

1. Polcz 7. hét

1.1. feladat

$$\left(\frac{\sqrt[3]{x^2} + x^5 - \sqrt{x}}{x^2}\right)' = \left(x^{-\frac{4}{3}} + x^3 - x^{-\frac{3}{2}}\right)' = -\frac{4}{3}x^{-\frac{7}{3}} + 3x^2 + \frac{3}{2}x^{-\frac{5}{2}}$$

1.2. feladat

$$\left(2x + \frac{x-1}{x} + \frac{1}{\sqrt[3]{x^7}}\right)' = \left(2x + 1 - x^{-1} + x^{-\frac{7}{3}}\right)' = 2 + x^{-2} - \frac{7}{3}x^{-\frac{10}{3}}$$

1.3. feladat

$$\left(\frac{x-4}{x-2} + \arctg x\right)' = \left(1 - \frac{2}{x-2} + \arctg x\right)' = \frac{2}{(x-2)^2} + \frac{1}{x^2+1} = \frac{3x^2 - 4x + 6}{(x-2)^2 \cdot (x^2+1)}$$

1.4. feladat

$$\left((e^x + 3) \sin x\right)' = (e^x \sin x)' + (3 \sin x)' = e^x \sin x + e^x \cos x + 3 \cos x$$

1.5. feladat

$$\left(\frac{\ln x}{x^2}\right)' = \frac{\frac{1}{x} \cdot x^2 - 2x \ln x}{x^4} = \frac{1 - 2 \ln x}{x^3}$$

1.6. feladat

$$\left(\lg x(x^2 + 6x + 1)\right)' = \frac{1}{x \ln 10} \cdot (x^2 + 6x + 1) + \lg x \cdot (2x + 6) = \frac{x^2 + 6x + 1 + (2x + 6) \ln x}{x \ln 10}$$

1.7. feladat

$$\left(e^{2x} + 3^{2-x} + \frac{1}{10^{3x}}\right)' = 2e^{2x} - \ln 3 \cdot 3^{2-x} - 3 \ln 10 \cdot 10^{-3x}$$

1.8. feladat

$$\begin{aligned} \left(\sin(2x) + \sin^2 x + \sin\left(\frac{x}{2} + 2\right)\right)' &= 2 \cos(2x) + 2 \sin x \cos x + \frac{1}{2} \cos\left(\frac{x}{2} + 2\right) = \\ &= 2 \cos(2x) + \sin(2x) + \frac{1}{2} \cos\left(\frac{x}{2} + 2\right) \end{aligned}$$

1.9. feladat

$$\begin{aligned} \left(x^2 \operatorname{arctg} \sqrt{x} + \arcsin \frac{3x-2}{5} \right)' &= 2x \operatorname{arctg} \sqrt{x} + x^2 \cdot \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} + \frac{3}{5} \cdot \frac{1}{\sqrt{1 - \left(\frac{3x-2}{5} \right)^2}} = \\ &= 2x \operatorname{arctg} \sqrt{x} + \frac{x^{\frac{3}{2}}}{2(1+x)} + \frac{3}{5} \cdot \frac{1}{\sqrt{1 - \left(\frac{3x-2}{5} \right)^2}} \end{aligned}$$

1.10. feladat

$$\begin{aligned} \left(\cos^2(2x-6) + \sin^3(\sqrt{x}-1) + \operatorname{tg}(x-7)^2 \right)' &= \\ = -4 \sin(2x-6) \cos(2x-6) + \frac{3 \sin^2(\sqrt{x}-1) \cos(\sqrt{x}-1)}{2\sqrt{x}} + \frac{2(x-7)}{\cos^2(x-7)^2} = \\ = -2 \sin(4x-12) + \frac{3 \sin^2(\sqrt{x}-1) \cos(\sqrt{x}-1)}{2\sqrt{x}} + \frac{2x-14}{\cos^2(x-7)^2} \end{aligned}$$

1.11. feladat

$$\begin{aligned} \left(\frac{e^x + e^{-x}}{e^x - e^{-x}} \right)' &= \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2} = \frac{-4}{(e^x - e^{-x})^2} = \frac{-4e^{2x}}{(e^{2x} - 1)^2} \\ \left((\operatorname{cth} x)' \right) &= -\frac{1}{\operatorname{sh}^2 x} \end{aligned}$$

