HOMEWORK 5.

HF1 f, g are measurable

$\Downarrow$

$\forall a \ \{x \in \mathbb{R} \mid f(x) < a\} \in \mathcal{M}$

$\forall a \ \{x \in \mathbb{R} \mid g(x) < a\} \in \mathcal{M}$

We stated on the lecture that:

"The usual ways of combining functions preserve measurability."

This means: $\forall a \ \{x \in \mathbb{R} \mid \max\{f(x), g(x)\} < a\} \in \mathcal{M}$

$\Downarrow$

$\forall a \ \{x \in \mathbb{R} \mid f \vee g(x) < a\} \in \mathcal{M}$

HT2 f is measurable

$f \in \mathcal{L} \Leftrightarrow |f| \in \mathcal{L}$

if $f \in \mathcal{L}$: $\int_E f \, d\mu = \int_E f_+ \, d\mu - \int_E f_- \, d\mu$

and

$$\int_E |f| \, d\mu = \int_E f_+ \, d\mu + \int_E f_- \, d\mu$$

since $\int_E f_+ \, d\mu \geq 0$ and $\int_E f_- \, d\mu \geq 0$

$$\int_E f \, d\mu \leq \int_E |f| \, d\mu$$