Csoma Borbála

HF1 (3) Show that $f - \hat{f} \perp \varphi_k \quad k = 1, 2 \dots n$

$$\hat{f} = \sum_{i=1}^{n} \langle f, \varphi_i \rangle \cdot \varphi_i$$

$$\text{if} \quad (f - \hat{f}) \perp \varphi_k \Leftrightarrow \langle f - \hat{f}, \varphi_k \rangle = 0$$

$$\langle f - \hat{f}, \varphi_k \rangle = \langle f, \varphi_k \rangle - \langle \hat{f}, \varphi_k \rangle =$$

$$= \langle f, \varphi_k \rangle - \left\langle \sum_{i=1}^{n} \langle f, \varphi_i \rangle \varphi_i, \varphi_k \right\rangle =$$

$$= \langle f, \varphi_k \rangle - \sum_{i=1}^{n} \langle f, \varphi_i \rangle \underbrace{\langle \varphi_i, \varphi_k \rangle}_{\substack{\text{if } i \neq k \to 0 \\ \text{if } i = k \to 1}} = \langle f, \varphi_k \rangle - \langle f, \varphi_k \rangle = 0$$

HF2 (7) Using the polynomial formula of T_0 and T_2 check that T_0 and T_2 are indeed orthogonal.

$$T_0 = 1 \qquad T_2 = 2x^2 - 1$$

$$\langle T_0, T_2 \rangle_g = 0$$

$$\int_{-1}^{1} 1 \cdot (2x^2 - 1) \frac{1}{\sqrt{1 - x^2}} \, dx = \int_{-1}^{1} \frac{2x^2 - 1}{\sqrt{1 - x^2}} \, dx =$$

$$= \Big[(-x) \cdot \sqrt{1 - x^2} \Big]_{-1}^{1} = 1 \sqrt{0} - (-1)\sqrt{0} = \underline{\underline{0}}$$

$$\Downarrow$$

Thus, they are orthogonal