1.12. feladat

$$\left(\sqrt{\frac{x+1}{x-1}} \right)' = \left(\sqrt{1 + \frac{2}{x-1}} \right)' = \frac{1}{2\sqrt{1 + \frac{2}{x-1}}} \cdot \frac{-2}{(x-1)^2} = -\frac{1}{\sqrt{1 + \frac{2}{x-1}} \cdot (x-1)^2}$$

1.13. feladat

$$(\ln \cos x)' = \frac{1}{\cos x} \cdot (-\sin x) = -\operatorname{tg} x$$

1.14. feladat

$$\begin{aligned} \left(\frac{\sqrt[3]{x^2}}{2^{\cos^2 x}} + \operatorname{tg} \frac{\pi}{4} \right)' &= \frac{\frac{2}{3} x^{-\frac{1}{3}} 2^{\cos^2 x} - x^{\frac{2}{3}} \ln 2 \cdot 2^{\cos^2 x} \cdot 2 \cos x (-\sin x)}{2^{\cos^2 x}} = \\ &= \frac{1}{3} 2^{1-\cos^2 x} x^{-\frac{1}{3}} + x^{\frac{2}{3}} \ln 2 \cdot 2^{1-\cos^2 x} \cos x \sin x = \frac{1}{3} x^{-\frac{1}{3}} 2^{\sin^2 x} + \frac{1}{2} x^{\frac{2}{3}} \ln 2 \cdot 2^{\sin^2 x} \sin(2x) \end{aligned}$$

1.15. feladat

$$\left(\frac{\operatorname{tg}^2(2x)}{x-1} + \sqrt{\operatorname{sh} x} \right)' = \frac{2 \operatorname{tg}(2x) \frac{1}{\cos^2(2x)} 2(x-1) - \operatorname{tg}^2(2x)}{(x-1)^2} + \frac{1}{2\sqrt{\frac{e^x - e^{-x}}{2}}} \cdot \frac{e^x + e^{-x}}{2}$$

$$\left((\sqrt{\operatorname{sh} x})' = \frac{\operatorname{ch} x}{2\sqrt{\operatorname{sh} x}} \right)$$

1.16. feladat

$$\left(\frac{1}{2\pi} e^{-\frac{x^3}{4}} \right)' = \frac{1}{2\pi} e^{-\frac{x^3}{4}} \cdot \left(-\frac{3}{4} \right) x^2 = -\frac{3}{8\pi} e^{-\frac{x^3}{4}} x^2$$

1.17. feladat

$$\left(\sin^2(\sqrt{x} \cdot e^{-x}) \right)' = 2 \sin(\sqrt{x} \cdot e^{-x}) \cos(\sqrt{x} \cdot e^{-x}) \left(\frac{1}{2\sqrt{x}} \cdot e^{-x} - \sqrt{x} \cdot e^{-x} \right) =$$

$$= \sin(2\sqrt{x} \cdot e^{-x}) \left(\frac{1}{2\sqrt{x}} \cdot e^{-x} - \sqrt{x} \cdot e^{-x} \right)$$

1.18. feladat

$$\left(\log_2 \sqrt{\frac{x+5}{x+7}} \right)' = \left(\log_2 \sqrt{1 - \frac{2}{x+7}} \right)' = \frac{1}{\ln 2 \cdot \sqrt{1 - \frac{2}{x+7}}} \cdot \frac{1}{2\sqrt{1 - \frac{2}{x+7}}} \cdot \frac{2}{(x+7)^2} =$$

$$= \frac{1}{\ln 2 \cdot (x+5)(x+7)}$$

1.19. feladat

$$\left(e^{x^2+5x+1} \cdot \ln x \right)' = e^{x^2+5x+1} \cdot (2x+5) \cdot \ln x + e^{x^2+5x+1} \frac{1}{x} = e^{x^2+5x+1} \left(\ln x^{2x+5} + \frac{1}{x} \right)$$

