HT1 ⑥ $a > 0$

is $\|x\| = \max\{|2x_1 - x_2|, |a x_1|\}$ a norm on $\mathbb{R}^2$?

1) non-negative?

because of the absolute values ✓ True

2) $x = (x_1, x_2) \to x = (0,0) = 0$

- $\|x\| = \max\{|2\cdot 0 - 0|, |a \cdot 0|\} = 0$
- $\|x\| = 0 \to x_1, x_2$ must be 0, because $a \neq 0$ } ✓

3) $\forall x \in \mathbb{R}^2 \quad \lambda \in \mathbb{R} \to \|\lambda x\| = |\lambda| \|x\|$ ✓ True

$\|\lambda x\| = \max\{|\lambda \cdot (2x_1 - x_2)|, |\lambda a x_1|\} = \lambda \max\{|2x_1 - x_2|, |a x_1|\} = \lambda$

4) triangle inequality ✓ True

$\|x + y\| \leq \|x\| + \|y\|$

$\|x + y\| = \max\{|(2x_1 - x_2) + (2y_1 - y_2)|, |a x_1 + a y_1|\} =$

~~$= \max\{|2x_1 + 2y_1 - x_2 - y_2|, |a x_1 + a y_1|\} = \leq$~~

(bec of tri inequality) $\leq \max\{|2x_1 - x_2| + |2y_1 - y_2|, |a x_1| + |a y_1|\} \leq$

$\leq \max\{|2x_1 - x_2|, |a x_1|\} + \max\{|2y_1 - y_2|, |a y_1|\} \leq \|x\| \|y\|$

All the properties are true, so $\|x\|$ is a norm on $\mathbb{R}^2$.

HT2 ⑨ $B_1(0) = \{x \in \mathbb{R}^2 \quad \|x\| \leq 1\}$ $\|x\| = \max\{|2x_1 - x_2|, |a x_1|\} \leq$

$\max\{|2x_1 - x_2|, |a x_1|\} \leq 1$

$|2x_1 - x_2| \leq 1$ and $|a x_1| \leq 1$

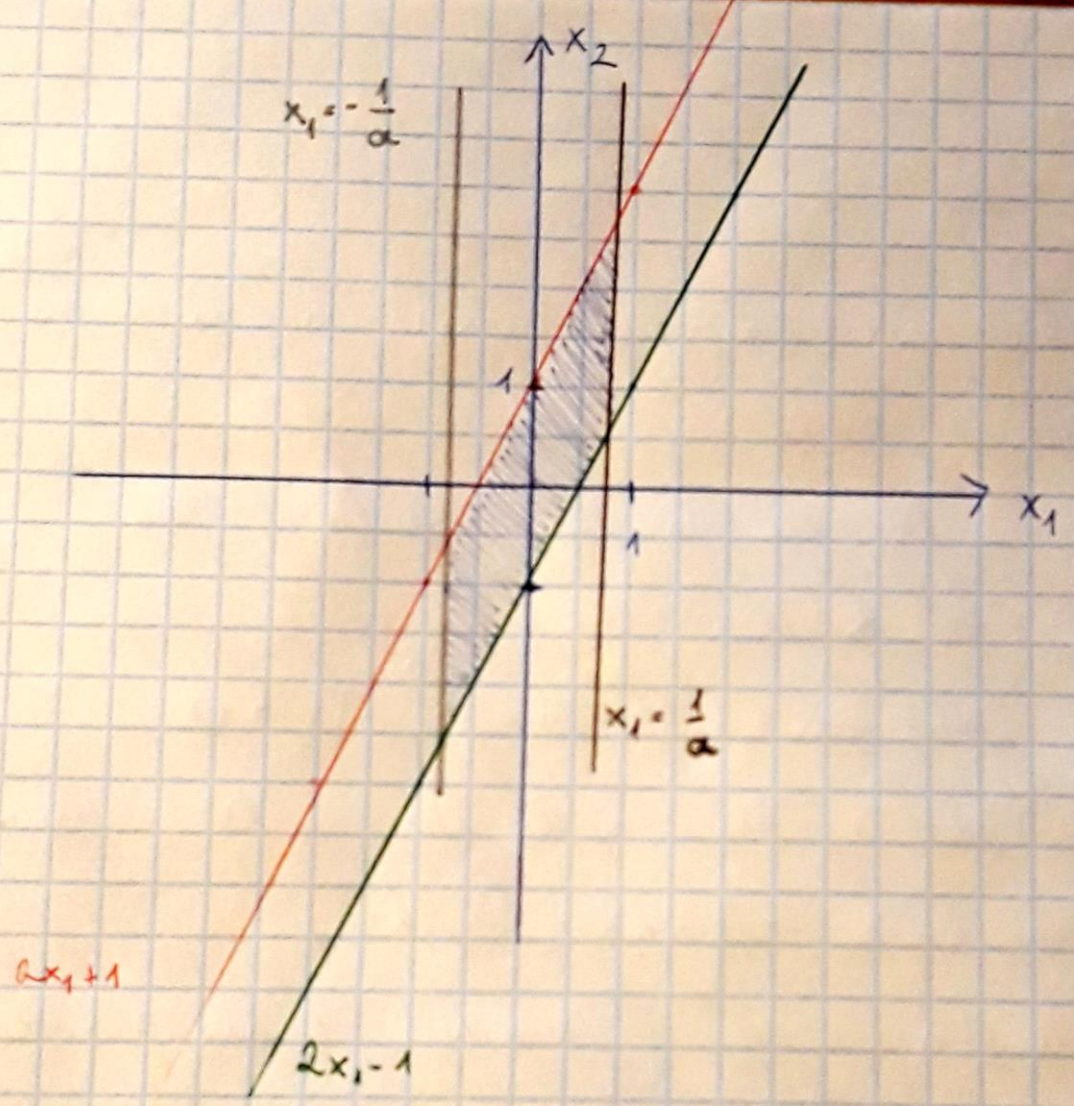
$2x_1 - x_2 \leq 1$ ↙ ↘ $-(2x_1 - x_2) \leq 1$; $a x_1 \leq 1$ ↙ ↘ $-a x_1 \leq 1$

$2x_1 - x_2 \leq 1$ → • $x_2 \geq 2x_1 - 1$

$-(2x_1 - x_2) \leq 1$ → $x_2 - 2x_1 \leq 1$ → • $x_2 \leq 1 + 2x_1$

$a x_1 \leq 1$ → $(a > 0)$ → • $x_1 \leq \frac{1}{a}$

$-a x_1 \leq 1$ → $(a > 0)$ → • $x_1 \geq -\frac{1}{a}$


x_2
$x_1 = -\frac{1}{a}$
1
1
1
x_1
$x_1 = \frac{1}{a}$
$2x_1 + 1$
$2x_1 - 1$