Példa $\mathbb{R}^3$-ban 2-dim sokaságon integrál

Sokaság : $M = \{(u,v,t(u,v)) : (u,v) \in D\}$

Diff forma $\omega = f\,dx\,dy + g\,dx\,dz + h\,dy\,dz$

(M) ω diff forma $\longleftrightarrow$ $F = (h, -g, f)$ vektormező

$$\int_M \omega = \iint_D \underbrace{\qquad\qquad}_{?}\, d(u,v)$$

Paraméterezés : $\phi(u,v) = \begin{pmatrix} u \\ v \\ t(u,v) \end{pmatrix}$

Jacobi mátrixa $D\phi(u,v) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ t'_u & t'_v \end{pmatrix}$

$$\omega(D\phi(u,v)) = f\,dx\,dy\,(D\phi) + g\,dx\,dz\,(D\phi) + h\,dy\,dz\,(D\phi) =$$

$$= f \cdot 1 + g \cdot t'_v + h(-t'_u)$$

$$\int_M \omega = \iint_D \Big(f(u,v,t(u,v)) + g(\;)\,t'_v(u,v) - h(\;)\,t'_u(u,v) \Big)\, du\,dv$$

$$\underbrace{\qquad\qquad}$$

$$\Big\langle F(u,v,t(u,v)),\ (-t'_u, -t'_v, 1) \Big\rangle$$

$$= \iint_S F(x,y,z)\,dS$$