1.20. feladat

$$\left(\sqrt{x + \sqrt{x + \sqrt{x}}} \right)' = \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \cdot \left(\frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(\frac{1}{2\sqrt{x}} + 1 \right) + 1 \right) =$$

$$= \frac{\frac{\frac{1}{2\sqrt{x}} + 1}{\sqrt{2\sqrt{x + \sqrt{x}}}} + 1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}}$$

1.21. feladat

$$\left(\ln \sqrt{\frac{e^{2x}}{1 + e^{2x}}} \right)' = \frac{1}{\sqrt{\frac{e^{2x}}{1 + e^{2x}}}} \cdot \frac{1}{2\sqrt{\frac{e^{2x}}{1 + e^{2x}}}} \cdot \frac{1}{(1 + e^{2x})^2} \cdot 2e^{2x} = \frac{1}{1 + e^{2x}}$$

1.22. feladat

$$\left(x^2 + 3x^4 + 5\sqrt{x}\right)' = 12x^3 + 2x + \frac{5}{2}x^{-\frac{1}{2}}$$

1.23. feladat

$$\left(1 - \frac{1}{x} + \frac{x^2}{2} - \frac{1}{3x^3}\right)' = x^{-2} + x + x^{-4}$$

1.24. feladat

$$\left(\frac{5}{\sqrt[3]{x^2}} - \frac{\sqrt[4]{x^3}}{3}\right)' = -\frac{10}{3}x^{-\frac{5}{3}} - \frac{1}{4}x^{-\frac{1}{4}}$$

1.25. feladat

$$\left((x+2)e^x\right)' = e^x(x+2) + e^x = e^x(x+3)$$

1.26. feladat

$$\left(\frac{3-x}{5x+1} + \frac{3}{x}\right)' = \left(-\frac{1}{5} + \frac{16}{5(5x+1)} + \frac{3}{x}\right)' = -\frac{16}{25(5x+1)^2} \cdot 25 - \frac{3}{x^2} = -\frac{91x^2 + 30x + 3}{x^2(5x+1)^2}$$

1.27. feladat

$$\begin{aligned} \left(\operatorname{th} x \cdot \ln x\right)' &= \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \ln x\right)' = \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x + e^{-x})^2} \cdot \ln x + \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{1}{x} = \\ &= \frac{4 \ln x \cdot x + e^{2x} - e^{-2x}}{(e^x + e^{-x})^2} \\ &\quad \left(\left(\operatorname{th} x\right)' = \frac{1}{\operatorname{ch}^2 x}\right) \end{aligned}$$

1.28. feladat

$$\begin{aligned} &\left(\frac{4x^2 \cdot \operatorname{arch} x}{e^x + 3}\right)' = \\ &= \frac{\left(8x \cdot \ln(x + \sqrt{x^2 - 1}) + 4x^2 \cdot \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left(1 + \frac{2x}{2\sqrt{x^2 - 1}}\right)\right)(e^x + 3) - 4x^2 \cdot \ln(x + \sqrt{x^2 - 1})}{(e^x + 3)^2} = \\ &= \frac{\left(8x \cdot \ln(x + \sqrt{x^2 - 1}) + \frac{4x^2}{\sqrt{x^2 - 1}}\right)(e^x + 3) - 4x^2 \ln(x + \sqrt{x^2 - 1})}{(e^x + 3)^2} \end{aligned}$$

$$\left((\operatorname{arch} x)' = \frac{1}{\sqrt{x^2 - 1}} \right)$$

1.29. feladat

$$\left((2x - 1)^2 + e^{2x} - \cos(2x) \right)' = 4(2x - 1) + 2e^{2x} + 2 \sin(2x)$$

1.30. feladat

$$\left((3^x + 2 \cdot 4^x) \cdot 5^{3x} \right)' = \left(375^x + 2 \cdot 500^x \right)' = \ln 375 \cdot 375^x + 2 \ln 500 \cdot 500^x$$

1.31. feladat

$$\left(\operatorname{tg} x \cdot (e^{3x} + 7) \right)' = \frac{e^{3x} + 7}{\cos^2 x} + 3 \operatorname{tg} x \cdot e^{3x}$$

1.32. feladat

$$\left(3e^{2x} \cdot \sin(2x) \right)' = 3 \left(2e^{2x} \cdot \sin(2x) + e^{2x} \cdot \cos(2x) \right) = 6e^{2x} (\sin(2x) + \cos(2x))$$

