HOMEWORK 4.

HF 1 Show that ℓ^∞ is complete.

Assume that $x^{(n)} \subset \ell^\infty$ is a Cauchy sequence

$\Rightarrow \forall \varepsilon > 0 \quad \exists N$ such that $n, m \geq N$

$$\| x^{(n)} - x^{(m)} \|_\infty < \varepsilon$$

$$\sup_{k \in \mathbb{N}} \{ | x_k^{(n)} - x_k^{(m)} | \} < \varepsilon \rightarrow \forall k \quad | x_k^{(n)} - x_k^{(m)} | < \varepsilon \rightarrow \lim_{n \to \infty} x_k^{(n)} = x_k$$

let's define $x = [x_1, x_2 \dots x_k]$

$$\sup_{k \in \mathbb{N}} \{ | x_k - x_k^{(m)} | \} < \varepsilon \Rightarrow \lim_{m \to \infty} x^{(m)} = x$$

HF 2 $A = \{ x \in (-1,1) ; x \notin \mathbb{Q} \} \rightarrow A = \{ (-1,1) \setminus \mathbb{Q} \}$

elements of A are countables therefore it is measurable

$$\left. \begin{array}{l} m((-1,1)) = 2 \\ m(\mathbb{Q}) = 0 \end{array} \right\} m(A) = 2$$