Computer Controlled Systems

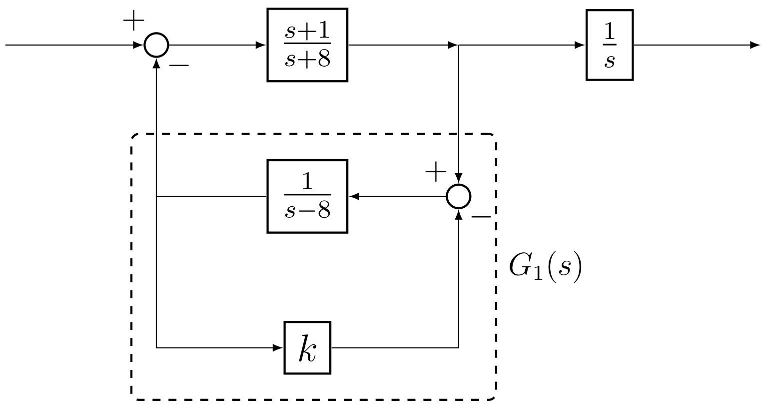
Homework 3.

Submission deadline: 30th of November midnight via Moodle

All solutions are expected to be calculated by hand. The submission can be handwritten or typed in in LaTeX. Computer programs (e.g., Matlab) can be used for self-verification, but all problems have to contain the detailed steps of solutions.

Problems

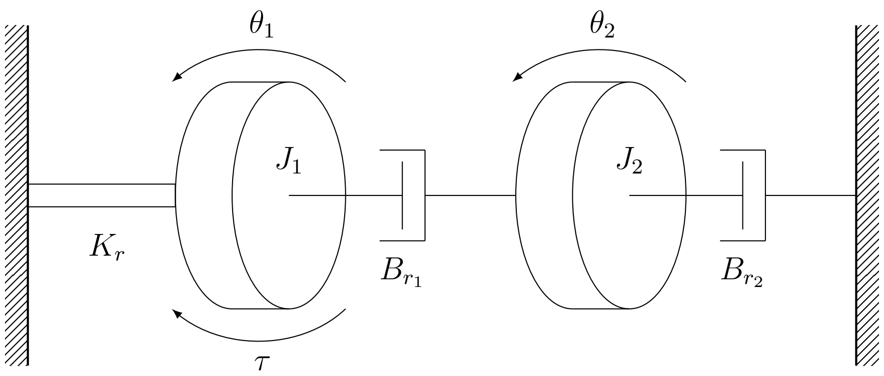
1. Let us consider the following block diagram:



   (a) Compute the resulting transfer function $G(s)$. It is advised to first determine the auxiliary transfer function $G_1(s)$ of the highlighted subsystem.

   (b) Choose the value of k such that resulting transfer function is stable.

2. Let us consider the following rotating system with two flywheels:



   The first flywheel has moment of inertia J_1 and is attached by a flexible shaft with spring constant K_r to the wall. We apply an external torque τ (which is the input). The second flywheel has moment of inertia J_2 is driven by friction between the two flywheels with friction coefficient B_{r_1} (which will be modeled with a damper). The second flywheel also has friction to the other wall with friction coefficient B_{r_2}. The output of the system is the first flywheel's angle, θ_1.
   Using D'Alembert's Law we know that the sum of all torques (including the initial torque) should be zero; that is, for the first flywheel we can write the following equation:

$$\tau + J_1\ddot{\theta}_1 + K_r\theta_1 + B_{r_1}\left(\dot{\theta}_1 - \dot{\theta}_2\right) = 0.$$

Similarly for the second flywheel:

$$J_2\ddot{\theta}_2 + B_{r_2}\dot{\theta}_2 - B_{r_1}\big(\dot{\theta}_1 - \dot{\theta}_2\big) = 0.$$

The above equations can be rearranged in the form:

$$\begin{aligned}\ddot{\theta}_1 &= \frac{1}{J_1}\big(-\tau - K_r\theta_1 - B_{r_1}\dot{\theta}_1 + B_{r_1}\dot{\theta}_2\big),\\ \ddot{\theta}_2 &= \frac{1}{J_2}\big(-(B_{r_1} + B_{r_2})\dot{\theta}_2 + B_{r_1}\dot{\theta}_2\big).\end{aligned}$$

Using the state variables, input and output functions

$$\begin{aligned}x_1 &= \theta_1,\\ x_2 &= \dot{\theta}_1,\\ x_3 &= \dot{\theta}_2,\\ u &= \tau,\\ y &= x_1\end{aligned}$$

we can rewrite the equations describing the dynamics of the system as

$$\begin{aligned}\dot{x}_1 &= x_2,\\ \dot{x}_2 &= \frac{1}{J_1}\big(-u - K_r x_1 - B_{r_1}x_2 + B_{r_1}x_3\big),\\ \dot{x}_3 &= \frac{1}{J_2}\big(-(B_{r_1} + B_{r_2})x_3 + B_{r_1}x_2\big),\\ y &= x_1\end{aligned}$$

and in matrix form

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ -\frac{K_r}{J_1} & -\frac{B_{r1}}{J_1} & \frac{B_{r1}}{J_1} \\ 0 & \frac{B_{r1}}{J_2} & -\frac{B_{r1}+B_{r2}}{J_2} \end{bmatrix} x + \begin{bmatrix} 0 \\ -\frac{1}{J_1} \\ 0 \end{bmatrix} u;\\ y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x.\end{aligned}$$

(a) Substitute the following numbers and rewrite the state space representation:

$$K_r = 1 \quad B_{r_1} = 1 \quad B_{r_2} = 4 \quad J_1 = 0.2 \quad J_2 = 2.$$

(b) Is the system exponentially stable?

(c) Explain why the impulse response of the system is oscillating for a while!

(d) Design a static state feedback controller such that the closed loop system will not oscillate at all.

(e) Design an observer gain L such that the error dynamics of the Luenberger observer tends exponentially to zero.

Compulsory only for TP students, ***but extra points for others***.

(f) The energy stored in a flexible shaft (rotational spring) and a rotating mass is

$$\begin{aligned}E_K &= \frac{1}{2}K\theta^2\\ E_J &= \frac{1}{2}J\dot{\theta}^2.\end{aligned}$$

Check that the total energy function $V = E_{K_r} + E_{J_1} + E_{J_2}$ is an appropriate Lyapunov function.