HOMEWORK 2

<u>HF1</u> $x_n = \dfrac{1}{2^{n-1}}$ $y_n = \dfrac{(-1)^n}{2^{n-1}}$

What is their distance in ℓ^1 and in ℓ^∞?

- if n is even

$$x_n - y_n = 0$$

- if n is odd

$$x_n - y_n = \frac{1}{2^{n-1}} - \frac{-1}{2^{n-1}} = \frac{2}{2^{n-1}} = \frac{\cancel{2}}{(2^{n-2})\cancel{2}^1} = \frac{1}{2^{n-2}}$$

we only add even 'n' values $\rightarrow n = (2n-1)$

<u>ℓ^1</u>

$$\|x-y\|_1 = \sum_{n=1}^{\infty} |x_n - y_n| = \sum_{n=1}^{\infty} \frac{1}{2^{n-2}} \overset{\downarrow}{=} \sum_{n=1}^{\infty} \frac{1}{2^{2n-3}} = \sum_{n=1}^{\infty} \frac{1}{2^n \cdot \frac{1}{2^3}} =$$

$$= 8 \cdot \sum_{n=1}^{\infty} \frac{1}{2^{2n}} = 8 \cdot \frac{\frac{1}{4}}{1-\frac{1}{4}} = 8 \cdot \frac{1}{\cancel{4} \cdot \frac{3}{4}} = \underline{\underline{\frac{8}{3}}}$$

$a_1 = \frac{1}{4}$

$q = \frac{1}{4}$

<u>ℓ^∞</u>

$$\|x-y\|_\infty = \max(|x_n - y_n|) = \max\left(\frac{1}{2^{n-2}}\right) = \underline{\underline{2}}$$

<u>HF2</u> $f(x) = x$ $g(x) = x^2$ $C[0,1]$

$$\|f-g\|_\infty = \sup(|f-g|) = \sup(|x-x^2|) = \sup(x-x^2) = \frac{1}{2} - \frac{1}{4} = \underline{\underline{\frac{1}{4}}}$$

if $x \in [0,1]$

$x - x^2 \geq 0$

$$(x-x^2)' = 1-2x$$

$1 - 2x = 0 \rightarrow$ where the first derivative is 0, we have

$x = \frac{1}{2}$ extremity values

$$\|f-g\|_2 = \sqrt{\int_0^1 (x-x^2)^2\,dx} = \sqrt{\int_0^1 (x^2 - 2x^3 + x^4)\,dx} = \sqrt{\left[\frac{x^3}{3} - 2\frac{x^4}{4_2} + \frac{x^5}{5}\right]} =$$

$$= \sqrt{\frac{1}{3} - \frac{1}{2} + \frac{1}{5}} = \underline{\frac{\sqrt{30}}{30}}$$