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HF₁

$$S: \ell^2 \rightarrow \ell^2$$

$$Sx = S(x_1, x_2, \dots) = \left(x_1, \frac{1}{5}x_2, \frac{2}{7}x_3, \dots, \frac{n-1}{2n+1}x_n, \dots\right)$$

$$Sx = \lambda x \rightarrow \text{true for } \forall \lambda = \frac{n-1}{2n+1}$$

$$\lambda_n = \frac{n-1}{2n+1} \text{ is bounded } \Rightarrow |\lambda_n| \leq \frac{1}{2}$$

$$\text{because } \lim_{n \rightarrow \infty} \frac{n-1}{2n+1} = \frac{1}{2}$$

The spectrum eigenvalues $\cup$ bound.

$\Downarrow$

$$\sigma(S) = \left\{ \frac{n-1}{2n+1} \mid n \in \mathbb{N} \right\} \cup \left\{ \frac{1}{2} \right\}$$

HF₂

$$X = C[0, 1]$$

$$T \in B(X) \quad (Tf)(x) = x^2 f(x)$$

$$T \text{ is invertible, the inverse is } (T^{-1}f)(x) = \frac{1}{x^2} f(x)$$

$\Downarrow$

$$x \neq 0$$

$$\sigma(T) = \{x^2 \mid x \in (0, 1]\}$$