$

$$10) \int \frac{\cos \theta}{\sin^2 \theta} \, d\theta = \int \left(\cos \theta \cdot \frac{1}{\sin^2 \theta} \right) d\theta = \cos^2 \theta + C$$

$$12) \int \sin(2x-5) \, dx = \frac{1}{2} \int \sin(2x-5) \cdot 2 \, dx = \frac{1}{2} \cos(2x-5) + C$$

$$13) \int \frac{2x-6}{x^2-6x+8} \, dx = \ln|x^2-6x+8| + C$$

$$14) \int \frac{1}{e^x + e^{-x}} \, dx = \frac{1}{2} \int \frac{2}{e^x + e^{-x}} \, dx = \frac{1}{2} \int \frac{1}{\cosh x} \, dx$$

$$(e^x + e^{-x})' = e^x + e^{-x} = e^x + \frac{1}{e^x} = \frac{e^{2x} + 1}{e^x}$$

$$15) \int x \sin x \, dx = (-x) \cos x + \int \cos x \, dx = (-x) \cos x + \sin x + C$$

$$g(x) = x \quad g'(x) = 1$$

$$f(x) = \sin x \quad f'(x) = \cos x$$

$$16) \int x \cdot e^{3x} \, dx = \frac{1}{3} x e^{3x} - \frac{1}{3} \int e^{3x} \, dx = \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C = e^{3x} \left(\frac{1}{3} x - \frac{1}{9} \right) + C$$

$$g(x) = x \quad g'(x) = 1$$

$$f(x) = e^{3x} \quad f'(x) = \frac{1}{3} e^{3x}$$

$$17) \int x^2 e^{3x} dx = \frac{1}{3} x^2 e^{3x} - \frac{2}{3} \int x e^{3x} dx = \frac{1}{3} x^2 e^{3x} - \frac{2}{3} x e^{3x} + \frac{2}{9} \int e^{3x} dx = \frac{1}{9} e^{3x} (3x^2 - 2x - 1) + C$$

$$\begin{array}{l|l} g(x) = x^2 & g'(x) = 2x \\ f'(x) = e^{3x} & f(x) = \frac{1}{3} e^{3x} \end{array} \quad \begin{array}{l|l} g(x) = x & g'(x) = 1 \\ f'(x) = e^{3x} & f(x) = \frac{1}{3} e^{3x} \end{array}$$

$$18) \int \ln x dx = \int 1 \cdot \ln x dx = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + C = x(\ln x - 1) + C$$

$$\begin{array}{l|l} g(x) = \ln x & g'(x) = \frac{1}{x} \\ f'(x) = 1 & f(x) = x \end{array}$$

$$19) \int x \ln x dx = \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x^2 \cdot \frac{1}{x} dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C = \frac{1}{2} x^2 \left(\ln x - \frac{1}{2} \right) + C$$

$$\begin{array}{l|l} g(x) = \ln x & g'(x) = \frac{1}{x} \\ f'(x) = x & f(x) = \frac{1}{2} x^2 \end{array}$$

$$20) \int \arctan \frac{1}{5} dx = \frac{1}{5} \arctan \frac{1}{5} x + \frac{1}{5} \int \frac{1}{1 + 25x^2} dx = \frac{1}{5} \arctan \frac{1}{5} x + \frac{1}{5} \int \frac{5x}{1 + 25x^2} dx = \frac{1}{5} \arctan \frac{1}{5} x + \frac{1}{10} \ln |25x^2 + 1| + C$$

$$\begin{array}{l|l} g(x) = \arctan \frac{1}{5} x & g'(x) = \frac{1}{1 + 25x^2} \\ f'(x) = 1 & f(x) = x \end{array}$$

21) $\int e^x \sin x dx \rightarrow$ man nimmt Integrationsform (1. Ableit 17. Ableit)

22) $\int \frac{x+1}{x^2+3x+2} dx = \int \frac{x+1}{(x+1)(x+2)} dx$?

$$21) \int e^x \cdot \sin x \, dx \Rightarrow \text{man öppen regelaltern (1. eller 17. flödet)}$$

$$22) \int \frac{x+1}{x^2-3x+2} \, dx = \int \frac{x+1}{(x-1)(x-2)} \, dx \quad ?$$

$$23) \int \frac{1}{(3x-5)^8} = \frac{1}{3} \int (3x-5)^{-8} \cdot 3 = \frac{1}{3} \cdot \left(-\frac{1}{7}\right) \cdot (3x-5)^{-7} = \frac{-1}{21(3x-5)^7} + C$$

$$24) \int \cos\left(\frac{2x-1}{3}\right) \, dx = \frac{3}{2} \int \cos\left(\frac{2}{3}x - 1\right) \cdot \frac{2}{3} = \frac{3}{2} \sin\left(\frac{2x-1}{3}\right) + C$$

$$25) \int (3x^2-2)(3x^3-2x)^7 \, dx = (3x^3-2x)^8 \cdot \frac{1}{8} + C$$

$$26) \int (2^x \cdot (2+2^x)) \, dx = \int (2^{x+1} + 2^{2x}) \, dx = \int 2^{x+1} \, dx + \int 2^{2x} \, dx \\ = \frac{1}{\ln 2} \int (2^x + 2) \cdot 2^x \cdot \ln 2 \, dx = \frac{1}{3} (2^x + 2)^{\frac{3}{2}} \cdot \frac{1}{\ln 2} + C$$

$$27) \int \frac{2 \sin x}{3 \cos^3 x} \, dx = \frac{2}{3} \int (\cos^{-3} x \cdot \sin x) \, dx = 2 \cos^{-2} x + C = \frac{2}{\cos^2 x} + C$$

$$28) \int \frac{1}{\sin^2 x} \, dx \Rightarrow \text{man öppen regelaltern (1. eller 25. flödet)}$$

$$29) \int \frac{\cos 2x}{\sin^2 x + 3} \, dx = \ln |\sin^2 x + 3| + C$$

$$30) \int \frac{e^{\tan x}}{\sin^2 x} \, dx = e^{\tan x} + C$$

31-40 lektionstid och övning