1.33. feladat

$$\left((9 - x^2) \cdot (e^{x^2} + 1) \right)' = -2x \cdot (e^{x^2} + 1) + 2x \cdot (9 - x^2) \cdot e^{x^2} = -2x \cdot (e^{x^2}(x^2 - 8) + 1)$$

1.34. feladat

$$(\operatorname{arctg}(x - 3) + 7)' = \frac{1}{(x - 3)^2 + 1} = \frac{1}{x^2 - 6x + 10}$$

1.35. feladat

$$\left(2^{\sin x} - 3^{\cos x} \right)' = \ln 2 \cdot 2^{\sin x} \cdot \cos x + \ln 3 \cdot 3^{\cos x} \cdot \sin x$$

1.36. feladat

$$\left(\frac{x^4 - 3x + 2\sqrt{x}}{\sqrt[3]{x}} \right)' = \left(x^{\frac{11}{3}} - 3x^{\frac{2}{3}} + 2x^{\frac{1}{6}} \right)' = \frac{11}{3}x^{\frac{8}{3}} - 2x^{-\frac{1}{3}} + \frac{1}{3}x^{-\frac{5}{6}}$$

1.37. feladat

$$\left((\sqrt{5} - 3x) \cdot \arcsin x \right)' = -3 \arcsin x + \frac{\sqrt{5} - 3x}{\sqrt{1 - x^2}}$$

1.38. feladat

$$\left(\frac{1}{\cos x}\right)' = \frac{1}{\cos^2 x} \cdot \sin x = \frac{\operatorname{tg} x}{\cos x}$$

1.39. feladat

$$\left(\frac{x}{1 - \ln x}\right)' = \frac{1 - \ln x + x \cdot \frac{1}{x}}{(1 - \ln x)^2} = \frac{2 - \ln x}{(1 - \ln x)^2}$$

1.40. feladat

$$\left(\frac{(x-1)^3}{3x^2}\right)' = \left(\frac{1}{3}x - 1 + x^{-1} - \frac{1}{3}x^{-2}\right)' = \frac{1}{3} - x^{-2} + \frac{2}{3}x^{-3}$$

1.41. feladat

$$\left(\frac{3x \cdot \arccos x}{x^2 + 1}\right)' = \frac{\left(3 \arccos x - \frac{3x}{\sqrt{1-x^2}}\right)(x^2 + 1) - 6x^2 \cdot \arccos x}{(x^2 + 1)^2}$$

1.42. feladat

$$\left(\operatorname{tg}^2\left(\frac{x}{4}\right) + \pi\right)' = 2 \operatorname{tg}\left(\frac{x}{4}\right) \cdot \frac{1}{\cos^2\left(\frac{x}{4}\right)} \cdot \frac{1}{4} = \frac{\operatorname{tg}\left(\frac{x}{4}\right)}{2 \cos^2\left(\frac{x}{4}\right)}$$

1.43. feladat

$$(\sin(2x) + \sin^2 x)' = 2 \cos(2x) + 2 \sin x \cos x = 2 \cos(2x) + \sin(2x)$$

1.44. feladat

$$\left(\cos^3 x + \cos(x^3)\right)' = -3 \cos^2 x \cdot \sin x - 3x^2 \cdot \sin(x^3)$$

1.45. feladat

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1.46. feladat

$$\left(\frac{\sqrt[5]{x}}{\sin x} + \frac{e^2}{e^x + 1}\right)' = \frac{\frac{1}{5}x^{-\frac{4}{5}} \cdot \sin x - x^{\frac{1}{5}} \cos x}{\sin^2 x} - \frac{e^{x+2}}{(e^x + 1)^2}$$

1.47. feladat

$$(\ln \sin \sqrt{x})' = \frac{1}{\sin \sqrt{x}} \cdot \cos \sqrt{x} \cdot \frac{1}{2\sqrt{x}} = \frac{\operatorname{ctg} \sqrt{x}}{2\sqrt{x}}$$

1.48. feladat

$$(x^2 \cdot e^x \cdot \sin x)' = (2x \cdot e^x + x^2 \cdot e^x) \cdot \sin x + x^2 \cdot e^x \cdot \cos x$$