Time-invariant linear system

We focus on discrete-time, time-invariant linear systems defined by

\[ y(t) = \sum_{k=0}^{\infty} g(k) u(t-k), \quad t \geq 0 \]

(2.6)

Here \( u(t) \) is a scalar input signal and \( y(t) \) is a scalar output signal. The mapping from \( u \) to \( y \) is clearly linear and shift-invariant, hence called time-invariant. The system above is also causal, since in the sense that for any \( t \), the output \( y(t) \) depends only on \( u \) up to that time \( t \).
In the derivation above we may assume that \( u(0) = 0 \) for \( t < 0 \). Alternatively, we may also assume that \( u(t) \) is defined in a non-trivial way for \( -\infty < t < +\infty \). In this latter case we need to take care to impose appropriate conditions on \( g \) and \( u \) to ensure that (2.6) makes sense.

The Note holds

The observation that

Equation (2.6) can be also read in a

bit of a symmetric way.

In mathematical analysis the operation

that maps the \( \phi \) pair
Sequence $y$ and $v$ into the $2/3$.

Sequence $y$ is called a convolution.

The shorthand notation for a convolution is

$$y = g * v,$$

with the tacit assumption that $g(k) = 0$ for $k \leq 0$. 
In most cases the output is also affected by signals beyond our control. These are called disturbances. The simplest source of a disturbance is measurement noise. Thus we come to the following model:

$$y(t) = \sum_{k=1}^{\infty} g(k) u(t-k) + v(t)$$

A major feature of a disturbance is that its value is not known beforehand.

A key point in modelling is the statistical characterization of the statistical properties of the noise.