1. Polcz 10. hét

1.1. feladat

$$\int \frac{x^2-7x+8}{x^2}dx = \int 1dx - 7\int \frac{1}{x}dx + 8\int \frac{1}{x^2}dx = x - 7\ln|x| - \frac{8}{x} + c$$

1.2. feladat

$$\int e^x \operatorname{sh} x dx = \int e^x \cdot \frac{e^x - e^{-x}}{2}dx = \int \frac{e^{2x}-1}{2}dx = \frac{e^{2x}}{4} - \frac{x}{2} + c$$

1.3. feladat

$$\int e^{5x+7}dx = \frac{1}{5}\int 5e^{5x+7}dx = \frac{e^{5x+7}}{5} + c$$

1.4. feladat

$$\int \frac{x-1}{x^2-6x+27}dx = \frac{1}{2}\int \frac{2x-6+4}{x^2-6x+27}dx = \frac{1}{2}\int \frac{2x-6}{x^2-6x+27}dx + 2\int \frac{1}{x^2-6x+27}dx =$$

$$= \frac{1}{2}\ln\left|x^2-6x+27\right| + 2\int \frac{1}{(x-3)^2+18}dx = \frac{1}{2}\ln\left|x^2-6x+27\right| + \frac{2}{18}\int \frac{1}{\left(\frac{x-3}{3\sqrt{2}}\right)^2+1}dx =$$

$$= \frac{1}{2}\ln\left|x^2-6x+27\right| + \frac{2}{3\sqrt{2}}\int \frac{\frac{1}{3\sqrt{2}}}{\left(\frac{x-3}{3\sqrt{2}}\right)^2+1}dx = \frac{1}{2}\ln\left|x^2-6x+27\right| + \frac{2}{3\sqrt{2}}\operatorname{arctg}\frac{x-3}{3\sqrt{2}} + c$$

1.5. feladat

$$\int_0^{\frac{\pi}{2}} \sin(3x)dx = -\frac{\cos(3x)}{3}\Big|_0^{\frac{\pi}{2}} = \frac{1}{3}$$

1.6. feladat

$$\int_{-1}^{1} \frac{1}{2x-4}dx = \frac{1}{2}\ln|2x-4|\Big|_{-1}^{1} = -\frac{\ln 3}{2}$$

1.7. feladat

$$\int_0^{\pi} \sin^3 x \cdot \cos x dx = \frac{1}{4}\int 4\sin^3 x \cdot \cos x dx = \frac{1}{4}\sin^4 x\Big|_0^{\pi} = 0$$

1.8. feladat

A függvény nem integrálható ezen az intervallumon, mert nem korlátos.

1.9. feladat

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 x \cdot \sin x dx = -\frac{1}{3}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -3\cos^2 x \cdot \sin x dx = -\frac{1}{3}\cos^3 x\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$$

1.10. feladat

$$\int_2^3 \frac{1}{(2x-1)^3}dx = -\frac{1}{4}\int_2^3 -4(2x-1)^{-3}dx = -\frac{1}{4}(2x-1)^{-2}\Big|_2^3 = \frac{4}{225}$$

1.11. feladat

$$\int_0^1 \frac{1}{x+1}dx = \ln|x+1|\Big|_0^1 = \ln 2$$

1.12. feladat

$$\int_0^1 x \cdot e^{\frac{x^2}{2}}dx = e^{\frac{x^2}{2}}\Big|_0^1 = e^{\frac{1}{2}} - 1$$

1.13. feladat

Lásd 1.5 feladat.

1.14. feladat

Lásd 1.6 feladat.

1.15. feladat

Lásd 1.7 feladat.

1.16. feladat

Lásd 1.8 feladat.

1.17. feladat

Lásd 1.9 feladat.

1.18. feladat

Lásd 1.10 feladat.

1.19. feladat

Lásd 1.11 feladat.

1.20. feladat

Lásd 1.12 feladat

1.21. feladat

$$\int_0^4 \sqrt{3x+4}dx = \frac{2}{9}\int_0^4 \frac{9}{2}(3x+4)^{\frac{1}{2}}dx = \frac{2}{9}(3x+4)^{\frac{3}{2}}\Big|_0^4 = \frac{112}{9}$$